OPTICAL DETERMINATION OF THE RADIUS AND POSITION OF SPHERICAL OBJECTS BY MONOLENS CAMERA SYSTEM

Jan Hošek, Ph.D.
Department of Instrumentation and Control Engineering,
Division of Precision Mechanics and Optics,
Faculty of Mechanical Engineering,
Czech Technical University in Prague,
Technická 4, 166 07 Prague 6
Czech Republic

ABSTRACT
This paper presents a new optical system, which allow to determinate radius and position in 3D-observation space of a number of spherical shape objects with using the only one monolens camera system. The principle of the system is based on anamorphic optical system and appropriate data processing algorithm. Both are detailed explained in the paper. The precision and repeatability of the system were tested by measurement of a glass and steel sphere mounted on x, y, z movable support. The first measurement and data processing are demonstrated in multiphase bubble-liquid system with different bubble density. We expect the presented system will be able to use for the determination of the objects velocity as well as dimension change in the case of application of time-resolved photography.

Keywords spherical object, position, radius, determination, anamorphic optical system

INTRODUCTION
Determination of spherical object in space is very important in many technical or production applications. The spherical objects may consist of solid particles, spray of the liquid droplets or spherical bubbles of gas in liquid or solid phase. Observation and determination of dimension and position of these objects can affect or be the bases for optimization of the technological processes like chemical reactors, where homogeneous distribution of bubbles is often demanded in multiphase reacting systems. On the other hand, there are technologies where the presence of multiphase system can indicate a technological problem, for example in hydrodynamic systems or in glass production industry.

Different experimental systems, which allow determination of the position and radius of the spherical objects in 3D space, have been under development since the 1970s. A lot of experimental methods are applied to measure a special case of the spherical objects – bubbles or drops. These methods are often based on particle image velocimetry (PIV) technique like measurement [1, 2, 3] and others. It is necessary to use stereo PIV based on two or more camera and laser systems for in 3D space measurement.

There are other more sophisticated systems still in development. The properties of spherical objects can be measured with interference method application [4, 5], laser light scattering method [6] or holographic [7, 8]. The main disadvantage of the referred methods is the necessity of using expensive devices like high speed or resolution cameras, laser systems or special experimental conditions. It decreases usage convenience of these methods for industrial and non scientific applications. Due to this fact new methods to determine object properties in 3D space measured with only one camera with simple illumination system were presented recently.

These methods are based on numerical computation of position data from temporal [9] or optical information [10] of the captured images. The most convenient method of bubble measurement for industrial and technical purposes seems to be the defocusing PIV method presented by [11] and developped to the digital method (DDPIV) applied to bubbly flows [12, 13]. This method uses only one camera lens and mask with two or more apertures. The sensor is out of lens focusing plane, so that each light source gives the same number of spots on the sensor as the number of apertures used.

This paper presents a new optical system, which is able to determinate radius and position of a number of spherical objects in 3D-observation space using only one monolens camera system. The principle of the system is based on anamorphic optical system and appropriate data processing algorithm.

PRINCIPLE OF THE METHOD
Presented experimental method [14] is based on anamorphic optical system and known shape of the measured object. Anamorphic optical system is characterized with astigmatism - different magnification in different planes along the optical axis. The typical representative of the anamorphic optical system is a cylindrical lens. This system imagines the object with different magnification in different directions perpendicular to the optical axis. Due to the fact, that objects magnification is a function of the object distance it is possible to
unambiguously determinate object distance from the ratio of magnifications different planes of anamorphic system and direction to the object. For the image magnifications determination it is necessary to know the ratio of dimension of original object. This value is well known for the spherical objects, where the ratio of dimensions is equal to 1 in all directions.

This experimental method can be used for the measurement of all kinds of spherical objects. It can be bubbles in liquids, droplets in gases, droplets of different density in liquids or solid spheres in gases or liquids. Real measurement can be realized with three different kinds of experimental instruments. It can be used special anamorphic lens instead of common camera lens. The second possibility is to use common camera lens and anamorphic lens placed to the optical system. The third kind of set-up is to use the common camera lens and a cylindrical tube used as an anamorphic lens. All three kinds of instrument set-up are shown in the figure 1.

![Figure 1. Three different instrumental set-ups of devised system.](image)

This last mentioned set-up of the devised method seems to be the most profitable application in industry due to its easy installation, since the majority of pipes in industry are of cylindrical tube shape and presented method employs this tube as a part of set-up. Optical scheme of this set-up is shown in the figure 2. There are two views shown of the optical system. The upper one in the plain contains optical axis of the system, marked // and perpendicular to the cylindrical tube axis. There is shown the part of the tube on the left and camera system represented with main planes of the camera lens H and H' and image plane I on the right. The main rays corresponding to the bubble margin in this plane are depicted with black lines. The bottom part of the figure shows the main rays along the central – blue line in the plane perpendicular to the upper part plane, marked ⊥. The angles of the main rays are determined from the camera-captured image and position of each ray on the tube wall is computed with the basic geometrical optics. Angles marked θ are measured from optical axis, angles marked α are measured from normal and angles marked δ are measured around tube curvature centroid C.
Distance of intersections of the main rays can be solved in both planes by equations:

$$ a + b = \sqrt{(H - H')^2 + (X - X')^2} $$

$$ d = \frac{b + l_z}{\cos \omega_{\pm S}} $$

It is easy to compute the radius of the sphere in both planes by:

$$ R_y = a \sin \theta $$

$$ R_z = d \sin(\omega_{\pm 2} - \omega_{\pm S}) $$

Spherical objects have constant radius in all directions, so that both radii have to be equal. Due to this condition the sphere radius and its position is computed by equations:

$$ R = \sqrt{(H - H')^2 + (X - X')^2 + l_z} $$

$$ \frac{1}{\sin \theta} + \frac{\cos \omega_{\pm S}}{\sin(\omega_{\pm 2} - \omega_{\pm S})} $$

$$ X = \bar{X} + b \cos(\delta_S - \alpha_s) $$

$$ Y = \bar{Y} + b \sin(\delta_S - \alpha_s) $$

$$ Z = Z + b \tan \omega_{\pm S} $$

where

$$ b = \frac{R \cos \omega_{\pm S}}{\sin(\omega_{\pm 2} - \omega_{\pm S})} - l_z $$

Coordinate origin of determined sphere position lies in the intersection of the tube inner surface and system optical axis O.
The angles $\omega_w$ and $\theta_w(\omega_w)$ can be determined from the known image length of the camera lens and spherical object image ellipticity $e = \frac{d_z}{d_y}$, where $d_z$ and $d_y$ are spherical object image dimension in Z and Y plane. A numerical simulation of the ellipticity as a function of viewing angle $\omega_w$ and depth X in the glass tube for the 2 mm diameter sphere in plane $Z = 0$ and glass tube radius $R = 52$ mm and lens distance $S = 270$ mm is shown in the figure 5.

**Figure 5.** Sphere image ellipticity as a function of depth in the glass tube and viewing angle $\omega_w$.

3D chart of the ellipticity function is shown in the figure 6:

**Figure 6.** 3D function of sphere image ellipticity as a function of depth in the glass tube and viewing angle $\omega_w$.

The cylindrical tube serves like a magnifying glass in this set-up configuration due to the inner part of the tube lying in front of the rear focal plane of the tube. The tube imagines the spherical object to the virtual magnified image and this image is observed with camera optics. Distance of the tube focal plane from the first tube wall can be determined with:

$$s_f = \frac{n_i n_f r_1 r_2 - n_i n_f (n_2 - n_1) t}{n_2 r_2 (n_2 - n_1) + (n_3 - n_2) [n_2 r_2 - (n_2 - n_1) t]}, \quad (10)$$

where $n_i$ are refractive indexes, $r_i$ are curvature radii and $t$ is tube wall thickness. This distance is always greater than tube diameter for glass tube of any diameter filled with water. This condition is fulfilled for the inert liquid refractive index up to 1.95 for 50 mm tube diameter. It means this method is suitable for any kind of liquid, high refractive indexes liquids or melted glasses included. Cylindrical tube filled with a liquid of refractive index higher then air magnifies spherical object image not only in one plane, but due to the refractive indexes difference in front of and behind the tube wall in both perpendicular planes. It contributes to better resolution of the measurement system.
The main requirement to the camera detection system is the ability to establish spherical object image margins. It means all measured spherical object images have to be sharp enough for its margin determination. Every optical system is able to imagine sharply objects lied in the limited distance called depth of field only. Depth of field is given by:

$$\Delta = \frac{2Dgd_0}{D^2 - d_0^2},$$

where D is entrance pupil diameter, $d_0$ is size of the accepted unsharpness in the object space and g is the object distance. It is necessary to set the object field of the optical system to contain virtual images of the bubble in both perpendicular planes.

**MEASUREMENT**

The goal of the presented measurement was to perform the first position and dimension measurement using devised technique. Next we try to evaluate the accuracy and repeatability of this technique for the instrumental set-up using cylindrical tube as an anamorphic optical part of the system.

A special experimental vessel was designed, constructed and mounted to perform experimental method testing. This experimental vessel is using the part of the glass cylindrical tube filled with distilled water like an anamorphic part of the optical system. Bubble imaginary image is observed with camera equipped by common lens. Optical parameters of this lens had to be proven. Experimental vessel was design to minimize unwanted illumination of the camera pictures caused by total reflection on the spheres objects of main illumination source rays. It is possible to use direct or side-indirect illumination scheme of the observed 3D space to minimize reflections. Experimental vessel was design like top and bottom open in order it was possible to locate any kind of spherical object inside there. The first kind of experimental vessel with XYZ movable support is shown in the figure 7.

![Figure 7. The first experimental vessel with XYZ movable support.](image)

In order for results of this experiment to be comparable with the user conditions, the glass tube part used as a cylindrical lens meniscus was made from drawn glass tube. This kind of tube does not have optical quality and it decreases experiment precision. Due to this fact the radius curvature error of the drawn tube was measured with 3D measurement machine Zeiss. Glass tube profiles shown in the figure 8 were measured in four slices separated by 10 mm.
Maximal diameter error was measured to 0.226 mm. We used the right side of the glass tube for the measurement only, where the maximal diameter deviation corresponds to 0.091 mm. This value limits the global accuracy of the application.

Due to this technical complication the precision and repeatability had to be measured. Special X, Y, Z, movable support was used for the precision and repeatability measurement as an auxiliary system of the experimental vessel. The possibility of accuracy and repeatability of the system position determinations was tested by measurement of a glass and steel sphere with diameters 1.62 mm and 2 mm, respectively, mounted on X, Y, Z movable support. Images of the measured volume with spherical object were taken with different camera systems. We used B/W CCD camera, and Olympus E-300 high-resolution 8Mpix camera. A review of glass sphere images and its shapes caused by anamorphic imagination optical system at different axial distances separated by 4 mm is shown in the figure 9:

**Figure 9.** Images of glass sphere inside the cylindrical glass tube filled with water at different axial distances. Axial position difference is 4 mm.

Captured images were processed with special algorithm written in Matlab for determination X, Y, Z position parameters and radius evaluation of each spherical object. These computed data were compared with the originally adjusted position coordinates and appropriate spherical object radius value.

The experimental set-up was arranged as shown in the figure 1c. Experiments show different resolution capability in different directions. We have used experimental optical set-up parameters: external radius of cylindrical glass tube $R = 25.08$ mm with average glass thickness $t = 1.9$ mm, image distance of the camera lens was $p = 46.7$ mm and distance $S = 321$ mm.

We performed more then 60 measurements for the each glass and steel sphere and we found out the Y-axis and Z-axis measurement repeatability and precision was ± 0.035 mm close to the system optical axis. Far from the optical axis the repeatability was determined to ± 0.06 mm, but accuracy was affected by offset up to 0.35 mm caused by glass tube radius error. The accuracy in X-axis (along the optical axis) was much lower about ± 3.25 mm only. This value is affected with the error of sphere margin determination using manual measurement ± 1.5 pixel. This value will be refined using special software for the sphere margin determination for the next measurements. The most precise is the spherical object radius determination with error only ± 0.03 mm. It was find out that computed radius value is about approximately 5% smaller then the real value.
We try to perform repeatability and precision measurement using spherical bubbles in water too. It was used a thin capillary stocked to the 0.8 mm needle mounted to the XYZ movable support. It releases spherical shaped bubbles, but its rising trajectory was too variable to be used for these measurements. On the other hand we find out to be used small bubbles stocked to the vessel front and rear walls shown in the figure 10 for the position precision measurement.

Figure 10. Image of small bubbles stick to the front (spheres) and rear (ellipsis) vessel walls.

The precision of this measurement corresponds to the value acquired from the previous measurement using glass and steel spheres. That resolution or slightly higher can be reached for the measurement industry application on the standard drawn tubes without any necessary measurement outgoings. Cause of the measurement set-up precision value is mainly affected with the geometrical quality of the drawn glass tube we will perform next measurement using optical glass quality cylindrical tube. Expected maximal resolution under the best experimental condition is estimated to: $\Delta y, \Delta z = 0.02 \, \text{mm}$, $\Delta R = 0.01 \, \text{mm}$ and $\Delta x = 0.3 \, \text{mm}$.

CONCLUSION

A new optical system, which allow to determinate radius and position in 3D-observation space of a number of spherical shape objects with using the only one monolens camera system was presented. There are shown three kinds of instrumentation set-ups of the devised methods. One kind of devised set-up using cylindrical tube like a anamorphotic part of the optical system was constructed and the first measurement and data processing are demonstrated using spherical glass and steel objects mounted to the XYZ movable support. The application to the measurement of spherical bubbles in water was tested too. The best measurement system accuracy appears for the sphere radius measurement. The worse measurement system accuracy was found out for the position measurement along the optical axis of the system. This system can be used for the low optical quality drawen glass tube too, but it decreases the accuracy of the measurement. We expect the presented system will be able to be used for the determination of the spherical objects velocity as well as dimension change in the case of application of time-resolved photography.

This technique is protected with patent application No 2005-672 to the Czech Patent and Trademark Office.

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