

# Processing Equipment Design

## 4. Pressure Vessels

Pressure vessels must be designed according proper standards:

ČSN 690010 – Czech State Standard

EN 13445-3 – European Standard; Unfired pressure vessels: Design and calculation

DIN – Deutsche Industrie Norm

ASME BPVC – American Society of Mechanical Engineers; Boiler and Pressure Vessel Code

GOST – Gosudarstvennyj Standart

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## Types of pressure vessels:

Reactors, autoclaves, heat exchangers, columns, boilers, tanks etc. are usually pressure vessels.

(remember our first lectures with equipments examples)

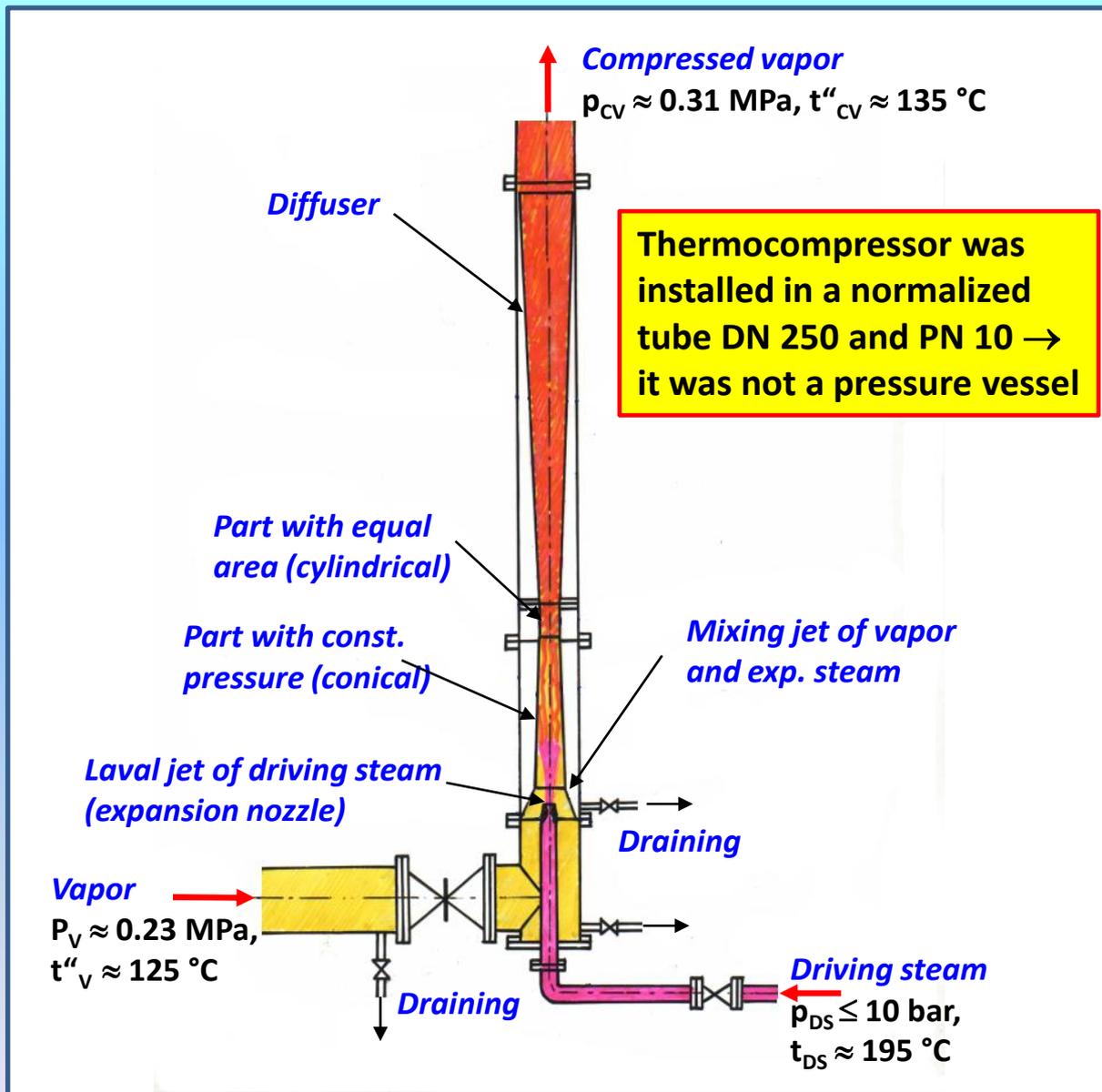
According the Czech Standard for pressure vessels ČSN 690010 - 1.1 and European standard ES 13445-1 pressure vessels are not

(→ rules of these standards need not be applied to these pressure vessels):

- **Pipes, tubes and their parts and vessels built in tubes produced according the ČSN for a given pressure**  
(e.g. pipe PN6 is designed for working pressure 6 bars → it is not necessary to check it again)
- **Pressure vessels made from tubes produced according the ČSN for a given pressure with maximal inside diameter 100 mm**  
(e.g. a vessel welded from such tubes)

- **Pressure vessels filled with a liquid (non aggressive, non toxic, non explosive) if the highest working temperature is lower than a boiling temperature corresponding to the overpressure 0.07 MPa (e.g. for water it is < 115 °C)**
- **Pressure vessels for that a product of volume (in dm<sup>3</sup>) and overpressure (in MPa) is less than 10 and the pressure is lower than 0.07 MPa**
- **Pressure vessels with volume less 1 dm<sup>3</sup> regardless of pressure**
- **Heat exchangers of type tube in tube with outside diameter < 100 mm**

**(Such vessels must be designed for these working parameters, but may not be approved by a relevant authority)**



It was not necessary to ask a relevant inspectorate for approval of the TC documentation (calculations and drawings) and the manufacturing authorization → time and money saving.

## Pressure vessels are usually designed from these geometrical shapes:

- Sphere
- Cylinder
- Cone
- Plate
- (Ellipsoid, torus ..)

## Brief repetition of previous knowledge:

- Rotary symmetrical vessels are from the stress point of view better than these ones made from flat plates.
- The most advantageous is sphere (remember the previous part).
- Theory of shells and membranes is used for solution of thin-walled vessels.

## Thin-wall shell is for ratio

$$k = \frac{d_e}{d_i} \leq 1.1 \text{ or } \leq 1.17$$

(with higher safety)      (according theory)

**Membrane** – in wall are only tensile or compression forces (stresses); stresses are calculated only from balance of forces in a section

**Rigid shell** – in wall are shear forces, bending and twist moments; for stresses calculation sometimes we need deformation conditions too.

Examples of shells whose dimensionless wall thickness is on the boundary between thin- and thick-walled shells.

For  $k = 1.1$  is wall thickness

$$s = (d_e - d_i) / 2 \leq (1.1 d_i - d_i) / 2 = 0.1 d_i / 2$$

*practical value*

For  $k = 1.17$  is wall thickness

$$s = (d_e - d_i) / 2 \leq (1.17 d_i - d_i) / 2 = 0.17 d_i / 2$$

*theoretical value*

$$d_i \text{ (mm)} = 100$$

$$500$$

$$1000$$

$$3000$$

$$k = 1.1$$

$$s \leq 5.0 \text{ mm} \rightarrow 11.7 \text{ MPa}$$

$$25.0 \rightarrow p_{imax} = 14.1 \text{ MPa}$$

$$50$$

$$150 \rightarrow p_{imax} = 15 \text{ MPa}$$

$$k = 1.17$$

$$s \leq 8.5 \text{ mm} \rightarrow p_{imax} = 22.7 \text{ MPa}$$

$$42.5$$

$$85$$

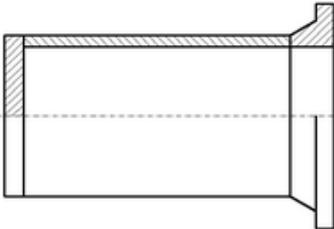
$$255 p_{imax} = 25.5 \text{ MPa}$$

→ A majority of vessels installed in industry are thin-walled shells

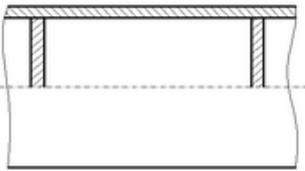
# Examples of shells according Czech Standards

Hladké válcové skořepiny  
*Smooth cylindrical shells*

Cylinder with flange and flat bottom

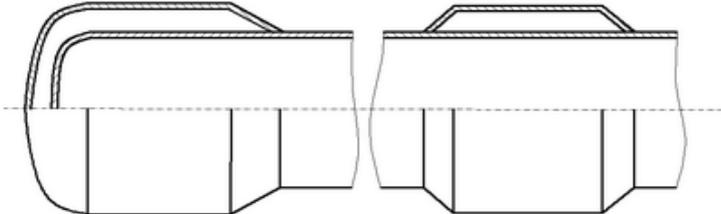


Cylinder with internal baffles



(flat bottom → easy production but bad from stress point of view)

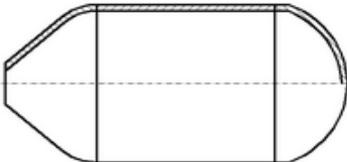
Hladká skořepina s duplikátorem  
*Smooth cylindr. shell with jacketed kettle*



Jacketed kettles are usually in vertical position

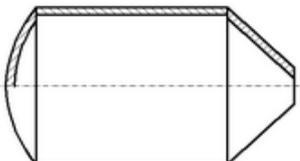
Hladká skořepina s klenutým nebo kuželovým dnem  
*Smooth cylindrical shell with dished or conical bottom*

S lemem  
With transition



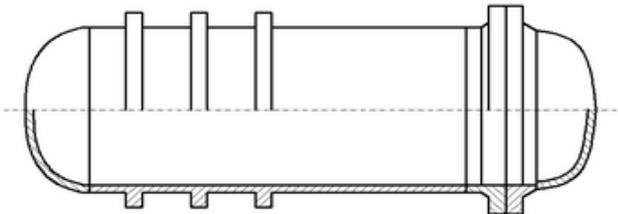
Smooth change of shape  
(toroidal transition)

Bez lemu  
Without transition



Sharp change of shape

Válcová skořepina vyztužená prstenci  
*Cyl. shell reinforced (stiffened) with rings*



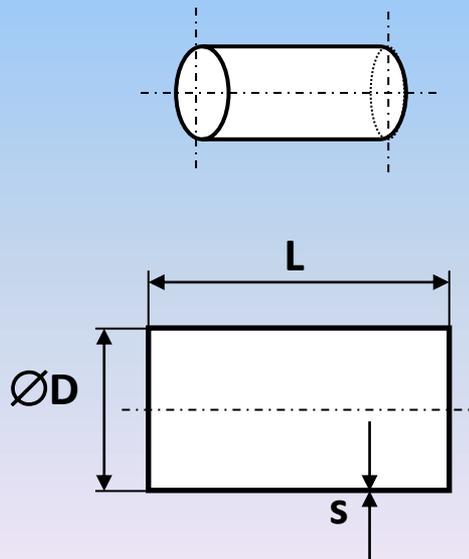
Reinforcing is often used for external pressure

# Pressure vessels calculation

According membrane theory and ČSN 690010

## Thin-walled cylinder with internal overpressure

(shell without bending moments etc. = membrane – see the part 3)



**Range of validity is for:**

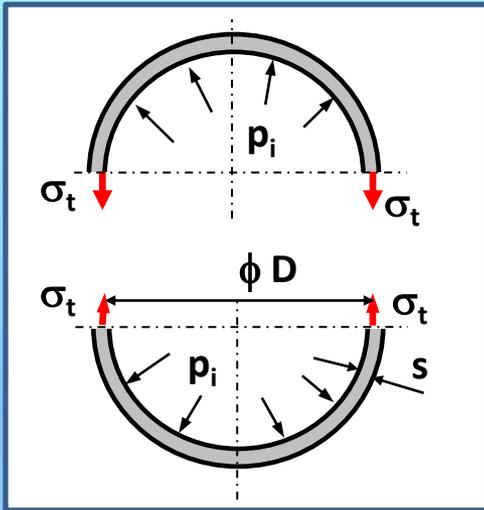
$$s_{\text{calc}} / D_i \leq 0.1 \quad \text{for } D_i \geq 200$$

$$s/D_i = (D_e/2 - D_i/2)/D_i = \frac{1}{2}(D_e/D_i - 1) = \frac{1}{2}(k-1)$$

$$\frac{1}{2}(k-1) \leq 0.1 \rightarrow k \leq 1.2 \quad \begin{array}{l} k_{\text{THEOR}} \leq 1.17 \\ k_{\text{PRACT}} \leq 1.10 \end{array}$$

$$s_{\text{calc}} / D_i \leq 0.3 \quad \text{for } D_i < 200$$

# Membrane theory (balance of external and internal forces) – remember part 3

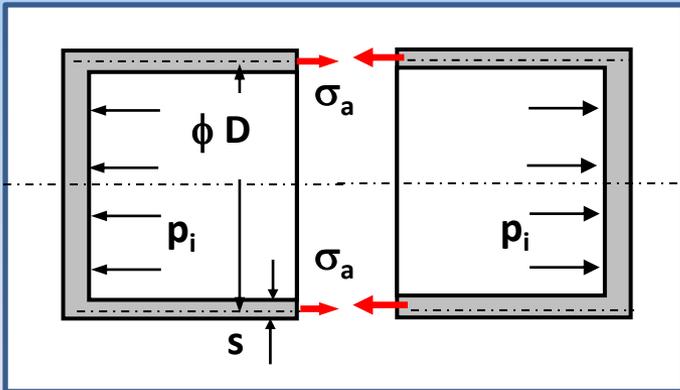


In tangential direction  $F_{et} = F_{it} \rightarrow$

$$p_i * D_i * L \approx 2 * L * s * \sigma_t$$

$$\sigma_t = p_i * D_i / 2s = p_i * r_i / s$$

$$s = p_i * D_i / 2 * \sigma_D = p_i * r_i / \sigma_D$$



In axial direction  $F_{ea} = F_{ia} \rightarrow$

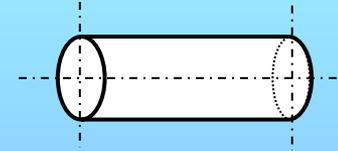
$$\pi * D_i^2 / 4 * p_i \approx \pi * D_i * s * \sigma_a$$

$$\sigma_a = p_i * D_i / 4s = p_i * r_i / 2s = \sigma_t / 2$$

$$s = p_i * D_i / 4 * \sigma_D = p_i * r_i / 2\sigma_D$$

# Cylindrical vessels or cylindrical parts of pressure vessels calculation according ČSN 690010 part 4.5.

## Loaded with internal pressure



### Range of validity:

$$k = \frac{D_e}{D_i} \leq 1.17 \approx 1.2$$

### Relative wall thickness (dimensionless)

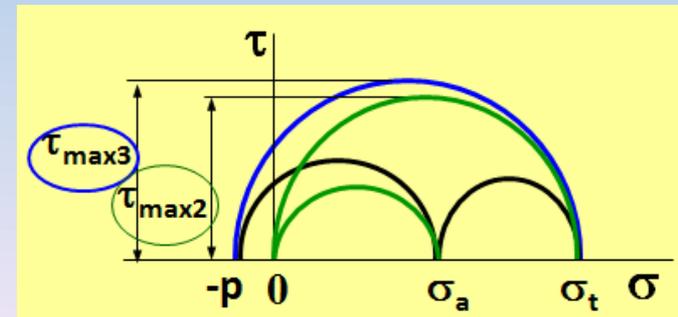
$$s_R / D_i = (s - c) / D_i \leq 0.1 \quad \text{for shells and tubes} \quad D_i \geq 200$$

$$s_R / D_i = (s - c) / D_i \leq 0.3 \quad \text{for tubes} \quad D_i < 200$$

### Maximal wall temperature:

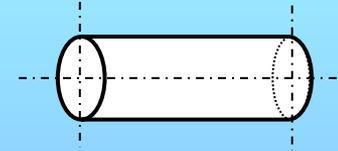
- 380 °C for carbon steels
- 420 °C for alloyed steels
- 525 °C for austenitic steels

Why ???



# Cylindrical vessels or cylindrical parts of pressure vessels calculation according ČSN 690010 part 4.5.

## Loaded with internal pressure



### Range of validity:

Relative wall thickness (dimensionless)

$$k = \frac{D_e}{D_i} \leq 1.17 \approx 1.2$$

$$s_R / D_i = (s - c) / D_i \leq 0.1$$

for shells and tubes

$$D_i \geq 200$$

$$s_R / D_i = (s - c) / D_i \leq 0.3$$

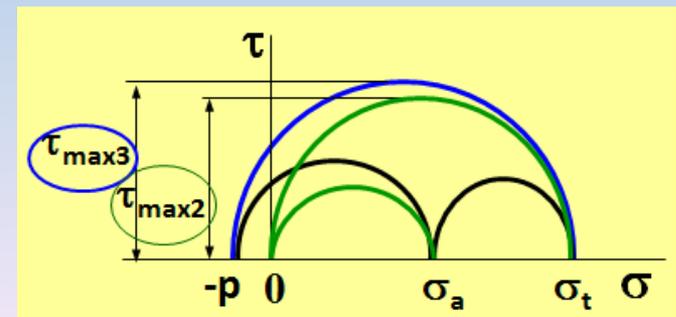
for tubes

$$D_i < 200$$

### Maximal wall temperature (creep):

- 380 °C for carbon steels
- 420 °C for alloyed steels
- 525 °C for austenitic steels

(For > °C → it is necessary to calculate such vessels according rules valid for creep)



## Calculated wall thickness

$$\sigma_t = p_i * D_i / 2s \rightarrow s = p_i * D / 2\sigma_t$$

$$s_R = \frac{p * D}{2 * \sigma_D * \varphi_P - p} \quad (\text{mm; MPa, mm, MPa, -})$$

(→ see Guest 3D)

$$\sigma_e = p_i * D / 2s + p_i \leq \sigma_D$$

## Allowed internal pressure for a wall with given thickness

$$p = \frac{2 * \sigma_D * \varphi_P * (s - c)}{D + (s - c)}$$

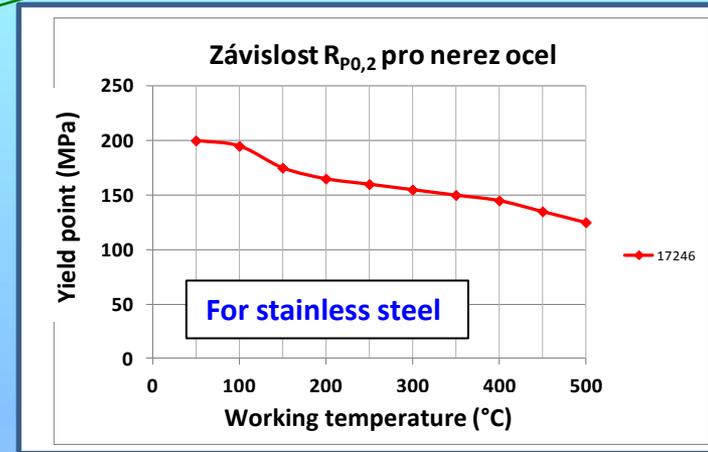
## Realized wall thickness

$$s \geq s_R + c$$

Where is:	p (MPa)	working overpressure
	D (mm)	internal diameter
	$\sigma_D$ (MPa)	allowable stress for working temperature
	$\varphi_P$ (-)	coefficient of weld weakening (or symbol $\mathbf{V}$ is used)
	c (mm)	sum of all allowances (for corrosion, manufact. tolerance ..)

Checking for a pressure test is not needed if a testing pressure is lower than

$$p_{PT} < p * (1.35 * \frac{\sigma_{D20}}{\sigma_D})$$



where is:

- $p$  calculated (working) pressure
- $\sigma_D$  allowable stress at calculated temperature
- $\sigma_{D20}$  allowable stress at temperature 20 °C *(pressure test temperature)*

$x_{WP} = 1.5$  safety coefficient for working pressure

$x_{PT} = 1.5 / 1.35 = 1.1$  safety coefficient for pressure test  $\rightarrow \sigma_{PT} = \sigma_Y / 1.1$

( $\rightarrow$  in places where are stress peaks  $\rightarrow \sigma_Y$  is reached there  $\rightarrow$  adaptation on this overloading – see the part about utilization of material plasticity)

# Cylindrical vessels loaded with external pressure

Calculated wall thickness is determined from formula

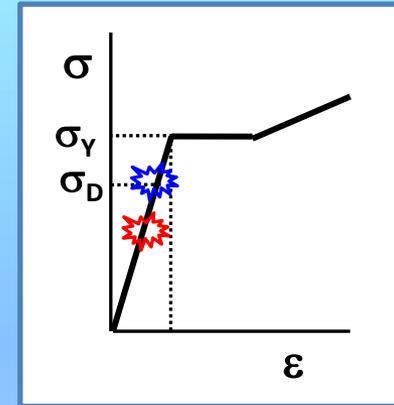
$$s_R = \text{Max} \left\{ K_2 * D * 10^{-2}; \frac{1.1 * p * D}{2 * \sigma_D} \right\}$$

stability point of view

stress point of view (modified membrane equation x 1.1)

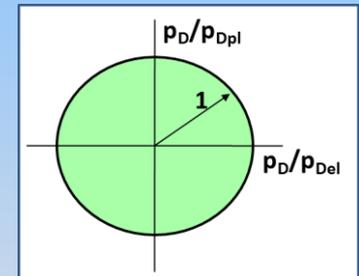
$$s \geq s_R + C$$

realized wall thickness



Coefficient  $K_2$  is determined from diagram in the standard (more in the part 8 – “Stability of beam, plate and cylinder”)

Wall thickness is checked from following formulas:



Allowable external overpressure

$$[p] = \frac{[p]_P}{\sqrt{1 + \left( \frac{[p]_P}{[p]_E} \right)^2}}$$

or

$$\left( \frac{p_D}{p_{Del}} \right)^2 + \left( \frac{p_D}{p_{Dpl}} \right)^2 = 1$$

PED-4 (where  $p_D = [p]$  = allowable ext. pressure)

where **allowable external overpressure in plastic state is (from strength condition)**

$$[p]_P = \frac{2 * \sigma_D * (s - c)}{D + (s - c)}$$

(calculated according the Guest hypothesis like for the internal pressure)

and **allowable external overpressure in elastic state is (from stability condition)**

$$[p]_E = \frac{20.8 * 10^{-6} * E * D}{n_U * B_1} * \frac{D}{L} * \left[ \frac{100 * (s - c)}{D} \right]^2 * \sqrt{\frac{100 * (s - c)}{D}}$$

$$B_1 = \text{Min} \left\{ 1.0; 9.45 * \frac{D}{L} * \sqrt{\frac{D}{100 * (s - c)}} \right\}$$

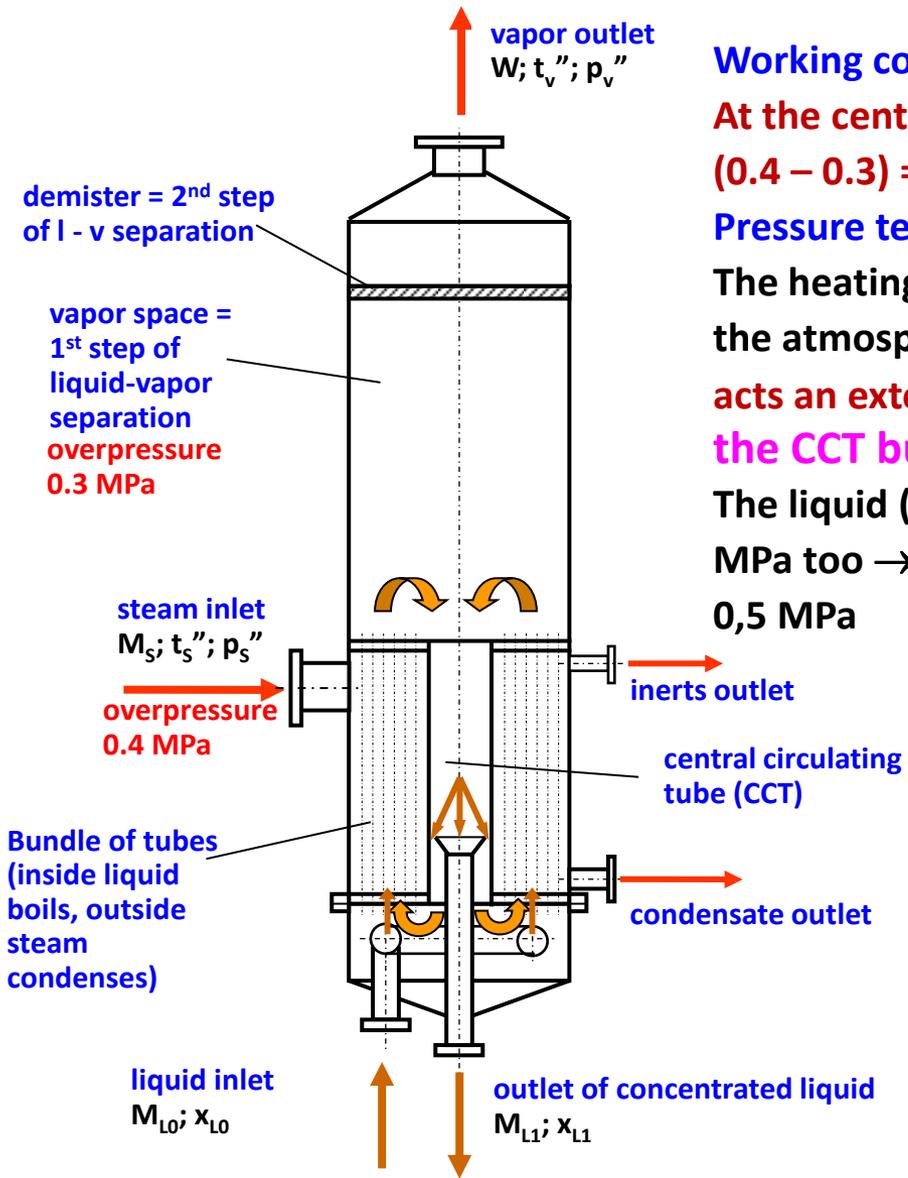
(equations are derived according the Mises theory – see „Stability...“)

where is: L (mm)      calculated length of smooth shell (cylinder)  
D (mm)      shell internal diameter  
s (mm)      realized shell wall thickness  
n<sub>U</sub> = 2.4      safety factor according stability loss in elastic state

## Notes:

- **Remember that the standard uses partially different symbols of parameters than are used in the other parts of the chapter.**
  - **The standard uses internal diameter instead external.** (the old ČSN used the ext. diameter)
- **Beware on the pressure test or working troubles, when for example in a heat exchanger in one space is not a fluid (or it has considerably lower pressure) → in such apparatus can be quite opposite pressure relations compared calculated one → for example instead supposed internal overpressure a part can be loaded with an external overpressure → danger of wall buckling!!**

# Tubular circulatory evaporator

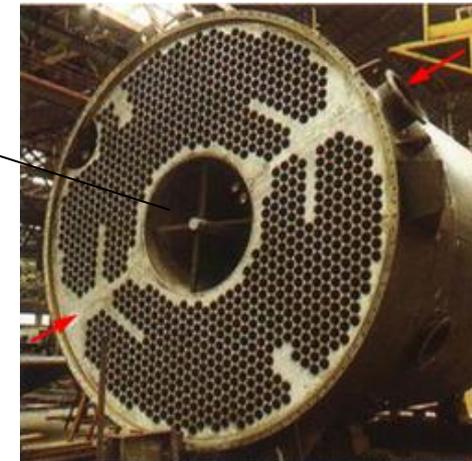


**Working conditions of the tubular circulatory evaporator**  
 At the central circulation pipe acts an external overpressure  $(0.4 - 0.3) = 0.1$  MPa

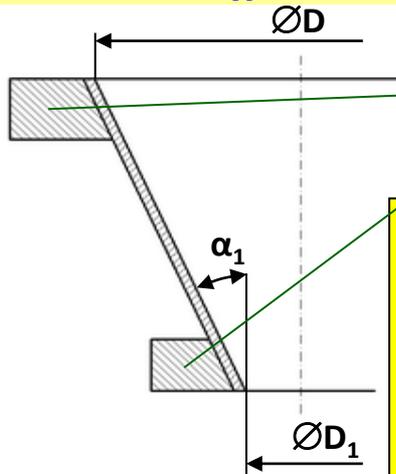
## Pressure test of the evaporator

The heating chamber is tested to 0.5 MPa, in other spaces is the atmospheric pressure → at the central circulation pipe acts an external overpressure 0.5 MPa → 5x > → danger of the CCT buckling.

The liquid (vapor) space is tested to the overpressure 0,5 MPa too → at the CCT and tubes acts the internal pressure 0,5 MPa



Kuželová skořepina s výztužnými prstenci  
*Conical shell stiffened with rings (flanges)*

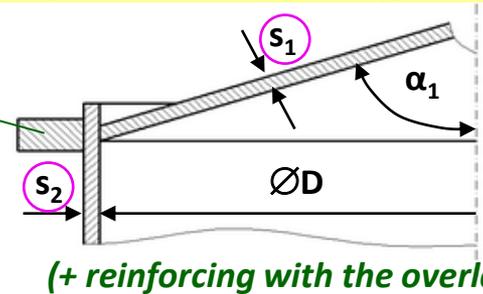


reinforcing rings or flanges that serve as the reinforcing

Kuželové skořepiny:  
 příklady řešení dle ČSN 690010

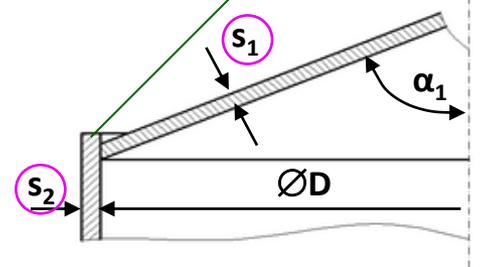
***Conical shells:***  
*some examples of their design*  
 according Czech Standards

Dno s ostrým přechodem s výztužným prstencem  
*Bottom with sharp transition and stiffening ring*

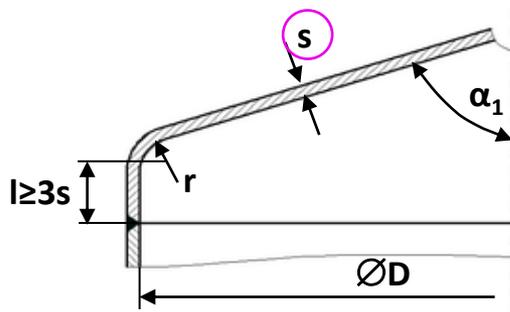


(+ reinforcing with the overlap)

Dno s ostrým přechodem bez vyztužení  
*Bottom with sharp transition without stiffening (reinforcing with the overlap)*



Dno s anuloidovým přechodem  
*Bottom with toroidal transition*



Plochá kuželová dna  
*Flat conical bottoms*

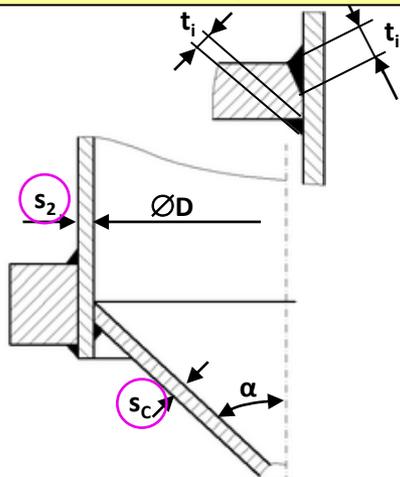
Výpočtové parametry:

Výpočtové délky přechodových částí se určují ze vzorců uvedených v ČSN 690010 pro příslušné tvary skořepin

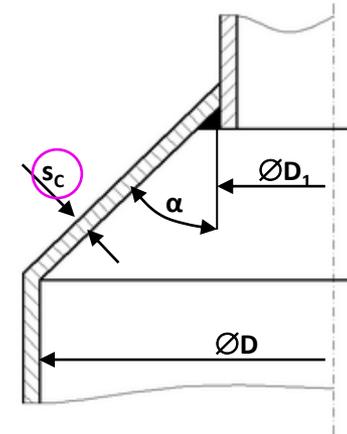
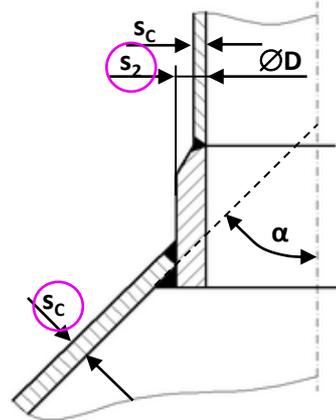
Calculated parameters:

Calculated lengths, thicknesses etc. for various shell shape are in the Czech Standard (calculation procedures are there)

**Spojení kuželové a válcové skořepiny s výztužným prstencem**  
*Joint of conical and cylindrical shells with stiffening ring*

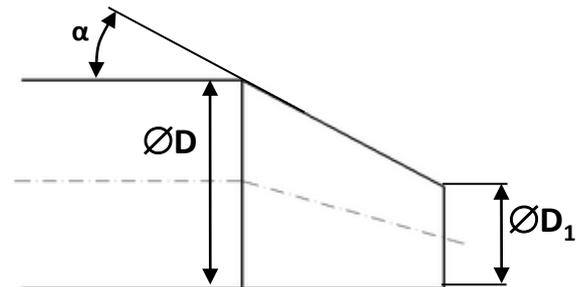
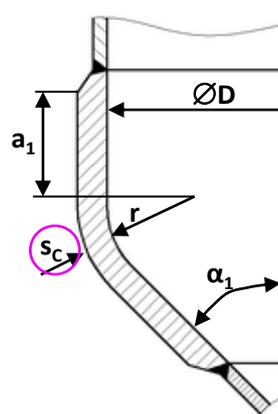
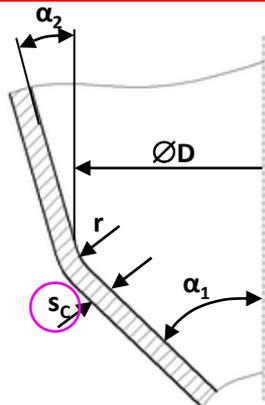


**Základní rozměry kuželového přechodu**  
*Basic dimensions of conical transition*



**Examples of designs that are solved in the Czech Standard**

**Spojení kuželové skořepiny s válcovou skořepinou menšího průměru**  
*Joint of conical shell with cylindrical one of smaller diameter*



**Spojení dvou kuželových skořepin**  
*Joint of two conical shells*

**Spojení s nesymetrickou kuželovou skořepinou**  
*Joint with non-symmetrical conical shell*

**Spojení kuželové a válcové skořepiny**  
*Joint of conical and cylindrical shells*

# Conical parts of vessels

Calculated cone diameter of smooth conical shell

$$D_C = D - 1.4 * a_1 * \sin \alpha_1$$

where is

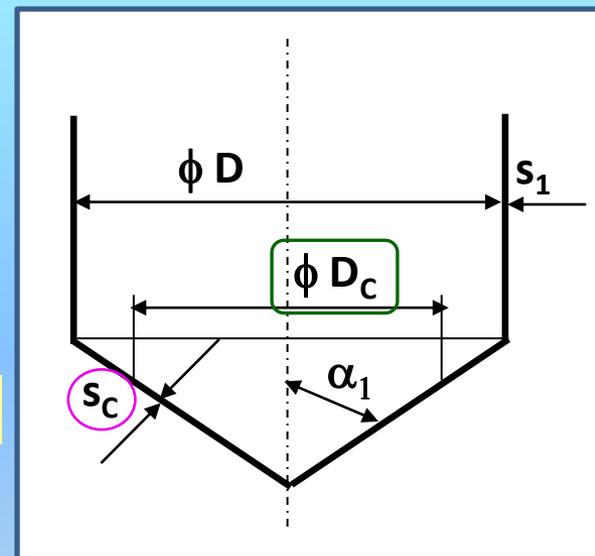
$$a_1 = 0.7 * \sqrt{\frac{D}{\cos \alpha_1} * (s_1 - c)}$$

(remember the reach of a stress peak)

$$L_K = 1.65 * \sqrt{D * s}$$

These calculations are valid for

$$0.001 \leq (s_1 * \cos \alpha_1 / D) \leq 0.05$$



Internal overpressure (calculations are valid for  $\alpha_1 \leq 70^\circ$ )

Calculated wall thickness of conical shell is

$$s_{CR} = \frac{p * D_C}{2 * \sigma_D * \varphi_P - p} * \frac{1}{\cos \alpha_1} = s'_{cyl} * \frac{1}{\cos \alpha_1}$$

Limit states:

$\alpha = 0^\circ$

$\alpha = 90^\circ$

What are results?

$$s_C \geq s_{CR} + c$$

realized wall thickness

Note:

For  $\alpha_1 = 0^\circ$  is  $\cos \alpha_1 = 1 \rightarrow s_{Cone} = s_{cylinder}$   
 $\alpha_1 = 90^\circ$  is  $\cos \alpha_1 = 0 \rightarrow s_{Cone} = \rightarrow \infty$

Therefore is the equation valid only for  $\alpha_1 \leq 70^\circ$

$\rightarrow$  we have to use calculations valid for plates

	$\alpha_1$	$D_C$	$s_{CR}$
cylinder	0	1000	2.73
	45	957	3.70
	60	937	5.13
	70	917	7.38
plate	90	0	$\infty$

Data for the table calculation:

$D = 1000$  mm;  $p_i = 0.6$  MPa;  
 $\sigma_D = 110$  MPa;  $\varphi_p = 1$

x  $s_{plate} = 37$  mm

Allowable internal overpressure for a conical shell with the wall thickness  $\underline{s}$

$$p_{cyl} = \frac{2 * \sigma_D * \varphi_p * (s - c)}{D + (s - c)}$$

$$p_{Dcone} = \frac{2 * \sigma_D * (s_C - c)}{\frac{D_C}{\cos \alpha_1} + (s_C - c)}$$

# External overpressure

Calculations are again valid for  $\alpha_1 \leq 70^\circ$ .

In the 1<sup>st</sup> iteration is the wall thickness determined as a wall thickness for cylinder multiplied by  $1/\cos \alpha_1$  (see above calculation for internal pressure).

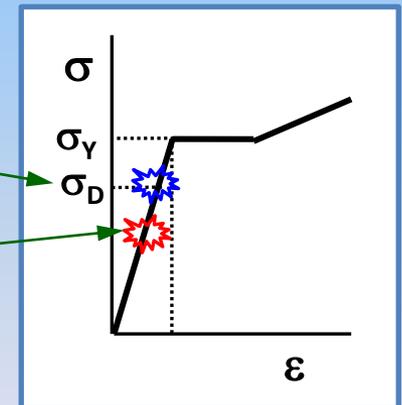
$$S_{CR1it} = S_R / \cos \alpha_1 \quad (\text{check of max. allowable external pressure} \rightarrow \text{wall is usually oversized} \rightarrow \text{new estimation} \rightarrow \text{iterative method})$$

Allowable external overpressure for this  $s_{c1it}$  is checked from formula (analogous to the previous + see exercises)  $\rightarrow s_{CR2it} \rightarrow$  new  $[p]$  etc.

$$[p] = \frac{[p]_P}{\sqrt{1 + \left(\frac{[p]_P}{[p]_E}\right)^2}}$$

stress point of view  
(danger of plastic def.)

stability point of view  
(danger of def. in elastic region)



In the ČSN or ES are all needed formulas for the calculation.

Analogous there are in the standard calculations for a shell created from 2 cones with various angles with or without toroidal transition or with and without stiffening rings etc. – examples see previous pages 16 and 17.

# Reinforcement of openings (holes) according ČSN 690010, part 4.12

It is important for big openings (holes, necks).

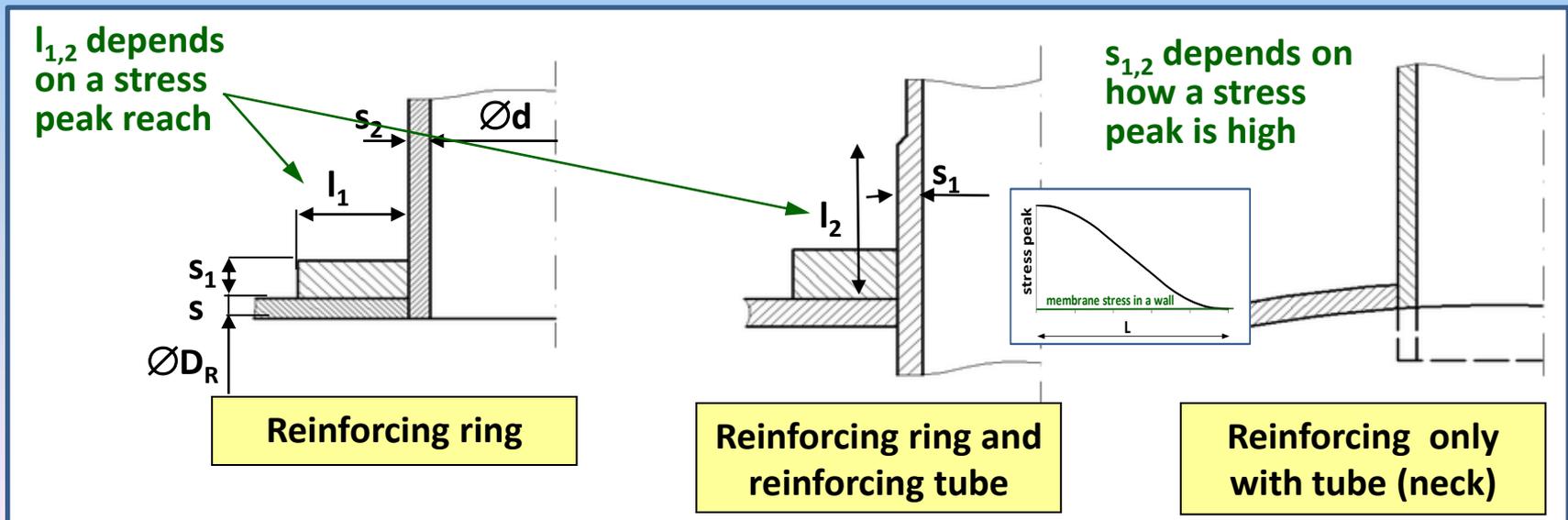
Ways of the reinforcement are:

Holes are the shell wall weakening = there is no material in the place → the load is transferred to the surrounding material → we must check if this part of a shell withstands a given loading

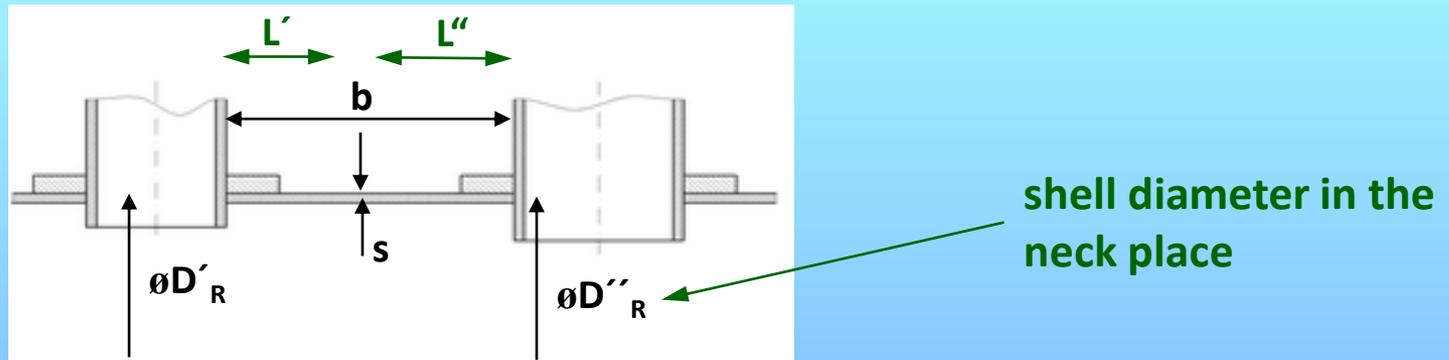
1. Reinforcement with rings
2. Reinforcement with tube or thicker tube – tubular reinforcing
3. Combination

For small openings reinforcement with tube (neck) is usually sufficient.

Examples of holes (necks) reinforcement:



# Interaction of two or more near openings



Openings are considered to be isolated if is their distance

$$b \geq \sqrt{D_R' * (s - c)} + \sqrt{D_D'' * (s - c)}$$

$\leftarrow L'$ 
 $\leftarrow L''$

→ stress peaks must not interact

For  $b <$  than the value we have to check allowable overpressure for a “bridge” between holes. → thicker wall must be there

Reach of a stress peak

$$L_{theor} = 0.55 * \sqrt{D * s} \quad L_{pract} = 1.65 * \sqrt{D * s}$$

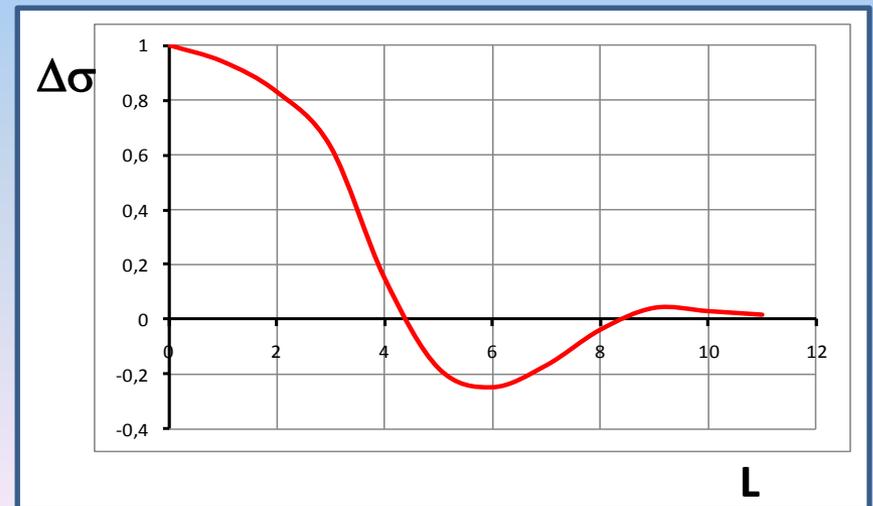
# Specification of reinforcing elements size:

- A thickness of reinforcing tubes or rings depends on a vessel wall thickness, diameter  $D$  and pressure  $p \rightarrow$  stress peak size.
- A length of the tube reinforcing or a width of the reinforcing ring depends on a reach of the stress peaks due to the hole.

Roughly speaking it is:

$$L \sim \sqrt{D * s}$$

theoretically x 0.55  
practically x 1.65



**Example:**

**Cylindrical shell** ( $D = 1000 \text{ mm}$ ;  $s_2 - c = 5 \text{ mm}$ ;  $p_i = 0.4 \text{ MPa}$ ;  $c = 1 \text{ mm}$ ;  $\sigma_D = 130 \text{ MPa}$ ) in which we want to install these necks:

allowance for corrosion etc.

**• What necks (made from tube DN xxx) have to be reinforced?**

Tube	tube nominal pressure (bar)	tube ext. diameter	tube wall thickness	minimal tube thickness (for hole reinforcing)	minimal required tube thickness (hole reinforcing)	Note
				$S_1 - c$	$S_1$	
DN 100; PN 6		$\phi 108$	4	1.5 mm	2.5 mm	→ no reinforcement
DN 300; PN 6		$\phi 324$	4	2.7 mm	3.7 mm	→ no reinforcement
DN 400; PN 10		$\phi 426$	5	4.2 mm	5.2 mm	→ ? reinforcement
DN 400; PN 16		$\phi 426$	6			→ OK (for SS $c \approx 0 \rightarrow \text{OK}$ )
DN 500; PN 6		$\phi 530$	5	6.0 mm	7.0 mm	→ reinforcement is needed
DN 500; PN 16		$\phi 530$	7			

*without allowance for corrosion*      *with allowance for corrosion*

**• What is a minimal distance between 2 necks (without mutual interaction)?**

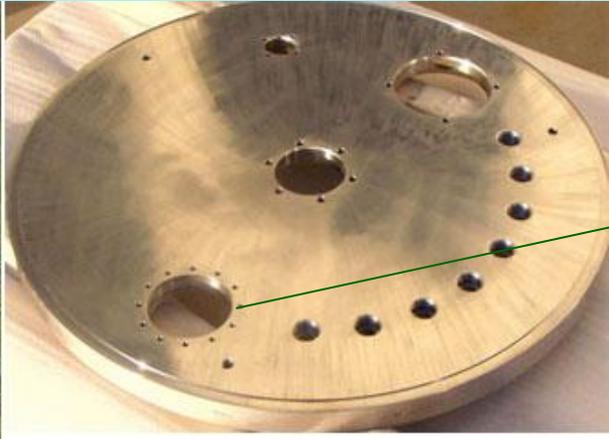
$$b \geq \sqrt{1000 * 5} + \sqrt{1000 * 5} = 141 \text{ mm}$$

# Příklady tlakových nádob

# Examples of pressure vessels



sight glass  
(level indicator)



flat cover  
with flanges  
connections

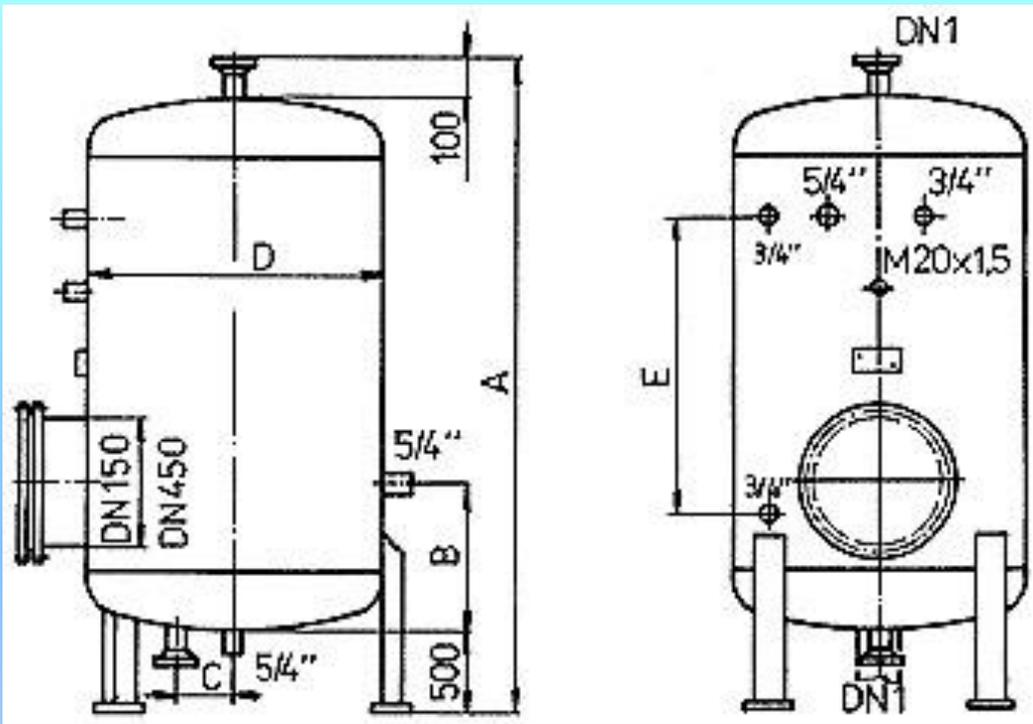
high pressure vessel  
with flat cover



For what purpose this pressure vessel is used?  
And what fluids pass through it?

vaulted  
cover

saddle  
support



stirrer drive

condenser

Size	Type 1	Type 2	etc.
A			
B			
C			
D			
E			



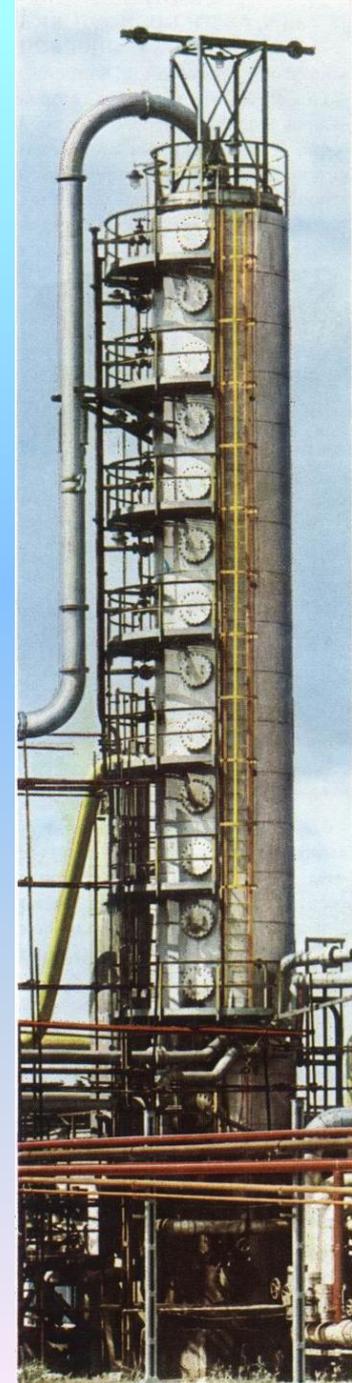
glass tube (level indicator)

Sketch of a pressure vessel that is produced in several sizes (manhole, necks for treated material and e.g. for thermometer, pressure gauge, level control ...)

check if it is necessary to reinforce these places



**A chemical reactor with heat exchanger**

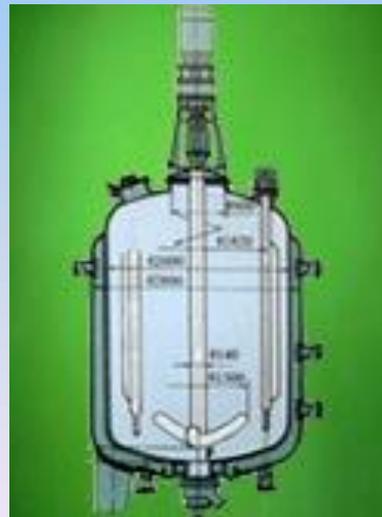




## Cryogenic vessels for LN2 storage (-196 °C)

The vessel has 2 shells and inside is vacuum + thermal insulation wound on the internal shell (layers from special paper + shiny Al foil) → <<< heat loss due to convection, conduction and radiation





[http://www.tradenote.net/reactor\\_2/](http://www.tradenote.net/reactor_2/)