SBS

Approaches to modeling of bio (and other) systems

Models based on

Analysis

- estimating governing laws
- differential equations
  $\ldots + a_1 \dot{y}(t) = b_0 u(t)$
- difference equations
  $\ldots + a_1 y(k) = b_0 u(k)$
  $t = k * \Delta t$
- population dynamics, ...

...

Measured data

- estimating structure of model and then optimizing its parameters
- adaptive models
- neural networks
- ⋮
- time-series prediction (respiration data, ECG, ...)

...

Models

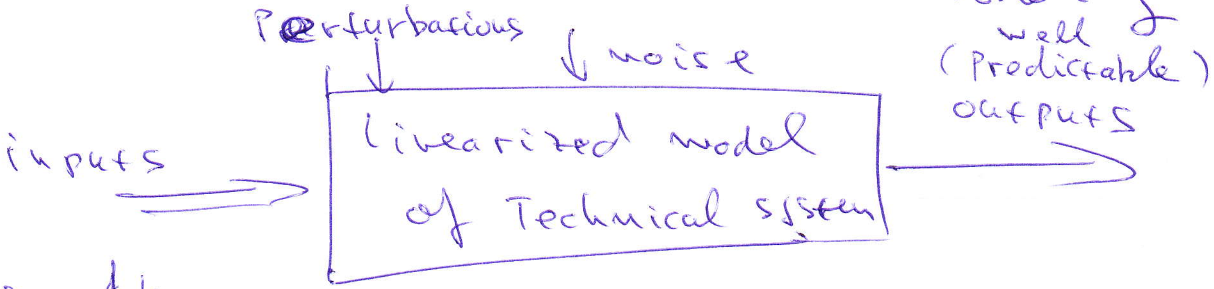
- Static models (do not involve time)
- dynamical models (time)
  - linear
  - nonlinear ⇕ deterministic chaos
- space + time models (Finite element methods ...)

Models

- stochastic
  - Bayes approaches
- deterministic
  - dif. equations
  - neural networks

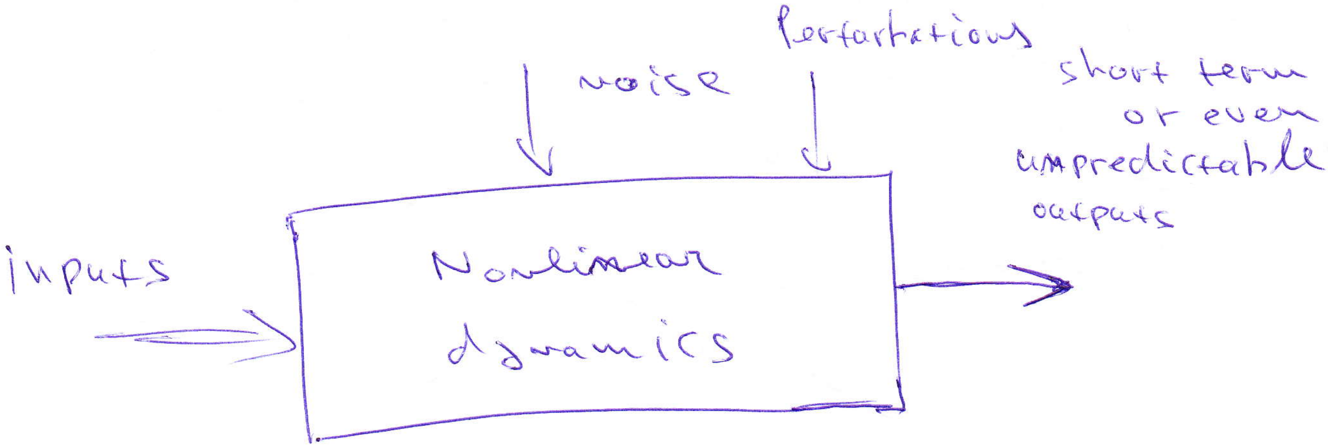
Technical systems

- are more suitable for math-physical analysis than other real-world systems
- are often suitable for linear modeling



(Perturbations ... known or unknown changes of parameters)

Real-World Models



Nonstationarity: - real systems change its dynamics in time
- we are able to have often only temporary valid models

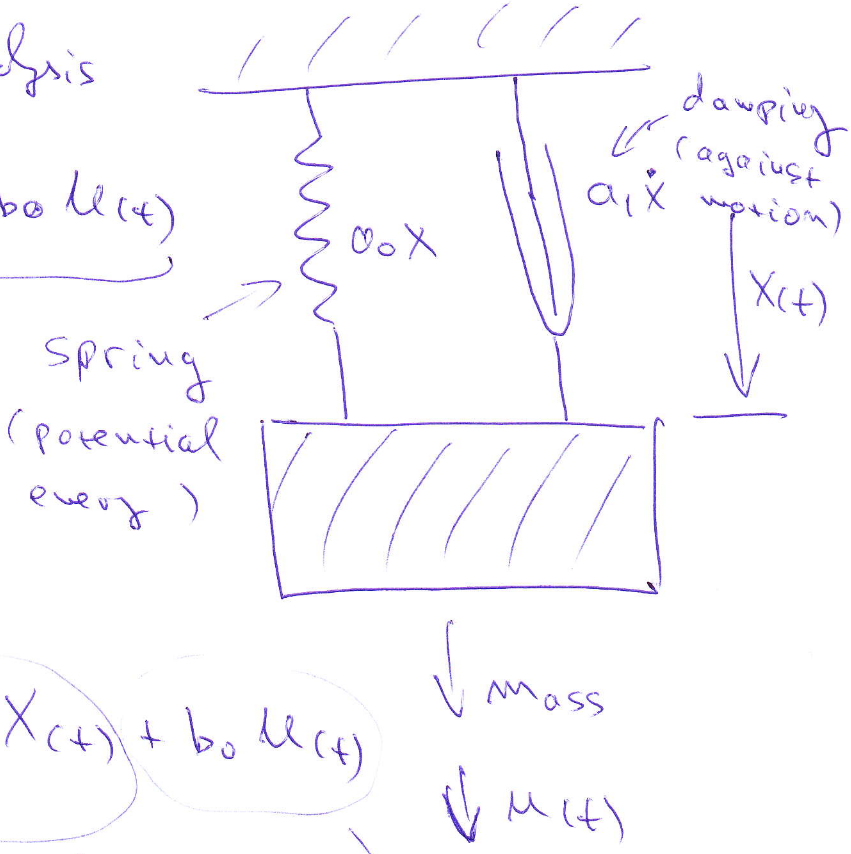
From Continuous-time linear model of a technical system toward population models

Tech. System - by analysis

$$\ddot{X}(t) + a_1 \dot{X}(t) + a_0 X(t) = b_0 u(t)$$

balance of energies

t ... time



$$\ddot{X}(t) = -a_1 \dot{X}(t) - a_0 X(t) + b_0 u(t)$$

damping against motion

potential energy as a function of position

external input

Continuous time linear and nonlinear models of population dynamics

$$\dot{X}(t) = \oplus b X(t) \ominus m X(t)$$

$X(t)$ ... population in time t

$$\dot{X}(t) = b\,X(t) - m\,X(t) \qquad (4)$$

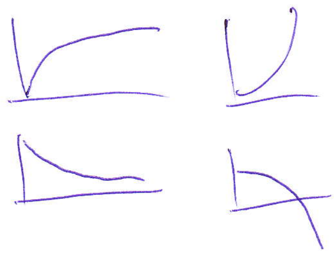
rate of population change per time unit [amount/time]

relative birth rate [% / time unit]

relative mortality rate [% / time unit]

Malthus equation

$$\dot{X}(t) = \underbrace{(b - m)}_{\text{relative coefficient of growth}} X(t) \Rightarrow X(t) = X_0 e^{(b-m)t}$$



The above is a simple liner model of a siangle population

Nonlinear models e.g.: b, m can be function of X

$$\dot{X}(t) = \big(b(X) - m(X)\big)\,X(t)$$

e.g.:

Linear model with time variant coefficients: b, m are funtion of time

$$\dot{X}(t) = \underbrace{\big(b(t) - m(t)\big)}_{\text{relative coefficient of growth}} X(t)$$

Fundamental population models (continuous-time)

Malthus equation: $\dot{X}(t) = (b-m)\,X(t)$

Logistic equation (Pearl – Verhulst)

- Considers capacity of environment

C ... optimal population for an environment

$$\dot{X}(t) = (b-m)\,X(t)\underbrace{\left(1 - \frac{X(t)}{C}\right)}_{\text{becomes zero for optimal population}}$$

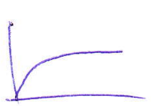
$X(t) = \text{const} \Leftarrow \dot{X}(t) = 0 \Leftarrow$ becomes zero for optimal population

Gompertz's Equation $\dot{X}(t) = \mu(x) \cdot X(t) =$

$$X_{t \to +\infty} = X(0)\, e^{a/k} \Leftarrow \; = \left(a - k \ln \frac{X(t)}{X(0)}\right) X(t)$$

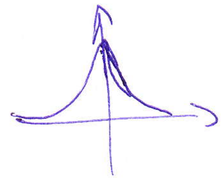
Fundamental nonlinearities for coefficients:

1) $1 - e^{-x}$

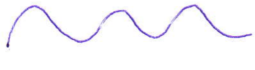


2) $\dfrac{1}{1 + e^{-x}}$

3) $\dfrac{1}{1 + x^2}$

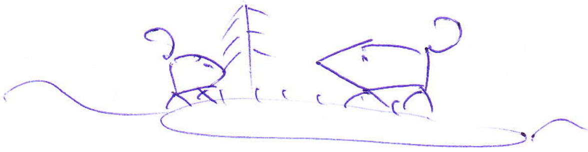


4) $\sin(\omega x + \varphi)$



⑥ Assignment for PC Lab

Design and implement your own continuous-time model of an island with wild pigs



<u>Variables:</u> $M(t)$ ... males

t ... time $F(t)$ ... females

$R(t)$ ... resources

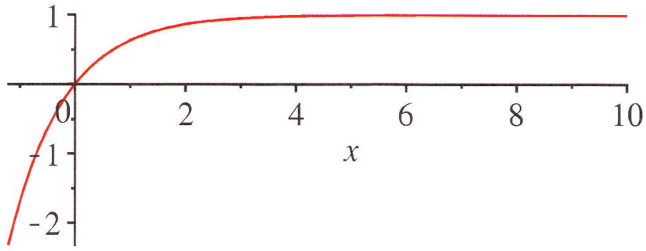
<u>coefficients</u> b_M, b_F ... birth rates of Males of Females

d_M, d_F ... death rates of ...

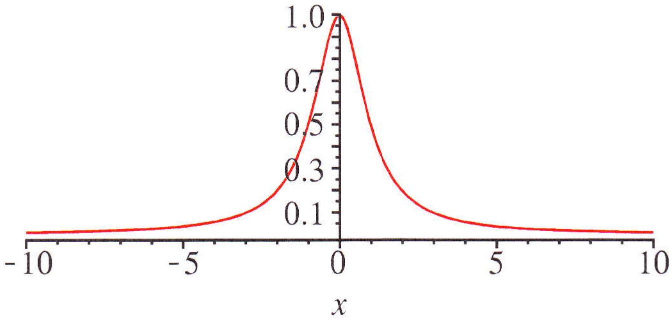
$A(M,F,R,t)$ ... Aggressivity

Think of various models and sketch how coefficients might depend on time or other variables. E.g., how aggressivity may depend ~~on~~ on total population? How does it affect death rate? How total population affects growth of resources?

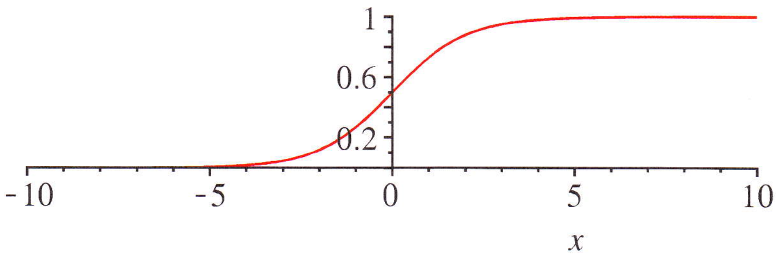
$plot(1 - \exp(-x), x = -1.2..10)$



$plot\left(\frac{1}{(1+x^2)}, x = -10..10\right)$



$plot\left(\frac{1}{1+\exp(-x)}, x = -10..10\right)$



$plot\left(\frac{2}{1+\exp(-x)} - 1, x = -10..10\right)$