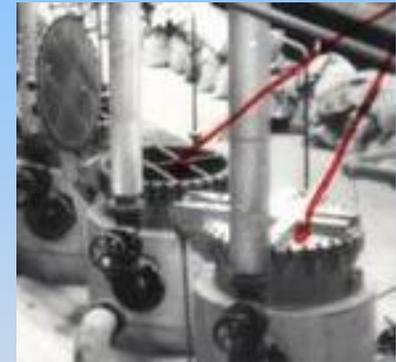


Processing Equipment Design

Exercise No. 5:

Reinforcement of flat cover of shell and tube heat exchanger



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<http://fsinet.fsid.cvut.cz/cz/U218/peoples/hoffman/index.htm>

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Design of reinforcement of flat cover of shell and tube heat exchanger (HE)

Solved as a fixed beam

Given:

Shell inner diameter

Calculated diameter of cover sealing

Pitch diameter of cover bolts

Cover external diameter

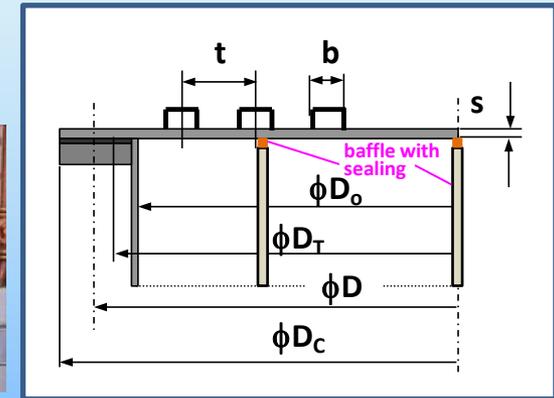
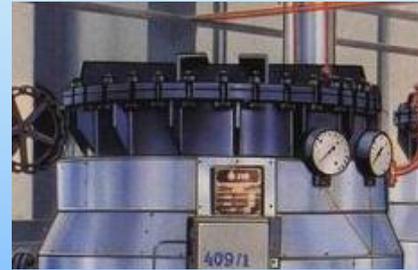
Overpressure in HE

Maximal allowable cover deflection

Type and size of reinforcing profiles

Section modulus of the profile for bending

Moment of inertia of the profile



$$D_o = 1000 \text{ mm}$$

$$D_t = 1050 \text{ mm}$$

$$D_r = 1085 \text{ mm}$$

$$D_c = 1120 \text{ mm}$$

$$p = 300 \text{ kPa}$$

$$y = 1 \text{ mm}$$

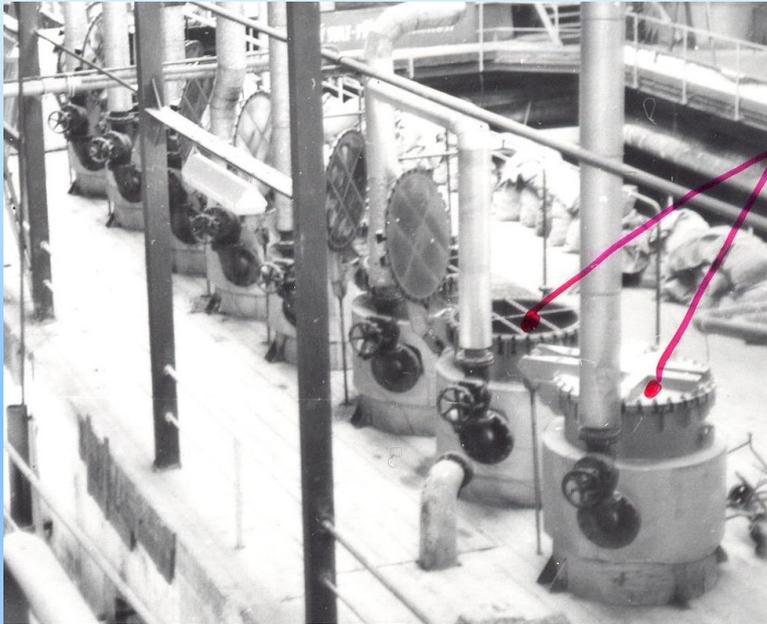
(tightness of
baffles sealing)

$$I 140$$

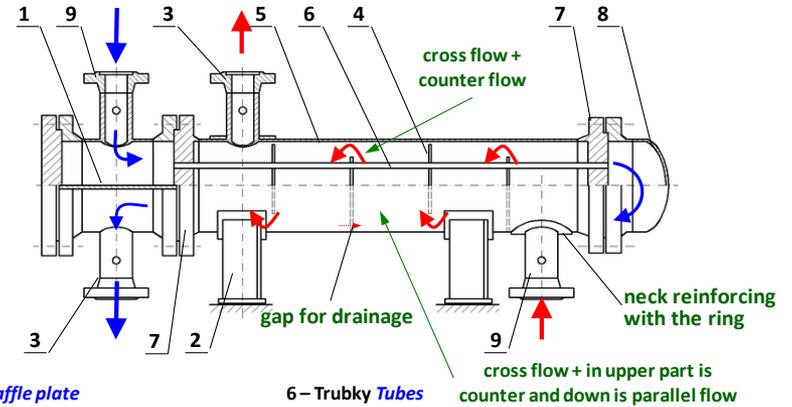
$$W_o = 81,9 \text{ cm}^3$$

$$J = 573 \text{ cm}^4$$

Examples of tubular HEs with several passes, flat cover and baffles

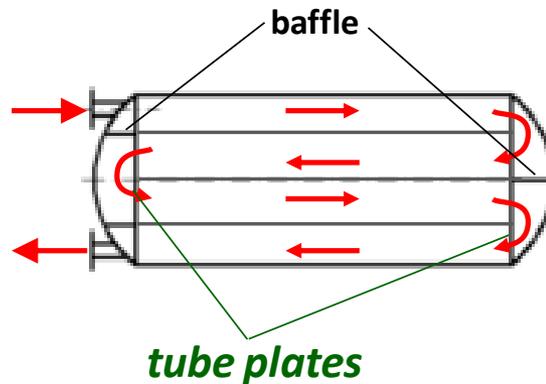


VT s pevnými trubkovicemi, přepážkami v mezitrubkovém prostoru a dvěma tahy v trubkách
 HE with fixed tube-plates and baffles in inter mediate tube space and 2 passes in tubes

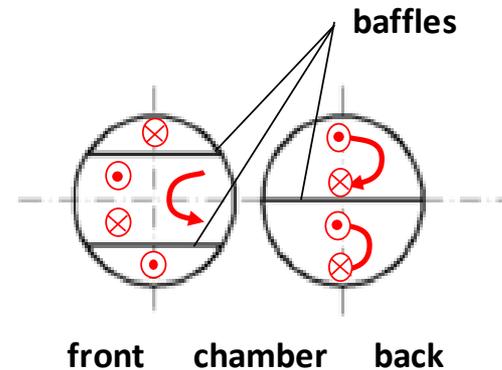


- 1 – Přepážka *Baffle plate*
- 2 – Patka *Support, footing*
- 3 – Výstupní hrdlo *Outlet neck*
- 4 – Segmentová přepážka *Segment baffle*
- 5 – Plášť *Shell*
- 6 – Trubky *Tubes*
- 7 – Pevná trubkovnice *Fixed tube plate (Welded like flange)*
- 8 – Příruba *Cover with flange*
- 9 – Vstupní hrdlo *Inlet neck*

Four passes HE



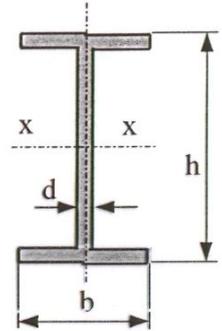
baffles keep flow in proper tubes



Tyče průřezu I Beams of I profile and their parameters

J (cm⁴) moment setrvačnosti průřezu k ose ohybu x - x **moment of inertia – needed from the deflection point of view**
 Wo (cm³) průřezový modul k ose ohybu x - x **section modulus – needed from the strength point of view**

Hodnoty určeny podle starších ČSN 420076 a 425551, které mohly být revidovány. Použitelné pouze pro účely výuky.



Type		Mass				
Označ.	h	b	d	Hmot.	Jx	Wo
Ⓘ	(mm)	(mm)	(mm)	(kg/m)	(cm ⁴)	(cm ³)
80	80	42	3,9	5,95	77,8	19,5
100	100	50	4,5	8,32	171	34,2
120	120	58	5,1	11,15	328	54,7
140	140	66	5,7	14,40	573	81,9
160	160	74	6,3	17,90	935	117
180	180	82	6,9	21,90	1450	161
200	200	90	7,5	26,30	2140	214
220	220	98	8,1	31,09	3060	278
240	240	106	8,7	36,20	4250	354
260	260	113	9,4	41,90	5740	442
280	280	119	10,1	48,00	7590	542
300	300	125	10,8	54,24	9800	653
320	320	131	11,5	61,07	12510	782
340	340	137	12,2	68,14	15700	923
360	360	143	13,0	76,22	19610	1090
380	380	149	13,7	84,00	24010	1260
400	400	155	14,4	92,60	29210	1460
450	450	170	16,2	115,00	45850	2040
500	500	185	18,0	141,00	68740	2750

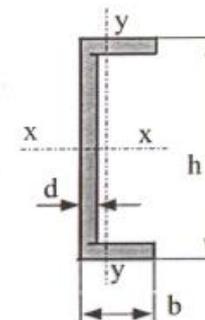
Type		Mass				
Označ.	h	b	d	Hmot.	Jx	Wo
IE	(mm)	(mm)	(mm)	(kg/m)	(cm ⁴)	(cm ³)
100	100	55	4,5	9,46	198	39,7
120	120	64	4,8	11,5	350	58,4
140	140	73	4,9	13,7	572	81,7
160	160	81	5,0	15,9	873	109
180	180	90	5,1	18,4	1290	143
200	200	100	5,2	21,0	1840	184
220	220	110	5,4	24,0	2550	232
240	240	115	5,6	27,3	3460	289
270	270	125	6,0	31,5	5010	371
300	300	135	6,5	36,5	7080	472
330	330	140	7,0	42,2	9840	597
360	360	145	7,5	48,6	13380	743

Napětí v nosníku; Průřezové moduly profilů
 P. Hoffman

Datum tisku: 10.10.200

Tyče průřezu U Beams of U profile and their parameters

J_x (cm ⁴)	moment setrv. průřezu k ose ohybu x - x	moment of inertia to bending axis x - x
W_{ox} (cm ³)	průřezový modul k ose ohybu x - x	section modulus to bending axis x - x
J_y (cm ⁴)	moment setrv. průřezu k ose ohybu y - y	moment of inertia to bending axis y - y
W_{oy} (cm ³)	průřezový modul k ose ohybu y - y	section modulus to bending axis y - y



Hodnoty určeny podle starších ČSN 420076 a 425571, které mohly být revidovány. Použitelné pouze pro účely výuky.

Type									Type								
Označ.	h	b	d	Mass Hmot.	J_x	W_{ox}	J_y	W_{oy}	Označ.	h	b	d	Mass Hmot.	J_x	W_{ox}	J_y	W_{oy}
U	(mm)	(mm)	(mm)	(kg/m)	(cm ⁴)	(cm ³)	(cm ⁴)	(cm ³)	UE	(mm)	(mm)	(mm)	(kg/m)	(cm ⁴)	(cm ³)	(cm ⁴)	(cm ³)
50	50	38	5,5	5,59	26,4	10,6	9,1	3,75	50	50	32	4,4	4,84	22,8	9,1	5,61	2,75
65	65	42	5,5	7,09	57,5	17,7	14,1	5,07	65	65	36	4,4	5,9	48,6	15	8,7	3,68
80	80	45	6,0	8,64	106	26,5	19,4	6,36	80	80	40	4,5	7,05	89,4	22,4	12,8	4,75
100	100	50	6,0	10,6	206	41,2	29,3	8,49	100	100	46	4,5	8,59	174	34,8	20,4	6,46
120	120	55	7,0	13,4	364	60,7	43,2	11,1	120	120	52	4,8	10,4	304	50,6	31,2	8,52
140	140	60	7,0	16,0	605	86,4	62,7	14,8	140	140	58	4,9	12,3	491	70,2	45,4	11,0
160	160	65	7,5	18,8	925	116	85,3	18,3	160	160	64	5,0	14,2	747	93,4	63,3	13,8
180	180	70	8,0	22,0	1350	150	114	22,4	180	180	70	5,1	16,3	1090	121	86	17,0
200	200	75	8,5	25,3	1910	191	148	27,0	200	200	76	5,2	18,4	1520	152	113	20,5
220	220	80	9,0	29,4	2690	245	197	33,6	220	220	82	5,4	21,0	2110	192	151	25,1
240	240	85	9,5	33,2	3600	300	248	39,6	240	240	90	5,6	24,0	2900	242	208	31,6
260	260	90	10,0	37,9	4820	371	317	47,7	260	260	95	6,0	27,7	4160	308	262	37,3
280	280	95	10,0	41,8	6280	448	399	57,2	300	300	100	6,5	31,8	5810	387	327	43,6
300	300	100	10,0	46,2	8030	535	495	67,8									

Profile width

Profile specific mass

Modulus of elasticity

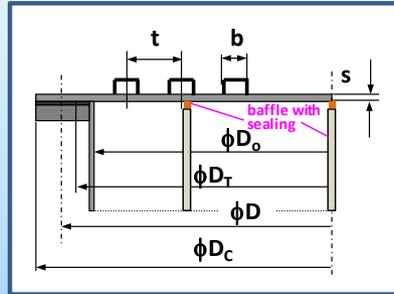
Maximal allowed stress

Material density

Minimal flat cover thickness specified

from technological reasons

(E.g. shell welding on the cover, flange strength ...)



$b = 66 \text{ mm}$

$m_{p1} = 14,4 \text{ kg/m}$

$E = 206 \text{ GPa}$

$\sigma_D = 156 \text{ MPa}$

$\rho = 7850 \text{ kg/m}^3$

$s = 10 \text{ mm}$

Task is to specify:

Pitch of reinforcing profiles → their number

Maximal bending stress

in central profiles

Maximal deflection of central

reinforcing profiles

Mass of cover together with reinforcing profiles

Comparison with unreinforced flat cover

$t = ? \text{ mm}$

$\sigma_{B_{\max}} = ? \text{ MPa}$

$y_{\max} = ? \text{ mm}$

$M_{\text{tot}} = ? \text{ kg}$

$\Delta M = ? \text{ kg}$

Appreciation of possibility of the reinforcement optimization

During our calculation we neglect a thin flat cover rigidity and will calculate only with rigidity of reinforcing profiles (it is that all forces acts only on these profiles).

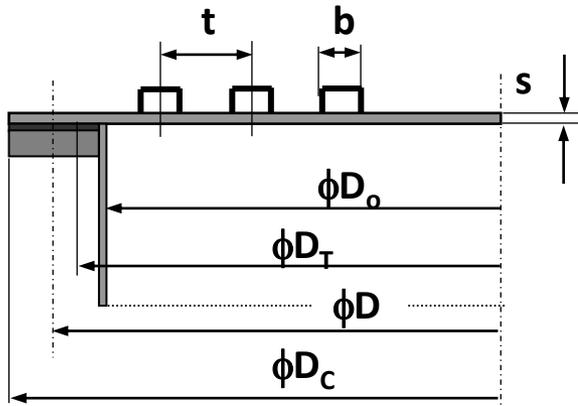
Our results will be on a side of better safety as the cover is able to withstand some loading.

We consider the case of a fixed beam.

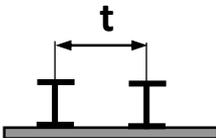
A sketch of the cover with profiles is on fig.1.

Fig.1 Sketch of flat circular cover reinforcement

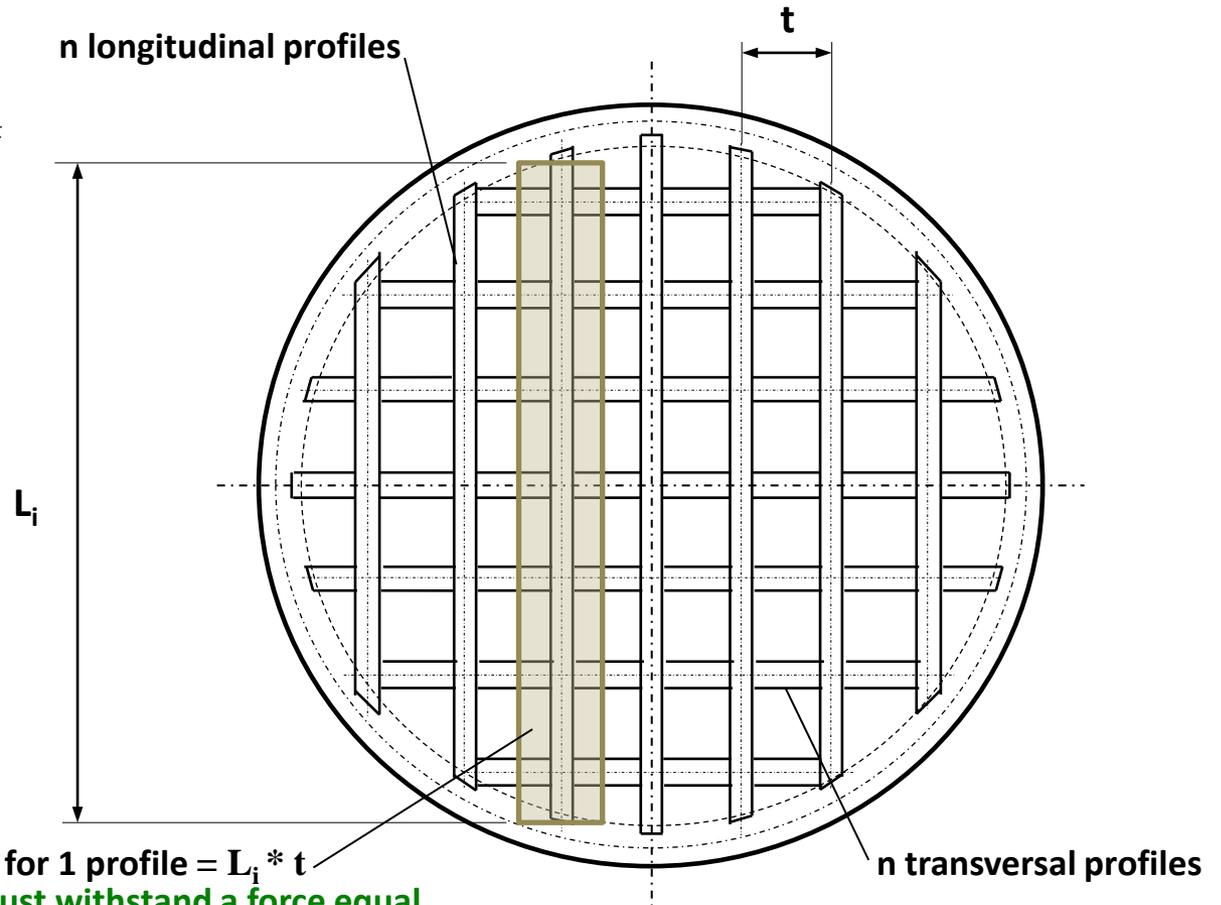
U profiles



I profiles

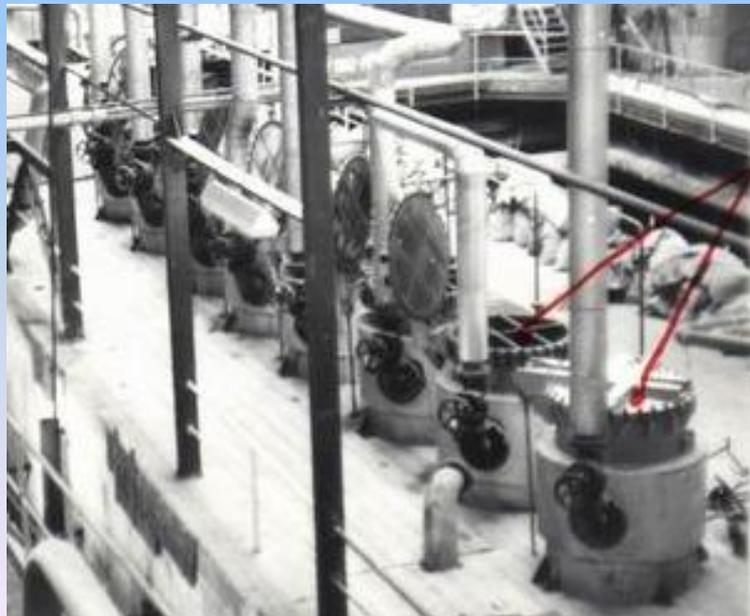
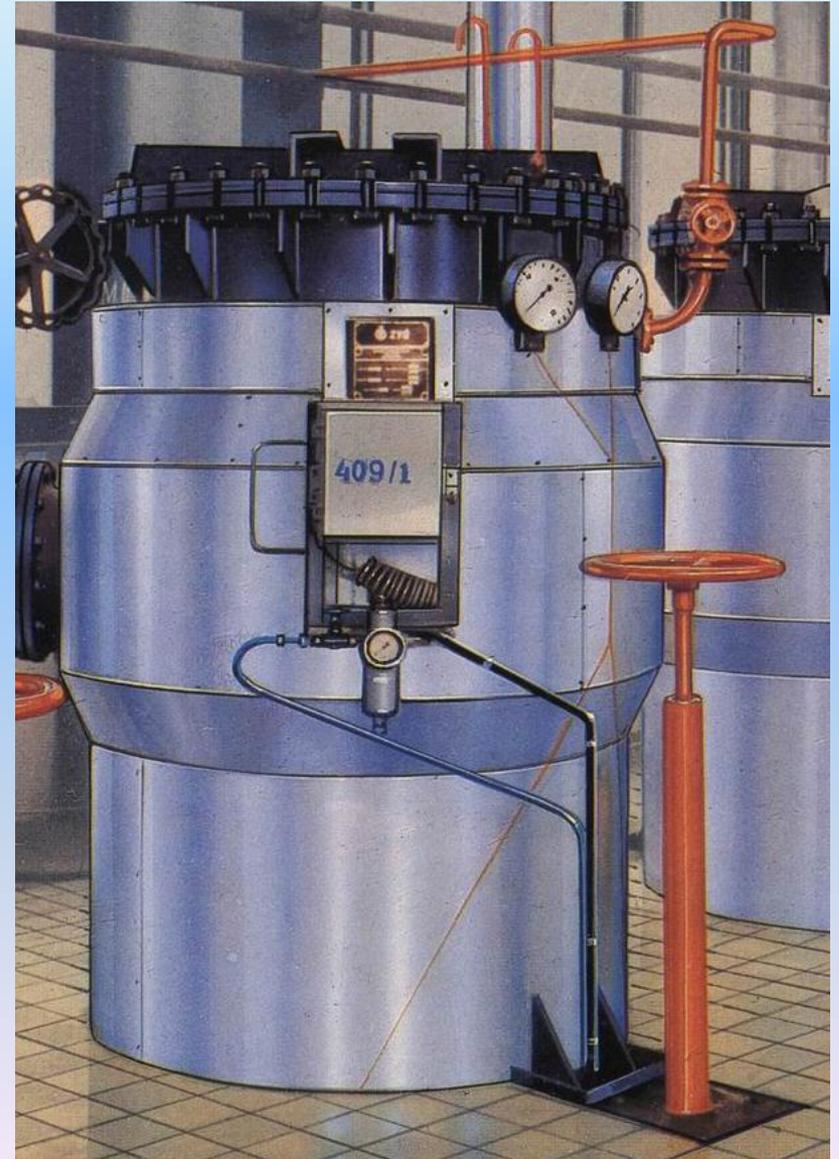
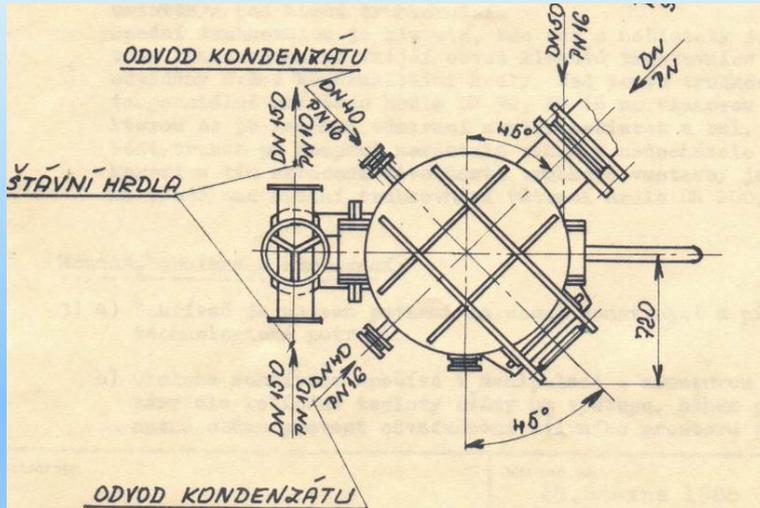


View of the longitudinal and transversal reinforcing beams



effective area for 1 profile = $L_i * t$
(the profile must withstand a force equal to the pressure acting on this surface)

Examples of flat covers of shell and tube HEs reinforcement

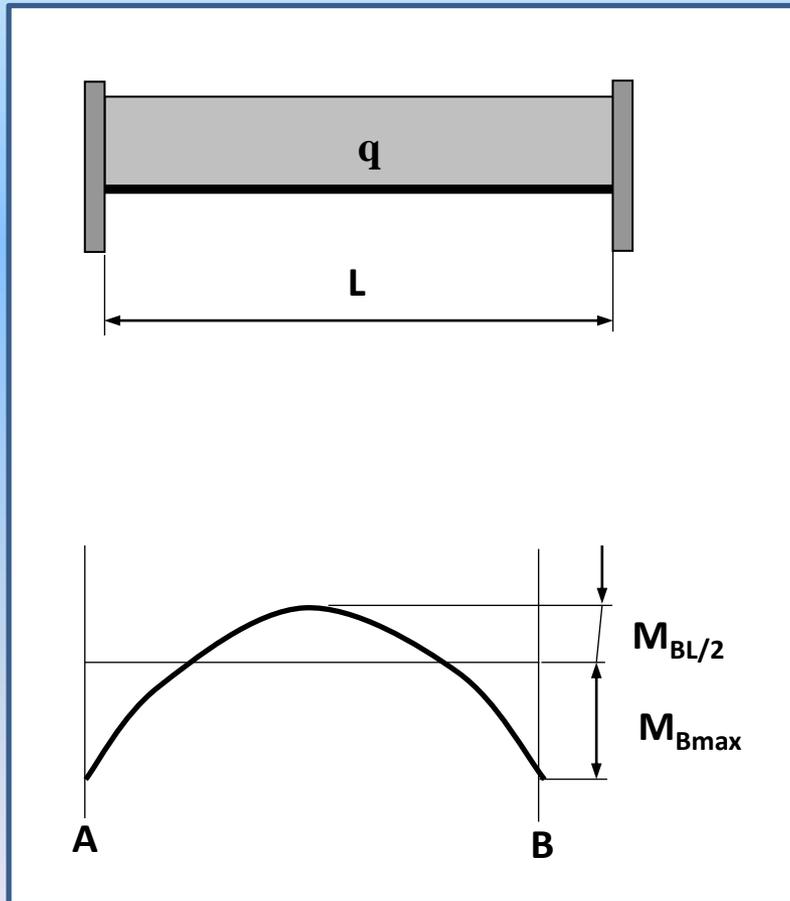


1. Loading course and equations used for fixed beam with length L and loaded with continuous load q.

fixed beam with continuous load
(in our example = internal overpressure)

For simplicity of our calculation we will consider the reinforcement as the fixed beam.
The real situation of the load will be between the fixed and supported beam closer to the fixed one.

course of bending moment in fixed beam with continuous load



Equations needed for the reinforcing beams calculation

Remember lectures of elasticity and strength

$$M_B(L/2) = \frac{q^* L^2}{24}$$

bending moment in the beam center

$$M_{B_{\max}} = M_A = M_B = -\frac{q^* L^2}{12}$$

bending moment in the place of the beam fixation

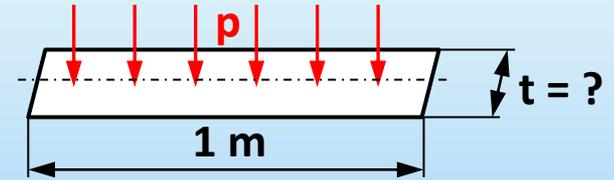
$$y(L/2) = \frac{q^* L^4}{384 * E * J}$$

deflection in the beam center

Continuous load of the beam

$$[q] = \text{N/m}^2 * m = \text{N/m}$$

2. Specification of beams (I profiles) pitch for the most loaded profile (near the axis)



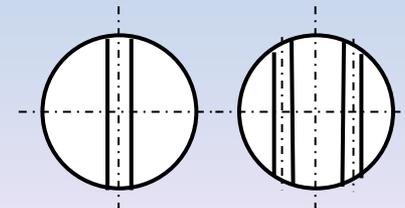
Note: Profiles are crosswise therefore is the loading divided by 2.

- Specification of the continuous loading referenced to 1 m of profile length

$$q = 1 / 2 * p * 1 * t \quad (\text{N/m; Pa, m, m})$$

- Maximal bending moment in the longest beam

$$M_{B_{\max}} = q * L_{\max}^2 / 12 \quad (\text{N*m; N/m, m}^2)$$



For the 1st approximation we suppose that $L_{\max} \approx D_t$ (for odd number of profiles it is valid, for even number is L_{\max} a few less than $D_t \rightarrow$ we are on the safety side)

- Real stress in beam must be equal or lower than allowed stress

Stress = bending moment / section modulus

$$\sigma_{\text{Breal}} = M_{\text{Bmax}} / W_O = (q * L_{\text{max}}^2) / (12 * W_O)$$

$$\sigma_{\text{Breal}} = 1 / 2 * (p * t * L_{\text{max}}^2) / (12 * W_O) \leq \sigma_D$$

- Theoretical maximal pitch of profiles is

$$t_{\text{max}} = 2 * (12 * \sigma_D * W_O) / (p * L_{\text{max}}^2)$$

$$t_{\text{max}} = 2 * (12 * 156 * 10^6 * 81,9 * 10^{-6}) / (300 * 10^3 * (1050 * 10^{-3})^2) =$$

$$t_{\text{max}} = 0.927 \text{ m} = 927 \text{ mm}$$

- Total maximal theoretical force acting on the longest profile

$$F_{\text{TPmax}} = p * t * L_1 / 2 \approx 300 * 0.927 * 1.050 / 2 = 146.0 \text{ kN}$$

- Total force acting on the cover is $\text{force} = \text{area} * \text{pressure}$

$$F_{TC} = \pi * D_t^2 / 4 * p = \pi * 1.05^2 / 4 * 300 = 259.8 \text{ kN}$$

- Maximal specific loading of the profile nearest the axis is (1st iteration = in the axis)

$$q_{\max} = p * 1 * t_{\max} / 2 = 300 * 1 * 0,927 / 2 = 139.1 \text{ kN/m}$$

- Maximal bending moment in the profile located nearest the axis is

$$M_{B_{\max}} = q_{\max} * L_{\max}^2 / 12 = 139.1 * 1.050^2 / 12 = 12.78 \text{ kN.m}$$

- Maximal theoretical bending stress in the profile located nearest the axis is

$$\sigma_{B_{\max}} = M_{B_{\max}} / W_o = 12.78 / (81.9 * 10^{-6}) = 156 \text{ 000 kPa} = \sigma_D$$

(checking if we counted correctly)

- Maximal theoretical deflection of the profile located nearest the axis is

$$y_{t_{\max}} = q * L_{\max}^4 / (384 * E * J) = 139.1 * 10^3 * 1.050^4 / (384 * 206 * 10^9 * 573 * 10^{-8})$$

$$y_{t_{\max}} = 0.00037 \text{ m} \approx 0.4 \text{ mm} < 1 \text{ mm} \quad \text{OK} \quad (\text{J} = \text{moment of inertia})$$

3. Specification of real number of profiles and loading

- Number of profiles in one direction

$$n_p = D_t / t_{\max} = 1050 / 927 = 1.13 \text{ pcs}$$

- Chosen number of profiles (minimally a next higher whole number)

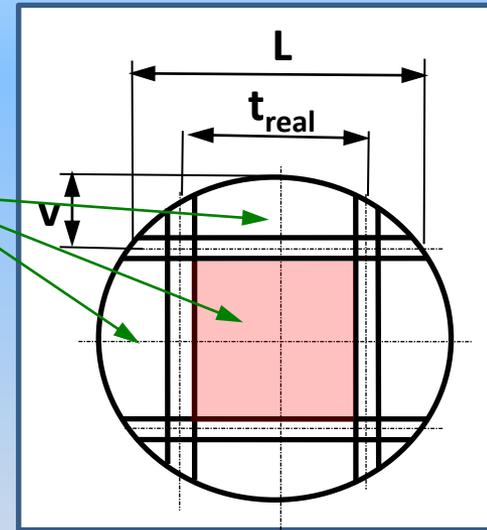
$$n_{p_{ch}} = 2 \text{ pcs}$$

- Real pitch of profiles

$$t_{\text{real}} = D_t / n_{p_{ch}} = 1050 / 2 = 525 \text{ mm}$$

- Length of the longest profile (near the axis) –

unreinforced parts
of the plate

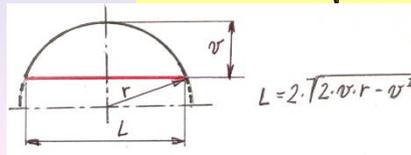


For even number of profiles: Equation for determining the length of the chord of the arc

For odd number of profiles

$$L_1 = 2 * \sqrt{2 * \left(\frac{D_t}{2} - 0,5 * t\right) * \frac{D_t}{2} - \left(\frac{D_t}{2} - 0,5 * t\right)^2}$$

$$L_1 = D_t$$



$$L = 2 * \sqrt{2 * v * r - v^2}$$

For our example is $n = 2$ even number. Then is a length of the longest profile

$$L_1 = 2 * \sqrt{2 * \left(\frac{1050}{2} - 0,5 * 525\right) * \frac{1050}{2} - \left(\frac{1050}{2} - 0,5 * 525\right)^2} = 909mm$$

• Real specific loading of the longest profile is (for the chosen No. of beams)

$$q_{\text{real}} = 1 / 2 * p * 1 * t_{\text{real}} = 1 / 2 * 300 * 1 * 0.525 = 78.75 \text{ kN/m}$$

• Real maximal bending moment in the longest profile is

$$M_{\text{Bmaxreal}} = q_{\text{real}} * L_1^2 / 12 = 78.75 * 0.909^2 / 12 = 5.42 \text{ kN.m}$$

• Real maximal bending stress in the longest profile is (for specified section modulus)

$$\sigma_{\text{Bmaxreal}} = M_{\text{Bmaxreal}} / W_o = 5.42 / 81.9 * 10^{-6} = 66000 \text{ kPa} = 66.0 \text{ MPa}$$

$$66 \text{ MPa} < 156 \text{ MPa} \quad \text{reserve } 2.36 \times$$

• Maximal real deflection of the longest profile is

$$y_{\text{maxreal}} = q_{\text{real}} * L_1^4 / (384 * E * J) = 78.75 * 10^3 * 0.909^4 / (384 * 206 * 10^9 * 573 * 10^{-8})$$

$$y_{\text{maxreal}} = 0.00012 \text{ m} = 0.12 \text{ mm} < 1 \text{ mm} \quad \text{O.K.} \quad \text{reserve } 8.3 \times$$

4. Specification of the reinforced cover mass

- Mass of the flat cover is

$$M_{FC} = \pi * D_C^2 / 4 * s * \rho = \pi * 1.12^2 / 4 * 0.010 * 7850 = 77.3 \text{ kg}$$

- Mass of longitudinal profiles

Length of i-th profile for odd number of profiles

$$L_i = 2 * \sqrt{2 * \left(\frac{D_t}{2} - 0,5 * t - (i-1) * t\right) * \frac{D_t}{2} - \left(\frac{D_t}{2} - 0,5 * t - (i-1) * t\right)^2}$$

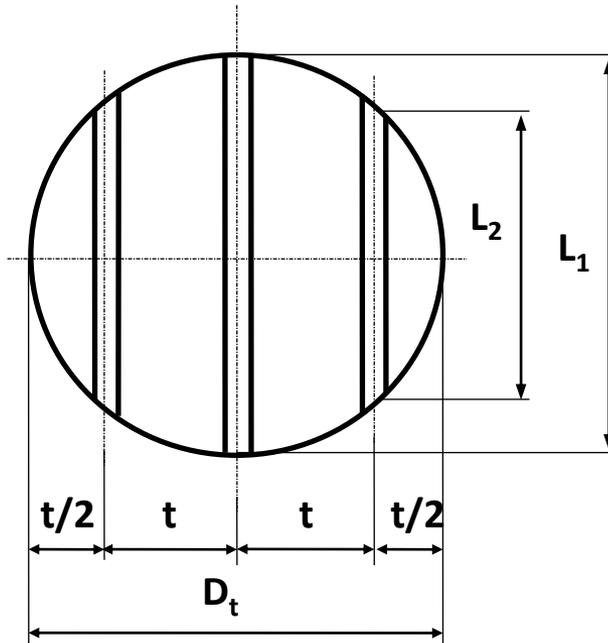


$$L = 2 * \sqrt{2 * v * r - v^2}$$

Length of i-th profile for even number of profiles

$$L_i = 2 * \sqrt{2 * \left(\frac{D_t}{2} - (i-1) * t\right) * \frac{D_t}{2} - \left(\frac{D_t}{2} - (i-1) * t\right)^2}$$

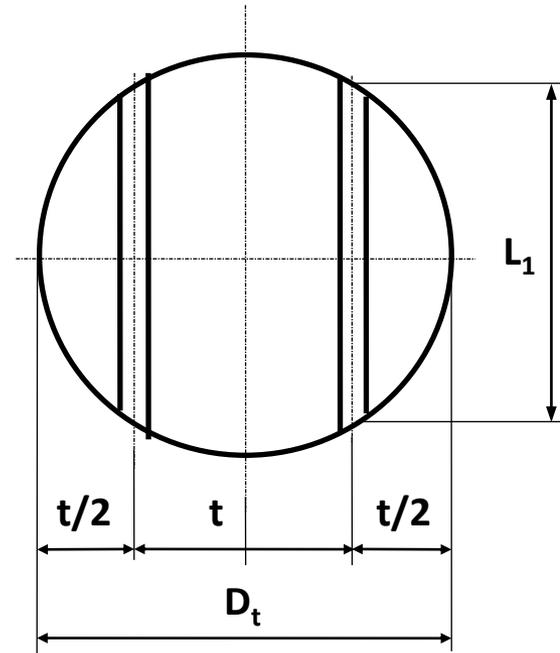
Real pitch of profiles



$n = 3$ (odd) number of profiles

$$t_{\text{real}} = D_t / n$$

$$L_{\text{max}} = L_1 = D_t$$



$n = 2$ (even) number of profiles

$$t_{\text{real}} = D_t / n$$

$$L_{\text{max}} = L_1 < D_t$$

In our example is $n = 2$ profiles, $i = 1$ (only 2 symmetric profiles) so that we can use the above calculated result $L_1 = 909 \text{ mm}$.

- Mass of the 1st profile

$$M_{p1} = L_1 * m_{p1} = 0.909 * 14.4 = 13.1 \text{ kg} \quad (\text{m} * \text{kg/m} = \text{kg})$$

- Mass of all longitudinal profiles

$$M_{plongit} = \sum L_i = 2 * L_1 = 2 * 13.1 = 26.2 \text{ kg}$$

- Analogical we can specify mass of transversal profiles (we must subtract places where is the material of longitudinal profiles – transversal profiles are welded among longitudinal ones)

$$M_{ptrans} \approx 24.5 \text{ kg}$$

- Total mass of reinforcing profiles

$$M_{Ptot} = M_{Plongit} + M_{Ptrans} = 26.2 + 24.5 = 50.7 \text{ kg}$$

- Mass of the flat cover is (circular plate)

$$M_{FC} = \pi * D_C^2 / 4 * s * \rho$$

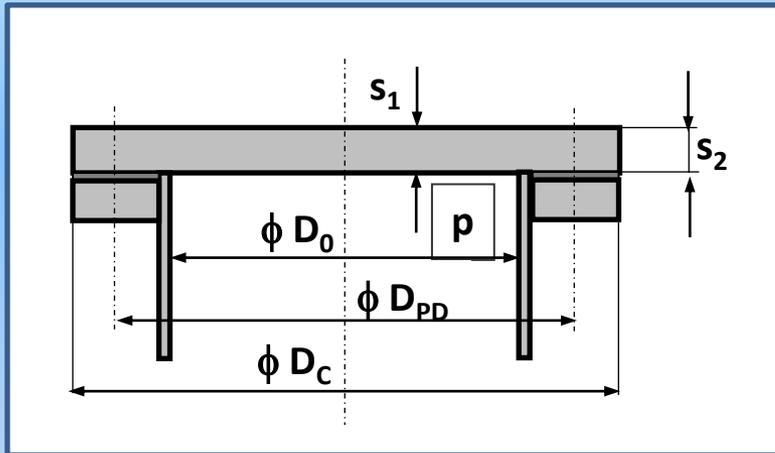
$$M_{FC} = \pi * 1.12^2 / 4 * 0.01 * 7850 = 77.3 \text{ kg}$$

- Total mass of the reinforced cover

$$M_{Ctot} = M_{FC} + M_{Ptot} = 77.3 + 50.7 = 128.0 \text{ kg}$$

5. Specification the of non-reinforced cover mass

In the case we can specify the minimal calculation thickness of the flat circular cover according Czech standard ČSN 690010, part 4.9. A sketch of the cover with marking according the ČSN is on the figure.



Equation for cover thickness specification

$$s_{1R} = K * K_0 * D_R * \sqrt{\frac{p}{\sigma_D * \varphi}}$$

[mm; -, -, mm, MPa, MPa, -]

- Calculated diameter (pitch diameter of flange bolts)
- Coefficient of cover weakening with holes for necks
(in the cover are not holes for necks)
- Coefficient of a type of cover periphery fixation
- Coefficient of cover weakening with weld
(no weld in the cover)

$$D_{PD} = 1085 \text{ mm}$$

$$K_0 = 1$$

$$K = 0.40$$

$$\varphi = 1$$

for circular
cover forming
flange

- Minimal calculated cover thickness

$$s_{1R} = 0,4 * 1 * 1085 * \sqrt{\frac{0,3}{156 * 1}} = 19,1mm$$

- Real cover thickness

$$s_1 = s_{1R} + c$$

c = allowances for corrosion etc.

$$s_1 \approx 21 \text{ mm}$$

- Mass of non-reinforced cover

$$M_{CN} = \pi * D_C^2 / 4 * s_1 * \rho$$

$$M_{CN} = \pi * 1.12^2 / 4 * 0.021 * 7850 = 162.3 \text{ kg}$$

- Mass reduction if reinforcing is used

$$\Delta M = M_{CN} - M_{Ctot} = 162.3 - 128.0 = 34.3 \text{ kg} \quad \text{or}$$

$$\Delta M = (M_{CN} - M_{Ctot}) / M_{CN} * 100$$

$$\Delta M = 100 * (162.3 - 128.0) / 162.3 = 21.1 \%$$

As you can see from previous results the **theoretical number of profiles was 1.1 and the chosen number was n = 2. Therefore are profiles too oversized.** It is affirmed by the calculated value of real stress $\sigma_{omaxreal} = 66.0 \text{ MPa}$ that is much lower than allowed value **156 MPa**. Therefore we can do an optimization of a choice of profiles size.

Calculations were performed in Excel and the result was that the best solution is using of profiles I 100.

6. Calculation for optimized size of reinforcing profiles I 100 instead original I 140.

Parameters of profile I 100:

for I 140

• section modulus of bending	$W_o = 34.2 \text{ cm}^3$	91.9
• moment of inertia	$J = 171 \text{ cm}^4$	573
• profile specific mass	$m_{p1} = 8.32 \text{ kg/m}$	14.4
• profile width	$b = 50 \text{ mm}$	66

6.1. Theoretical maximal pitch of profiles

$$t_{\max} = 2 * (12 * \sigma_D * W_o) / (p * L_{\max}^2) \leftarrow \begin{matrix} M_{B\max} = q * L_{\max}^2 / 12 = \sigma_B * W_o \\ q = p * t / 2 \quad \sigma_B \leq \sigma_D \end{matrix}$$

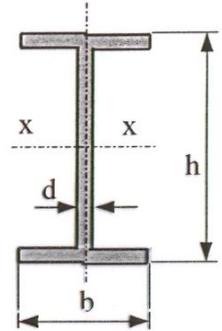
$$t_{\max} = 2 * (12 * 156 * 10^6 * 34.2 * 10^{-6}) / (300 * 10^3 * (1050 * 10^{-3})^2)$$

$$t_{\max} = 0.387 \text{ m} = 387 \text{ mm} \quad 927$$

Týče průřezu I Beams of I profile

J (cm⁴) moment setrvačnosti průřezu k ose ohybu $x - x$ **moment of inertia – needed from the deflection point of view**
 W_o (cm³) průřezový modul k ose ohybu $x - x$ **section modulus – needed from the strength point of view**

Hodnoty určeny podle starších ČSN 420076 a 425551, které mohly být revidovány. Použitelné pouze pro účely výuky.



Type		Mass				
Označ.	h	b	d	Hmot.	Jx	W _o
Ⓘ	(mm)	(mm)	(mm)	(kg/m)	(cm ⁴)	(cm ³)
80	80	42	3,9	5,95	77,8	19,5
100	100	50	4,5	8,32	171	34,2
120	120	58	5,1	11,15	328	54,7
140	140	66	5,7	14,40	573	81,9
160	160	74	6,3	17,90	935	117
180	180	82	6,9	21,90	1450	161
200	200	90	7,5	26,30	2140	214
220	220	98	8,1	31,09	3060	278
240	240	106	8,7	36,20	4250	354
260	260	113	9,4	41,90	5740	442
280	280	119	10,1	48,00	7590	542
300	300	125	10,8	54,24	9800	653
320	320	131	11,5	61,07	12510	782
340	340	137	12,2	68,14	15700	923
360	360	143	13,0	76,22	19610	1090
380	380	149	13,7	84,00	24010	1260
400	400	155	14,4	92,60	29210	1460
450	450	170	16,2	115,00	45850	2040
500	500	185	18,0	141,00	68740	2750

Type		Mass				
Označ.	h	b	d	Hmot.	Jx	W _o
IE	(mm)	(mm)	(mm)	(kg/m)	(cm ⁴)	(cm ³)
100	100	55	4,5	9,46	198	39,7
120	120	64	4,8	11,5	350	58,4
140	140	73	4,9	13,7	572	81,7
160	160	81	5,0	15,9	873	109
180	180	90	5,1	18,4	1290	143
200	200	100	5,2	21,0	1840	184
220	220	110	5,4	24,0	2550	232
240	240	115	5,6	27,3	3460	289
270	270	125	6,0	31,5	5010	371
300	300	135	6,5	36,5	7080	472
330	330	140	7,0	42,2	9840	597
360	360	145	7,5	48,6	13380	743

Napětí v nosníku; Průřezové moduly profilů
 P. Hoffman

Datum tisku: 10.10.200

- Total max. theoretical force acting on the profile nearest the axis

$$F_{TPmax} = p * t * L_1 / 2 \approx 300 * 0.387 * 1.050 / 2 = 61.0 \text{ kN}$$

146

- Total force acting on the cover

$$F_{CT} = \pi * D_t^2 / 4 * p = \pi * 1,05^2 / 4 * 300 = 259.8 \text{ kN}$$

259.8

- Maximal specific loading of the profile nearest the axis

$$q_{max} = p * 1 * t_{max} / 2 = 300 * 1 * 0.387 / 2 = 58.1 \text{ kN/m}$$

139.1

- Maximal bending moment in the profile located nearest the axis is

$$M_{omax} = q_{max} * L_{max}^2 / 12 = 58.1 * 1.050^2 / 12 = 5.34 \text{ kN.m}$$

12.78

- Maximal theoretical bending stress in the profile located nearest the axis is (for given section modulus)

1100

1140

$$\sigma_{Bmax} = M_{Bmax} / W_o$$

$$\sigma_{Bmax} = 5.34 / (34.2 * 10^{-6}) = 156\ 000\ \text{kPa} = 156\ \text{MPa} = \sigma_D$$

156

- Maximal theoretical deflection of the profile located nearest the axis is

$$y_{tmax} = q * L_{max}^4 / (384 * E * J)$$

$$y_{tmax} = 58.1 * 10^3 * 1.050^4 / (384 * 206 * 10^9 * 573 * 10^{-8})$$

$$y_{tmax} = 0.0005\ \text{m} \approx 0.5\ \text{mm} < 1\ \text{mm} \quad \text{OK}$$

0.4

6.2. Specification of real number of profiles and loading

- Number of profiles in one direction

100

140

$$n_p = D_t / t_{\max} = 1050 / 387 = 2.7 \text{ pcs}$$

1.13

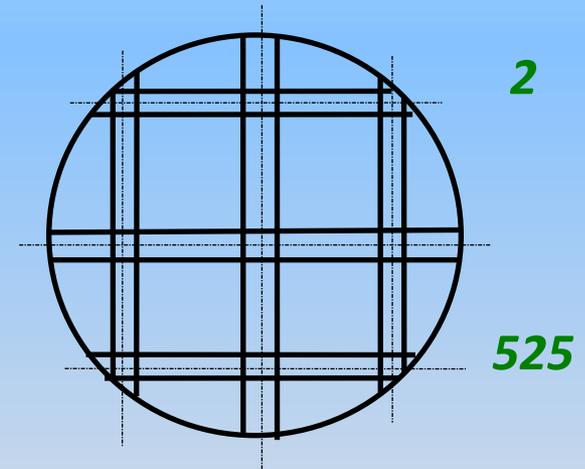
- Chosen number of profiles (minimally a next higher number)

$$n_{p_{ch}} = 3 \text{ pcs}$$

- Real pitch of profiles

$$t_{\text{real}} = D_t / n_{p_{ch}} = 1050 / 3 = 350 \text{ mm}$$

- Length of the longest profile (near the axis)



For odd number of profiles

$$L_1 = D_t$$

For even number of profiles

$$L_1 = 2 * \sqrt{2 * \left(\frac{D_t}{2} - 0,5 * t\right) * \frac{D_t}{2} - \left(\frac{D_t}{2} - 0,5 * t\right)^2}$$

For our example is $n = 3$... odd number. Then is a length of the longest profile

$$L_1 = D_t = 1050 \text{ mm}$$

100

140

- Real specific loading of the longest profile

$$q_{\text{real}} = 1 / 2 * p * 1 * t_{\text{real}} = 1 / 2 * 300 * 1 * 0.350 = 52.5 \text{ kN/m}$$

- Real maximal bending moment in the longest profile

78.75

$$M_{\text{Bmaxreal}} = q_{\text{real}} * L_1^2 / 12 = 52.5 * 1.05^2 / 12 = 4.82 \text{ kN.m}$$

5.42

- Real maximal bending stress in the longest profile

$$\sigma_{\text{Bmaxreal}} = M_{\text{Bmaxreal}} / W_O = 4.82 / 34.2 * 10^{-6} = 141000 \text{ kPa}$$

$$\sigma_{\text{Bmaxreal}} = 141.0 \text{ MPa} < 156 \text{ MPa} \quad \text{O.K.}$$

reserve is 1.1 x

66.0 < 156

- Maximal real deflection of the longest profile

reserve was 2.56 x

$$y_{\text{maxreal}} = q_{\text{real}} * L_1^4 / (384 * E * J) \leq 1 \text{ mm}$$

reserve is 2 x

$$y_{\text{maxreal}} = 52.5 * 10^3 * 1.05^4 / (384 * 206 * 10^9 * 171 * 10^{-8}) = 0.00047 \text{ m} \approx 0.5 \text{ mm}$$

better material utilization



reserve was 8.3 x

0.12

6.3. Specification of the reinforced cover mass

I 100

I 140

- Mass of the flat cover

$$M_{FC} = \pi * D_C^2 / 4 * s * \rho = \pi * 1.12^2 / 4 * 0.010 * 7850 = 77.3 \text{ kg}$$

77.3

- Mass of longitudinal profiles

Length of i-th profile for even number of profiles

$$L_i = 2 * \sqrt{2 * \left(\frac{D_t}{2} - 0,5 * t - (i-1) * t\right) * \frac{D_t}{2} - \left(\frac{D_t}{2} - 0,5 * t - (i-1) * t\right)^2}$$

Length of i-th profile for odd number of profiles

$$L_i = 2 * \sqrt{2 * \left(\frac{D_t}{2} - (i-1) * t\right) * \frac{D_t}{2} - \left(\frac{D_t}{2} - (i-1) * t\right)^2}$$

In our example is $n = 3$ profiles, $i = 2$ (only 1 profile in axis and 2 symmetrical profiles). For the axial profile we can use above calculated result $L_1 = 1050 \text{ mm}$.

- Mass of the 1st profile

100

140

$$M_{p1} = L_1 * m_{p1} = 1.050 * 8.32 = 8.7 \text{ kg}$$

13.1

- Length of the 2nd profile

$$L_2 = 2 * \sqrt{2 * \left(\frac{1050}{2} - (2-1) * 350\right) * \frac{1050}{2} - \left(\frac{1050}{2} - (2-1) * 350\right)^2}$$

$$L_2 = 783 \text{ mm}$$

0

- Mass of the 2nd profile

$$M_{p2} = L_2 * m_{p1} = 0.783 * 8.32 = 6.5 \text{ kg}$$

0

Mass of all longitudinal profiles

$$M_{plongit} = \sum L_i = 1 * L_1 + 2 * L_2 = 1 * 8.7 + 2 * 6.5 = 21.7 \text{ kg}$$

26.2

Analogical we can specify mass of transversal profiles (we must subtract places where is the material of longitudinal profiles – transversal profiles are welded among longit. ones)

$$M_{P_{trans}} \approx 20.2 \text{ kg}$$

1100 *1140*

24.5

• Total mass of reinforcing profiles

$$M_{P_{tot}} = M_{P_{longit}} + M_{P_{trans}} = 21.7 + 20.2 = 41.9 \text{ kg}$$

50.7

• Total mass of the reinforced cover

$$M_{C_{topt}} = M_{FC} + M_{P_{tot}} = 77.3 + 41.9 = 119.2 \text{ kg}$$

128.0

6.4. Total mass of non-reinforced cover

Is the same as in the previous case, it is $M_{CN} = 162.3 \text{ kg}$.

6.5. Mass reduction for this optimized variant

I 100

I 140

- Mass reduction if the variant is used compared with non- reinforced cover

$$\Delta M = M_{CN} - M_{Ctotopt} = 162.3 - 119.2 = 43.1 \text{ kg} \quad \text{or}$$

34.3

$$\Delta M = (M_{CN} - M_{Ctotopt}) / M_{CN} * 100$$

$$\Delta M = 100 * (162.3 - 119.2) / 162.3 = \underline{\underline{26.5 \%}}$$

21.1

or compared with reinforcing profiles I 140 it is

$$\Delta M = M_{Ctot} - M_{Ctotopt} = 128.0 - 119.2 = 8.8 \text{ kg} \quad \text{or}$$

$$\Delta M = (M_{Ctot} - M_{Ctotopt}) / M_{Ctot} * 100$$

$$\Delta M = 100 * (128.0 - 119.2) / 128.0 = \underline{\underline{6.9 \%}}$$

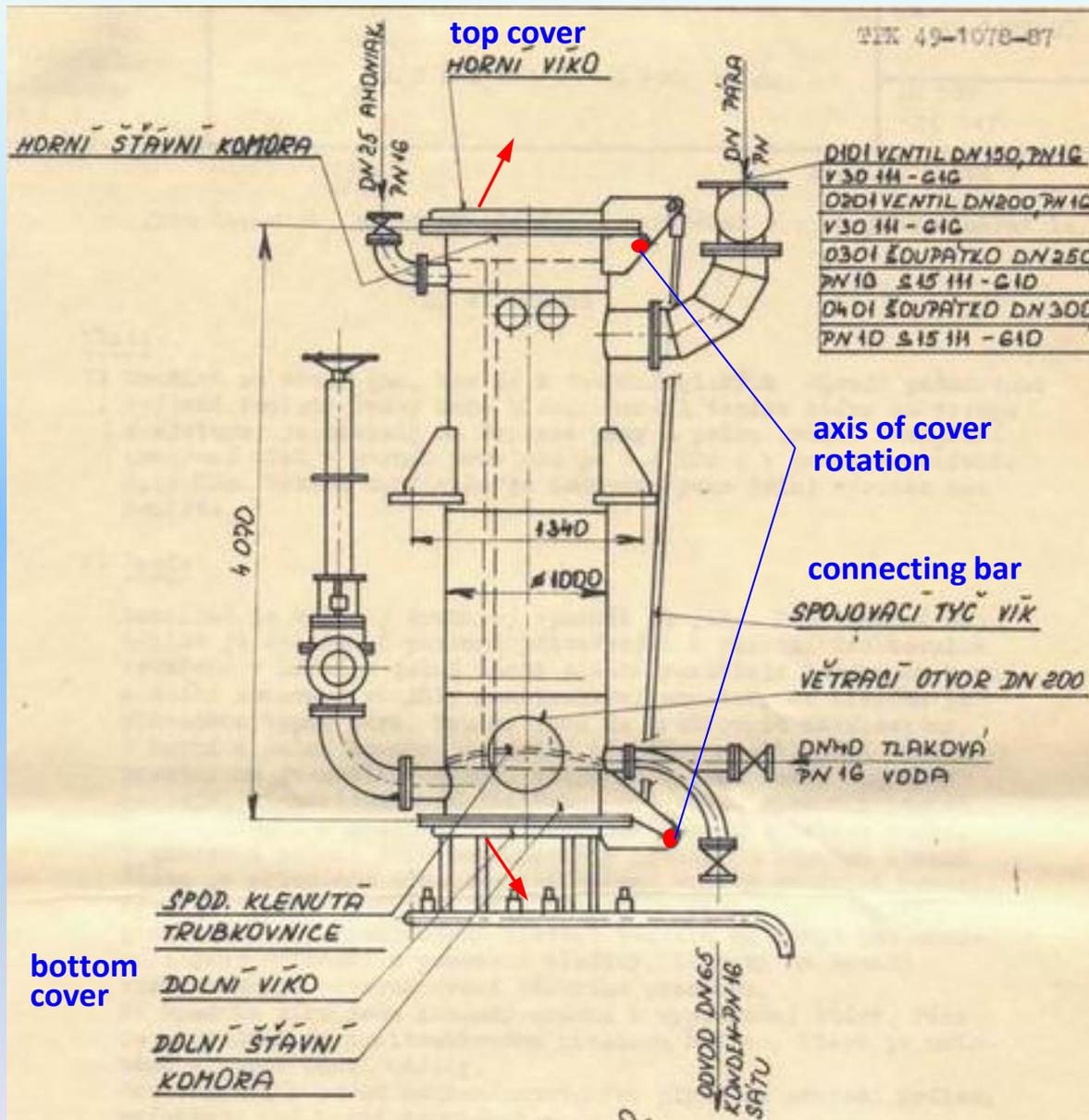
Recapitulation of results

Similar calculations can be performed for other reinforcing profiles.

Way of cover reinforcing	Total cover mass (kg)	
• Non-reinforced cover	162.3	
• I 140 – 2 x 2 pcs of profiles	128.0	
• I 100 – 3 x 3 pcs of profiles	119.2	least material
• I 160 – 1 x 1 pc of profile	124.4	smallest laboriousness
• U 200 – 3 x 3 pcs of profiles	171.9 kg	U profile has lower bending section modulus!
hemispherical cover $s_{\text{real}} = 0.5+1.0 = 1.5 \text{ mm}$	18.5	theory, in fact the thickness would have to be greater from technological aspects (welding of flange or baffles ...)

Note:

- *During a decision making it is necessary to take into account not only the material saving but for example a labor consumption of a variant (e.g. more welds).*
- *Therefore we must judge every variant (not only like it is in the very simple example) from more points of view.*



Drawing of the actual tube heater with basic dimensions (producer ZVU Hradec Králové)

Handling with heavy covers:

- Mass of the lower cover lifts the top cover (through the lever mechanism)
- At the top is the axis of the cover rotation before the connecting bar, down is the axis of the cover rotation behind the connecting bar

