

3.13 Solutions of exercises

Solution of Exercise 3.3.2

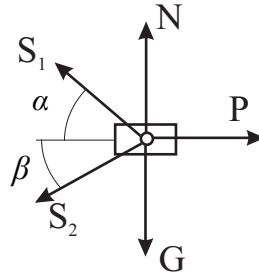


Figure 3.86: Exercise 3.3.2. Free-body diagram

Forces in springs:

$$S_1 = k_1(\sqrt{l_{01}^2 + x^2} - l_{01})$$

$$S_2 = k_2(\sqrt{l_{02}^2 + x^2} - l_{02})$$

Geometry:

$$\sin \alpha = \frac{l_{01}}{\sqrt{l_{01}^2 + x^2}}, \quad \cos \alpha = \frac{x}{\sqrt{l_{01}^2 + x^2}}$$

$$\sin \beta = \frac{l_{02}}{\sqrt{l_{02}^2 + x^2}}, \quad \cos \beta = \frac{x}{\sqrt{l_{02}^2 + x^2}}$$

Equations of equilibrium:

$$P - S_1 \cos \alpha - S_2 \cos \beta = 0$$

$$N + S_1 \sin \alpha - G - S_2 \sin \beta = 0$$

Solution:

$$P = S_1 \cos \alpha + S_2 \cos \beta$$

Result:

$$P = 77.6 \text{ N}$$

Notice: The second equation of equilibrium is not necessary for finding the force P . It can be used for determination of the reaction force N .

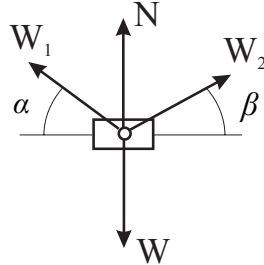
Solution of Exercise 3.3.3

Figure 3.87: Exercise 3.3.3. Free-body diagram

Geometry:

$$\sin \alpha = \frac{h}{\sqrt{h^2 + x^2}}, \quad \cos \alpha = \frac{x}{\sqrt{h^2 + x^2}}$$

$$\sin \beta = \frac{h}{\sqrt{(l-x)^2 + h^2}}, \quad \cos \beta = \frac{l-x}{\sqrt{(l-x)^2 + h^2}}$$

Equations of equilibrium:

$$W_2 \cos \beta - W_1 \cos \alpha = 0$$

$$N + W_1 \sin \alpha + W_2 \sin \beta - W = 0$$

Solution:

$$W_2 \frac{l-x}{\sqrt{(l-x)^2 + h^2}} - W_1 \frac{x}{\sqrt{h^2 + x^2}} = 0 \quad \Rightarrow x_{eq}$$

Notice: See Matlab file s213.m for numerical solution.

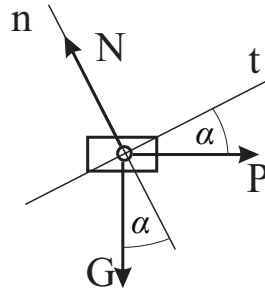
Solution of Exercise 3.3.4

Figure 3.88: Exercise 3.3.4. Free-body diagram

Geometry:

$$y = \frac{b}{a^2} x^2$$

$$y' = \tan \alpha = \frac{2b}{a^2} x$$

Equations of equilibrium:

$$P \cos \alpha - G \sin \alpha = 0$$

$$N - G \cos \alpha - P \sin \alpha = 0$$

From equation of equilibrium we have:

$$\tan \alpha = \frac{P}{G}$$

Solution:

$$x_{eq} = \frac{a^2 P}{2b G}$$

Result:

$$x_{eq} = 0.036 \text{ m}$$

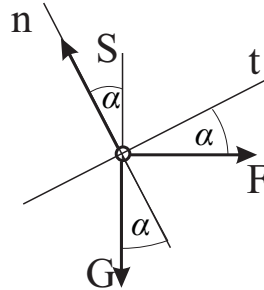
Solution of Exercise 3.3.5

Figure 3.89: Exercise 3.3.5. Free-body diagram

Geometry:

$$\tan \alpha = \frac{x_{eq}}{\sqrt{l^2 - x_{eq}^2}}$$

Equations of equilibrium:

$$F \cos \alpha - G \sin \alpha = 0$$

$$S - G \cos \alpha - F \sin \alpha = 0$$

Solution:

$$\tan \alpha = \frac{x_{eq}}{\sqrt{l^2 - x_{eq}^2}} = \frac{F}{G} \quad \rightarrow x_{eq}$$

Notice: See Matlab file S216.m for numerical solution.

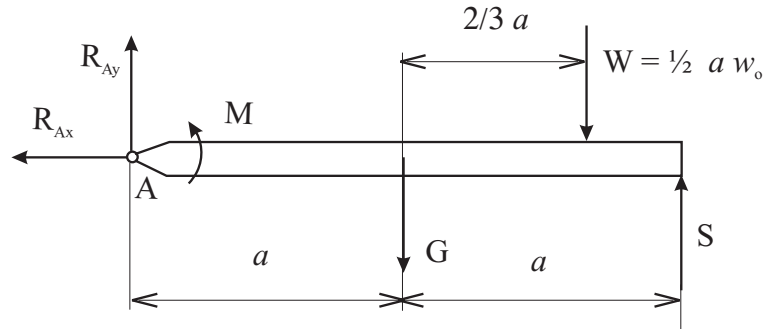
Solution of Exercise 3.4.2

Figure 3.90: Exercise 3.4.2. Free-body diagram

Equations of equilibrium:

$$\begin{aligned} \sum F_{ix} : \quad R_{Ax} &= 0 \\ \sum F_{iy} : \quad R_{Ay} + S - W - G &= 0 \\ \sum M_{iA} : \quad -G a - W \frac{5}{3} a + S 2 a + M &= 0 \end{aligned}$$

Solution:

$$\begin{aligned} S &= \frac{1}{2} G + \frac{5}{6} W - \frac{M}{2a} \\ l_0 &= 0.5 a + \xi = 0.5 a + \frac{S}{k} \\ R_A &= R_{Ay} = G + W - S \end{aligned}$$

Result:

$$\begin{aligned} S &= 603.4 \text{ N} \\ l_0 &= 0.112 \text{ m} \\ R_A &= 396.6 \text{ N} \end{aligned}$$

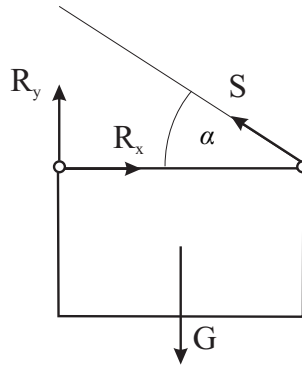
Solution of Exercise 3.4.3

Figure 3.91: Exercise 3.4.3. Free-body diagram

Geometry:

$$\sin \alpha = \frac{a}{\sqrt{4a^2 + a^2}} = \frac{1}{\sqrt{5}}$$

Equations of equilibrium:

$$\begin{aligned} \sum F_{ix} : \quad R_x - S \cos \alpha &= 0 \\ \sum F_{iy} : \quad R_y + S \sin \alpha - G &= 0 \\ \sum M_{iA} : \quad S \sin \alpha \cdot 2a - G a &= 0 \end{aligned}$$

Solution:

$$S = \frac{G}{2 \frac{1}{\sqrt{5}}} = \frac{\sqrt{5}}{2} G$$

$$k = \frac{S}{\xi} = \frac{S}{a\sqrt{5} - 2a} = \frac{167.7}{0.15\sqrt{5} - 0.3}$$

Result:

$$S = 167.7 \text{ N}$$

$$k = 4737 \text{ Nm}^{-1}$$

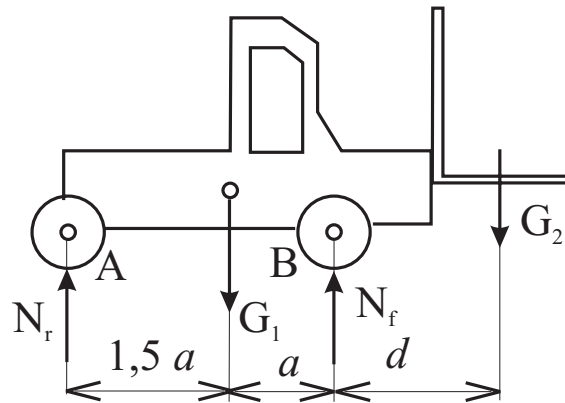
Solution of Exercise 3.4.4

Figure 3.92: Exercise 3.4.4. Free-body diagram

Equations of equilibrium:

$$\begin{aligned} \sum M_{iA} : \quad & N_f 2.5 a - G_1 1.5 a - G_2 (2.5 a + d) = 0 \\ \sum M_{iB} : \quad & G_1 a - N_r 2.5 a - G_2 d = 0 \end{aligned}$$

Solution:

$$\begin{aligned} N_f &= \frac{1.5 G_1 a + (2.5 a + d) G_2}{2.5 a} \\ N_r &= \frac{G_1 a + G_2 d}{2.5 a} \end{aligned}$$

Result:

$$N_f = 6700 \text{ N}$$

$$N_r = 300 \text{ N}$$

Solution of Exercise 3.4.5

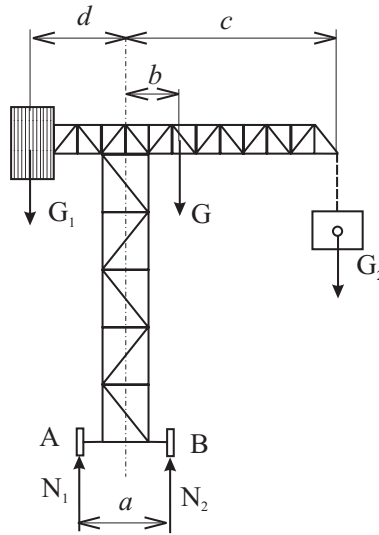


Figure 3.93: Exercise 3.4.5. Free-body diagram

a) Minimum counterweight $G_{1\min}$ for the crane not to lose its stability with $G_2 = 0$. We suppose $N_1 = 0$.

Equations of equilibrium:

$$\sum M_{iB} : G_{1\min} \left(d + \frac{a}{2} \right) - G \left(b - \frac{a}{2} \right) = 0$$

Solution:

$$G_{1\min} = \frac{b - \frac{a}{2}}{d + \frac{a}{2}} G = 1538 \text{ N}$$

b) Maximum weight $G_{2\max} = 0$ for the crane not to loose its stability with $G_{1\max}$.

First we determine $G_{1\max}$ from the condition $N_2 = 0$, $G_2 = 0$. Equations of equilibrium:

$$\sum M_{iA} : G_{1\max} \left(d - \frac{a}{2} \right) - G \left(b + \frac{a}{2} \right) = 0$$

$$G_{1\max} = \frac{b + \frac{a}{2}}{d - \frac{a}{2}} G = 11428 \text{ N}$$

Now we find $G_{2\max}$ supposing $N_1 = 0$.

Equations of equilibrium:

$$\sum M_{iB} : G_{1\max} \left(d + \frac{a}{2} \right) - G \left(b - \frac{a}{2} \right) - G_{2\max} \left(c - \frac{a}{2} \right) = 0$$

Solution:

$$G_{2\max} = \frac{G_{1\max} \left(d + \frac{a}{2}\right) - G \left(b - \frac{a}{2}\right)}{c - \frac{a}{2}} = 3475 \text{ N}$$

Solution of Exercise 3.5.2

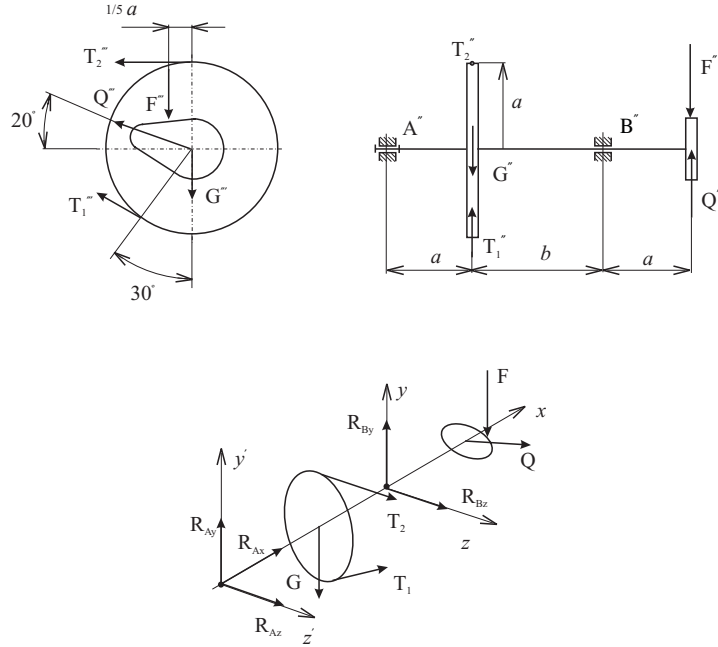


Figure 3.94: Exercise 3.5.2. Free-body diagram

Equations of equilibrium in a scalar form:

$$\begin{aligned}
 \sum F_{ix} : & R_{Ax} = 0 \\
 \sum M_{ix} : & (T_2 - T_1) a + F 0.2 a = 0 \\
 \sum M_{iy} : & T_2 b + R_{Az} (a + b) + T_1 \cos 30^\circ b - Q a \cos 20^\circ = 0 \\
 \sum M_{iz} : & -F a + Q a \sin 20^\circ - R_{Ay} (a + b) - T_1 b \sin 30^\circ + G_1 b = 0 \\
 \sum M_{iy'} : & -T_2 a - T_1 a \cos 30^\circ - R_{Bz} (a + b) - Q (a + b + a) \cos 20^\circ = 0 \\
 \sum M_{iz'} : & -G_1 a + T_1 a \sin 30^\circ + R_{By} (a + b) - F (a + b + a) + Q (a + b + a) \sin 20^\circ = 0
 \end{aligned}$$

Equation of equilibrium in matrix form:

$$\begin{bmatrix}
 1 & -5 & 0 & 0 & 0 & 0 \\
 -a & a & 0 & 0 & 0 & 0 \\
 b \cos 30^\circ & b & 0 & a + b & 0 & 0 \\
 -b \sin 30^\circ & 0 & -a - b & 0 & 0 & 0 \\
 -a \cos 30^\circ & -a & 0 & 0 & 0 & -a - b \\
 a \sin 30^\circ & 0 & 0 & 0 & a + b & 0
 \end{bmatrix}
 \begin{bmatrix}
 T_1 \\
 T_2 \\
 R_{Ay} \\
 R_{Az} \\
 R_{By} \\
 R_{Bz}
 \end{bmatrix}
 =$$

$$\begin{bmatrix} 0 \\ -F 0.2 a \\ Q a \cos 20^\circ \\ F a - Q a \sin 20^\circ - G_1 b \\ Q (a + b + a) \cos 20^\circ \\ G_1 a + F (a + b + a) - Q (a + b + a) \sin 20^\circ \end{bmatrix}$$

Result:

$$R_{Ax} = 0 \text{ N}, R_{Ay} = -946.2 \text{ N}, R_{Az} = -860 \text{ N}, R_A = 12786 \text{ N}, R_{By} = 4349.4 \text{ N}, \\ R_{Bz} = -581.9 \text{ N}, R_B = 43881 \text{ N}, T_1 = 10000 \text{ N}, T_2 = 2000 \text{ N}$$

Notice: See Matlab file s3415.m for numerical solution.

Solution of Exercise 3.5.3

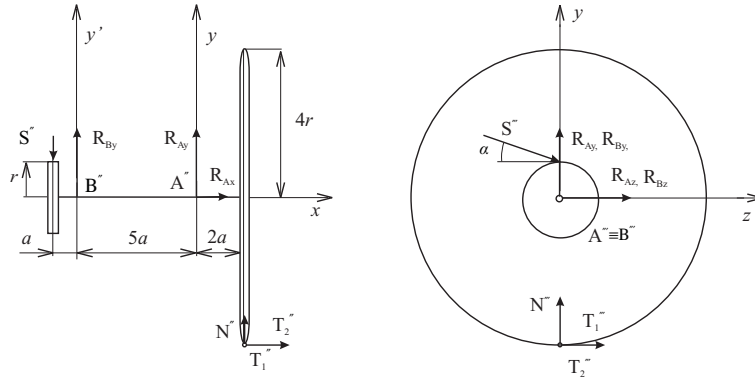


Figure 3.95: Exercise 3.5.3. Free-body diagram

Equations of equilibrium in scalar form:

$$\begin{aligned}
 \sum F_{ix} : & R_{Ax} + T_2 = 0 \\
 \sum M_{ix} : & S \cos \alpha r - T_1 4r = 0 \\
 \sum M_{iy} : & R_{Bz} 5a - T_1 2a + S \cos \alpha 6a = 0 \\
 \sum M_{iz} : & -R_{By} 5a + S \sin \alpha 6a + N 2a + T_2 4r = 0 \\
 \sum M_{iy'} : & -R_{Az} 5a + S \cos \alpha a - T_1 7a = 0 \\
 \sum M_{iz'} : & R_{Ay} 5a + S \sin \alpha a + N 7a + T_2 4r = 0
 \end{aligned}$$

Equation of equilibrium in matrix form:

$$\begin{bmatrix}
 1 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & \cos \alpha \\
 0 & 0 & 0 & 0 & 5 & 6 \cos \alpha \\
 0 & 0 & 0 & -5 & 0 & 6 \sin \alpha \\
 0 & 0 & -5 & 0 & 0 & \cos \alpha \\
 0 & 5 & 0 & 0 & 0 & \sin \alpha
 \end{bmatrix}
 \begin{bmatrix}
 R_{Ax} \\
 R_{Ay} \\
 R_{Az} \\
 R_{By} \\
 R_{Bz} \\
 S
 \end{bmatrix}
 =
 \begin{bmatrix}
 -T_2 \\
 4T_1 \\
 2T_1 \\
 -2N - 4.08T_2 \\
 7T_1 \\
 -7N - 4.08T_2
 \end{bmatrix}$$

Notice: See Matlab file S3421.m for numerical solution.

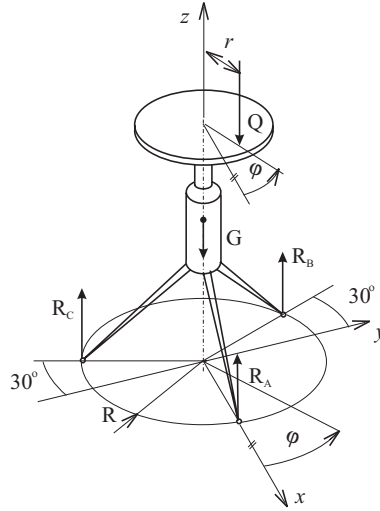
Solution of Exercise 3.5.4

Figure 3.96: Exercise 3.5.4. Free-body diagram

Equations of equilibrium:

$$\begin{aligned} \sum F_{iz} : & \quad -Q - G + R_A + R_B + R_C = 0 \\ \sum M_{ix} : & \quad -Q r \sin \varphi + R_B R \cos 30^\circ - R_C R \cos 30^\circ = 0 \\ \sum M_{iy} : & \quad Q r \cos \varphi - R_A R + R_B R \sin 30^\circ + R_C R \sin 30^\circ = 0 \end{aligned}$$

Solution:

$$\begin{aligned} R_A &= Q + G - (R_B + R_C) \\ R_B - R_C &= \frac{Q r \sin \varphi}{R \cos 30^\circ} = f_1(\varphi) \\ R_B + R_C &= \frac{(Q + G) R - Q r \cos \varphi}{R (1 + \sin 30^\circ)} = f_2(\varphi) \\ R_B &= \frac{f_1(\varphi) + f_2(\varphi)}{2} \\ R_C &= \frac{f_2(\varphi) - f_1(\varphi)}{2} \end{aligned}$$

Notice: See Matlab file s3423.m for numerical solution.

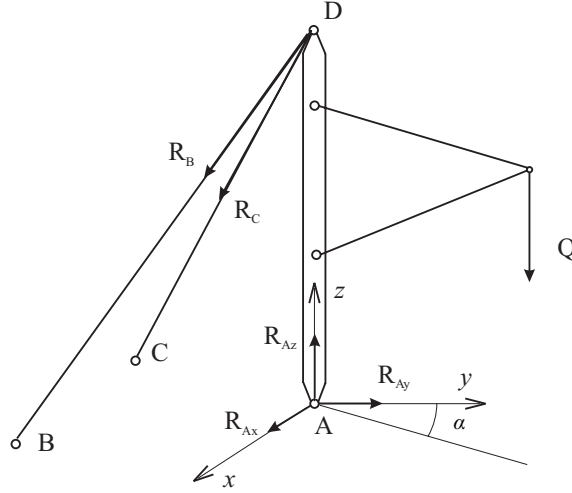
Solution of Exercise 3.5.5

Figure 3.97: Exercise 3.5.5. Free-body diagram

Equations of equilibrium:

$$\begin{aligned}
 \sum F_{ix} : & R_{Ax} + R_{Bx} + R_{Cx} = 0 \\
 \sum F_{iy} : & R_{Ay} + R_{By} + R_{Cy} = 0 \\
 \sum F_{iz} : & -Q + R_{Az} + R_{Bz} + R_{Cz} = 0 \\
 \sum M_{ix} : & -3aQ \cos \alpha - 5aR_{By} - 5aR_{Cy} = 0 \\
 \sum M_{iy} : & 3aQ \sin \alpha + 5aR_{Bx} + 5aR_{Cx} = 0
 \end{aligned}$$

Geometry:

$$\begin{aligned}
 \cos \alpha_B &= \frac{x_B - x_D}{BD}, & \cos \beta_B &= \frac{y_B - y_D}{BD}, & \cos \gamma_B &= \frac{z_B - z_D}{BD} \\
 \cos \alpha_C &= \frac{x_C - x_D}{CD}, & \cos \beta_C &= \frac{y_C - y_D}{CD}, & \cos \gamma_C &= \frac{z_C - z_D}{CD}
 \end{aligned}$$

Components of reaction forces:

$$\begin{aligned}
 R_{Bx} &= R_B \cos \alpha_B, & R_{By} &= R_B \cos \beta_B, & R_{Bz} &= R_B \cos \gamma_B \\
 R_{Cx} &= R_C \cos \alpha_C, & R_{Cy} &= R_C \cos \beta_C, & R_{Cz} &= R_C \cos \gamma_C
 \end{aligned}$$

Equations of equilibrium in matrix form:

$$\begin{bmatrix}
 1 & 0 & 0 & \cos \alpha_B & \cos \alpha_C \\
 0 & 1 & 0 & \cos \beta_B & \cos \beta_C \\
 0 & 0 & 1 & \cos \gamma_B & \cos \gamma_C \\
 0 & 0 & 0 & -5 \cos \beta_B & -5 \cos \beta_C \\
 0 & 0 & 0 & 5 \cos \alpha_B & 5 \cos \alpha_C
 \end{bmatrix}
 \begin{bmatrix}
 R_{Ax} \\
 R_{Ay} \\
 R_{Az} \\
 R_B \\
 R_C
 \end{bmatrix}
 =
 \begin{bmatrix}
 0 \\
 0 \\
 Q \\
 -3Q \cos \alpha \\
 -3Q \sin \alpha
 \end{bmatrix}$$

Notice: See Matlab file S3410.m for numerical solution.

Solution of Exercise 3.5.6

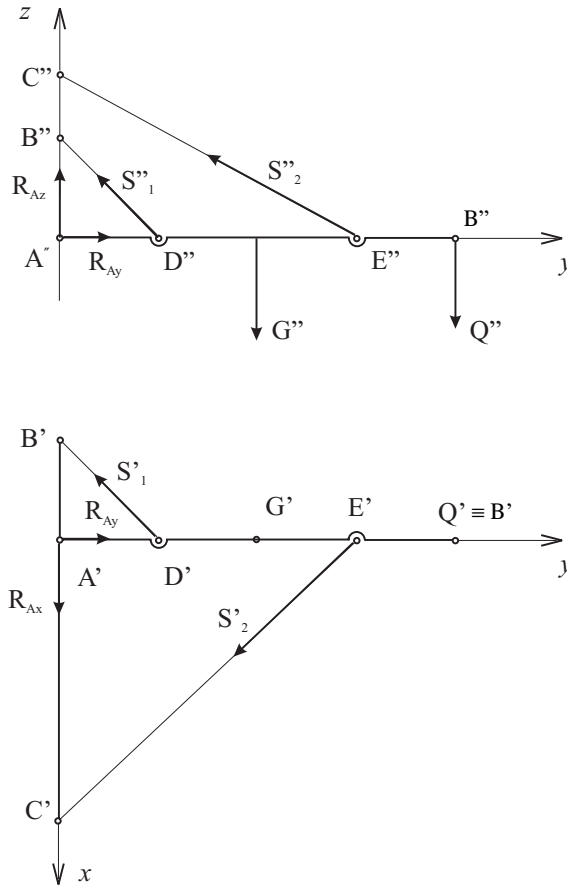


Figure 3.98: Exercise 3.5.6. Free-body diagram

Equations of equilibrium:

$$\begin{aligned}
 \sum F_{ix} : & \quad R_{Ax} + S_{1x} + S_{2x} = 0 \\
 \sum F_{iz} : & \quad R_{Ay} + S_{1y} + S_{2y} = 0 \\
 \sum F_{iy} : & \quad R_{Az} + S_{1z} + S_{2z} - Q - G = 0 \\
 \sum M_{ix} : & \quad -4aQ + aS_{1z} + 3aS_{2z} - 2aG = 0 \\
 \sum M_{iz} : & \quad -aS_{1x} - 3aS_{2x} = 0
 \end{aligned}$$

Geometry:

$$\begin{aligned}
 \cos \alpha_{DB} &= \frac{x_B - x_D}{DB}, & \cos \beta_{DB} &= \frac{y_B - y_D}{DB}, & \cos \gamma_{DB} &= \frac{z_B - z_D}{DB} \\
 \cos \alpha_{EC} &= \frac{x_C - x_E}{EC}, & \cos \beta_{EC} &= \frac{y_C - y_E}{EC}, & \cos \gamma_{EC} &= \frac{z_C - z_E}{EC}
 \end{aligned}$$

Components of forces:

$$S_{1x} = S_1 \cos \alpha_{DB}, \quad S_{1y} = S_1 \cos \beta_{DB}, \quad S_{1z} = S_1 \cos \gamma_{DB}$$

$$S_{2x} = S_2 \cos \alpha_{EC}, \quad S_{2y} = S_2 \cos \beta_{EC}, \quad S_{2z} = S_2 \cos \gamma_{EC}$$

Equations of equilibrium in matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & \cos \alpha_{DB} & \cos \alpha_{EC} \\ 0 & 1 & 0 & \cos \beta_{DB} & \cos \beta_{EC} \\ 0 & 0 & 1 & \cos \gamma_{DB} & \cos \gamma_{EC} \\ 0 & 0 & 0 & \cos \gamma_{DB} & 3 \cos \gamma_{EC} \\ 0 & 0 & 0 & \cos \alpha_{DB} & 3 \cos \alpha_{EC} \end{bmatrix} \begin{bmatrix} R_{Ax} \\ R_{Ay} \\ R_{Az} \\ S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ G + Q \\ 2G + 4Q \\ 0 \end{bmatrix}$$

Notice: See Matlab file S3411.m for numerical solution.

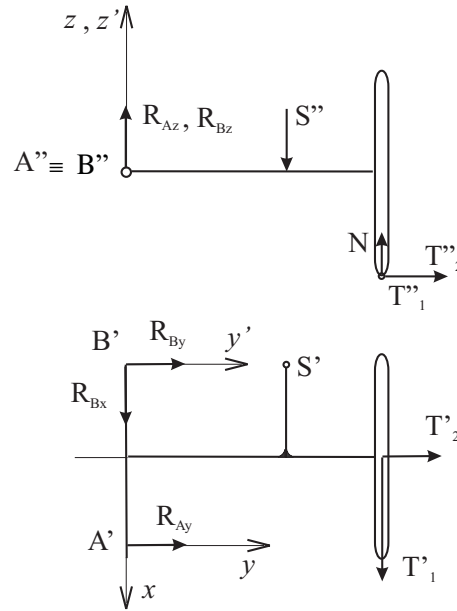
Solution of Exercise 3.5.7

Figure 3.99: Exercise 3.5.7. Free-body diagram

Equations of equilibrium:

$$\begin{aligned}
 \sum F_{ix} : & R_{Bx} + T_1 = 0 \\
 \sum M_{ix} : & -2aS + 3aN + rT_2 = 0 \\
 \sum M_{iy} : & aN - rT_1 - 2aS + 2aR_{Bz} = 0 \\
 \sum M_{iz} : & -3aT_1 - aT_2 - 2aR_{By} = 0 \\
 \sum M_{iy'} : & -aN - rT_1 - 2aR_{Az} = 0 \\
 \sum M_{iz'} : & aT_2 - 3aT_1 + 2aR_{Ay} = 0
 \end{aligned}$$

Equations of equilibrium in matrix form:

$$\begin{bmatrix}
 0 & 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 2 \\
 0 & 0 & 0 & 0 & 2 & -2 \\
 0 & 0 & 0 & -2 & 0 & 0 \\
 0 & -2 & 0 & 0 & 0 & 0 \\
 2 & 0 & 0 & 0 & 0 & 0
 \end{bmatrix}
 \begin{bmatrix}
 R_{Ay} \\
 R_{Az} \\
 R_{Bx} \\
 R_{By} \\
 R_{Bz} \\
 S
 \end{bmatrix}
 =
 \begin{bmatrix}
 -T_1 \\
 3N + 3.5T_2 \\
 3.5T_1 - N \\
 3T_1 + T_2 \\
 -N - 3.5T_1 \\
 3T_1 - T_2
 \end{bmatrix}$$

Notice: See Matlab file S3422.m for numerical solution.

Solution of Exercise 3.6.2

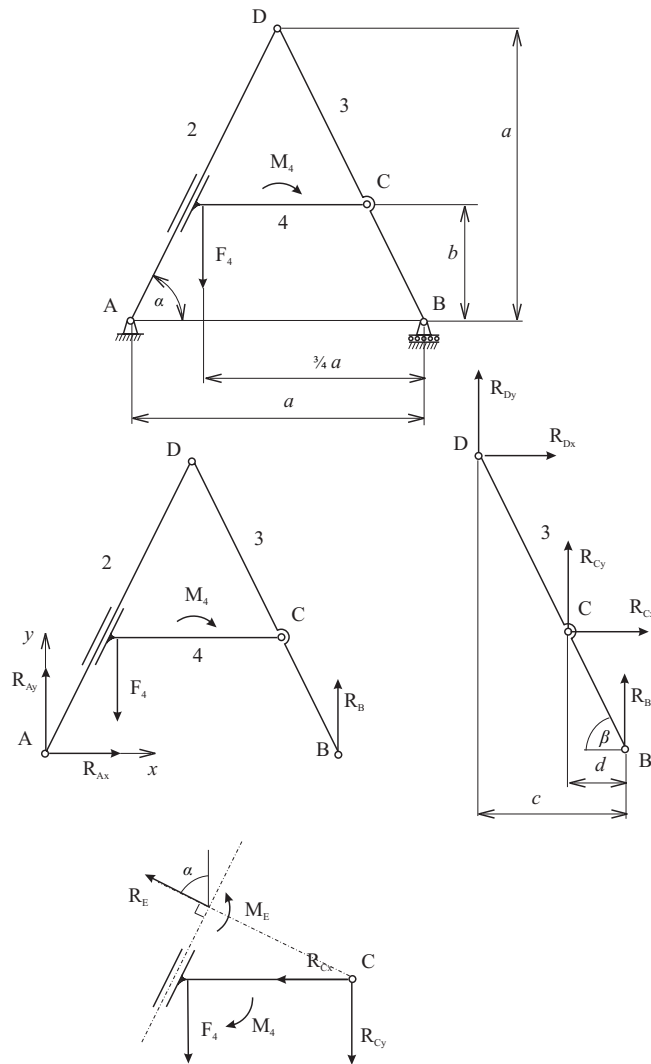


Figure 3.100: Exercise 3.6.2. Equilibrium of a structure

The structure has zero degree of freedom and it consists of three members. We sketch the free-body diagram for the structure as a whole, and for members 3, 4. Than, we write 3 equilibrium equations for each free structure or body. We have

Structure as a whole:

$$\begin{aligned}\sum F_{ix} : & R_{Ax} = 0 \\ \sum F_{iy} : & R_{Ay} + R_B - F_4 = 0 \\ \sum M_{iB} : & R_{Ay} \cdot a - F_4 \cdot \frac{3}{4}a + M_4 = 0\end{aligned}$$

Member 3:

$$\begin{aligned}\sum F_{ix} : & R_{Dx} + R_{Cx} = 0 \\ \sum F_{iy} : & R_{Dy} + R_{Cy} + R_B = 0 \\ \sum M_{iC} : & R_B d - R_{Dy} (c - d) - R_{Dx} (a - b) = 0\end{aligned}$$

Member 4:

$$\begin{aligned}\sum F_{ix} : & -R_{Cx} - R_E \sin \alpha = 0 \\ \sum F_{iy} : & R_E \cos \alpha - R_{Cy} - F_4 = 0 \\ \sum M_{iC} : & M_E + F_4 (3/4 a - d) - M_4 = 0\end{aligned}$$

Geometry yields

$$c = a - \frac{a}{\operatorname{tg} \alpha}, \quad d = \frac{b}{\operatorname{tg} \beta}, \quad \beta = \operatorname{arctg} \frac{a}{c}$$

Equation of equilibrium in matrix form:

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ a & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & d & 0 & 0 & b-a & d-c & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & \sin \alpha & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & \cos \alpha & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_{Ay} \\ R_B \\ R_{Cx} \\ R_{Cy} \\ R_{Dx} \\ R_{Dy} \\ R_E \\ M_E \end{bmatrix} = \begin{bmatrix} F_4 \\ \frac{3}{4}a F_4 - M_4 \\ 0 \\ 0 \\ 0 \\ 0 \\ F_4 \\ F_4 (d - \frac{3}{4}a) + M_4 \end{bmatrix}$$

Notice: See Matlab file SSB612.m for numerical solution.

Solution of Exercise 3.6.3

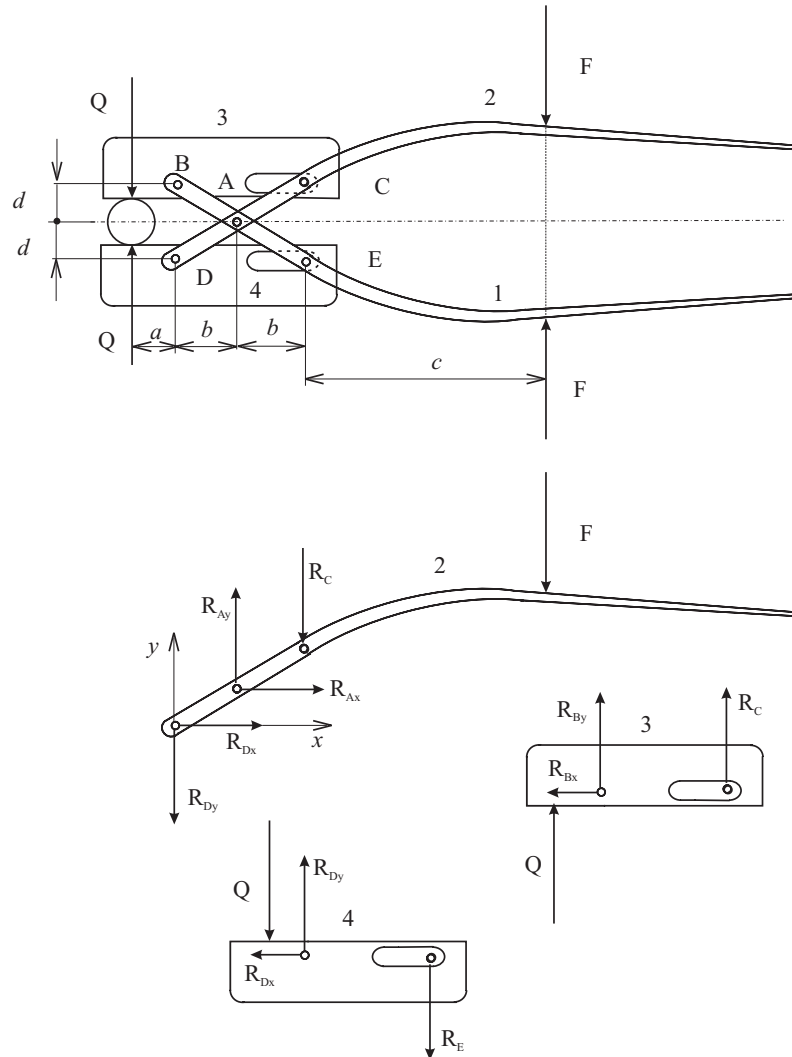


Figure 3.101: Exercise 3.6.3. Equilibrium of the pliers

We free bodies 2, 3, 4 and write the particular equations of equilibrium.
Member 2:

$$\begin{aligned} \sum F_{ix} : \quad & R_{Ax} + R_{Dx} = 0 \\ \sum F_{iy} : \quad & -R_{Dy} + R_{Ay} - R_C - F = 0 \\ \sum M_{iA} : \quad & R_{Dx} d + R_{Dy} b - R_C b - F(b + c) = 0 \end{aligned}$$

Member 3:

$$\begin{aligned}\sum F_{ix} : \quad R_{Bx} &= 0 \\ \sum F_{iy} : \quad Q + R_{By} + R_C &= 0 \\ \sum M_{iB} : \quad 2b R_C - Qa &= 0\end{aligned}$$

Member 4:

$$\begin{aligned}\sum F_{ix} : \quad R_{Dx} &= 0 \\ \sum F_{iy} : \quad -Q + R_{Dy} - R_E &= 0 \\ \sum M_{iD} : \quad Qa - 2b R_E &= 0\end{aligned}$$

Remark: You can see that it is not necessary to free the member 4. It is clear that $R_{Dx} = R_{Bx}$, $R_{Dy} = R_{By}$, $R_C = R_E$ from symmetry.

Using symmetry and excluding trivial scalar equations we have a system of only 6 equilibrium equation. These are in matrix form:

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & -1 \\ 0 & 0 & d & b & -b & -(b+c) \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 2b & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} R_{Ax} \\ R_{Ay} \\ R_{Dx} \\ R_{Dy} \\ R_C \\ F \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -Q \\ aQ \\ Q \end{bmatrix}$$

Notice: See Matlab file `SSB614.m` for numerical solution.

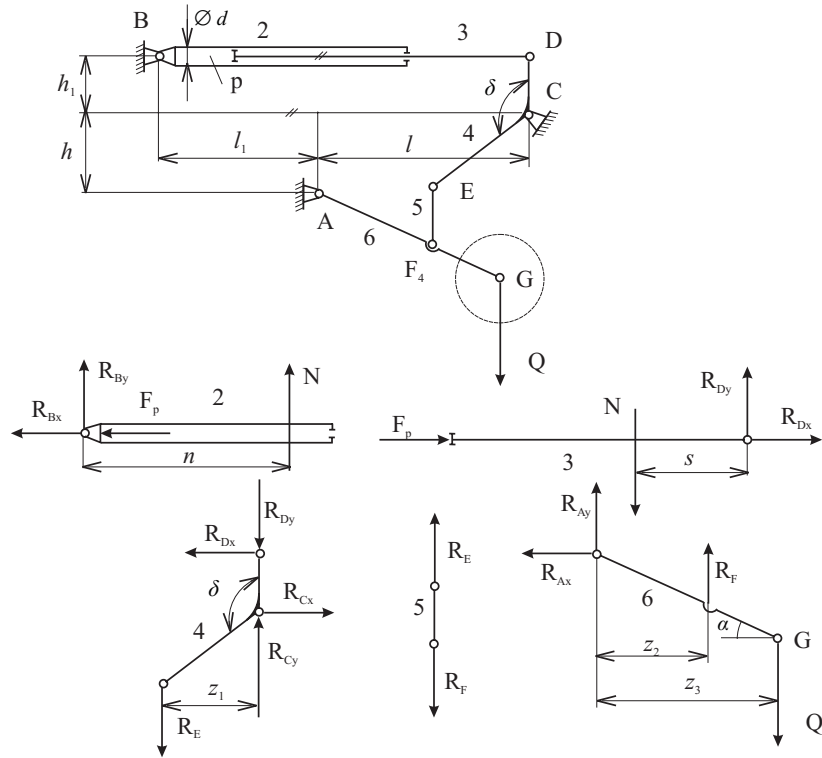
Solution of Exercise 3.6.4

Figure 3.102: Exercise 3.6.4. Equilibrium of the landing gear

First of all we express the geometrical dependencies.

$$z_1 = EC \cos \delta, \quad z_2 = AF \cos \alpha, \quad z_3 = AG \cos \alpha$$

Next we free bodies 2, 3, 4, 5, 6 and write the particular equations of equilibrium.

Member 2:

$$\begin{aligned} \sum F_{ix} : \quad & -R_{Bx} - F_p = 0 \\ \sum F_{iy} : \quad & R_{By} + N = 0 \\ \sum M_{iB} : \quad & N n = 0 \end{aligned}$$

Member 3:

$$\begin{aligned} \sum F_{ix} : \quad & F_p + R_{Dx} = 0 \\ \sum F_{iy} : \quad & -N + R_{Dy} = 0 \\ \sum M_{iD} : \quad & N n = 0 \end{aligned}$$

Member 4:

$$\begin{aligned}\sum F_{ix} : & -R_{Dx} + R_{Cx} = 0 \\ \sum F_{iy} : & -R_E - R_{Dy} + R_{Cy} = 0 \\ \sum M_{iC} : & R_{Dx} h_1 + R_E z_1 = 0\end{aligned}$$

Member 5:

$$\sum F_{iy} : R_E - R_F = 0$$

Member 6:

$$\begin{aligned}\sum F_{ix} : & -R_{Ax} = 0 \\ \sum F_{iy} : & R_{Ay} + R_F - Q = 0 \\ \sum M_{iA} : & R_F - z_2 - Q z_3 = 0\end{aligned}$$

Remark: You can see that it is not necessary to determinate angle α . The last equations can be rewritten to

$$R_F = Q \frac{BF}{AF}$$

.

Excluding trivial scalar equations we have system of only 8 equilibrium equation. These are in matrix form:

$$\begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & h_1 & z_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R_{Ax} \\ R_{Bx} \\ R_{Cx} \\ R_{Cy} \\ R_{Dx} \\ R_E \\ R_F \\ R_{Fp} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Q \\ Q z_3 / z_2 \end{bmatrix}$$

The pressure can be computed from equation:

$$p = \frac{F_p}{\frac{\pi d^2}{4}}$$

.

Notice: See Matlab file S615.m for numerical solution.

Solution of Exercise 3.6.5

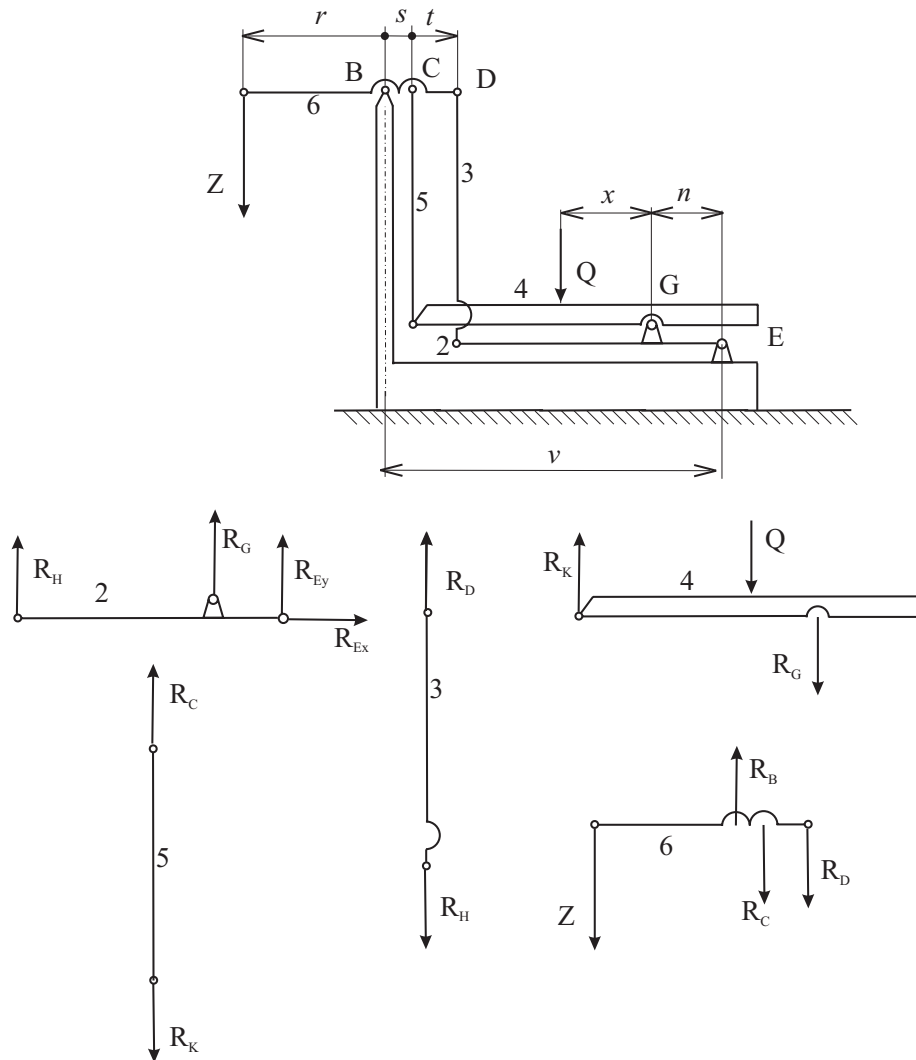


Figure 3.103: Exercise 3.6.5. Equilibrium of the decimal scales

We free bodies 2, 3, 4, 5, 6 and write the particular equations of equilibrium.

Member 2:

$$\begin{aligned} \sum F_{ix} : & R_{Ex} = 0 \\ \sum F_{iy} : & R_G + R_{Ey} + R_H = 0 \\ \sum M_{iH} : & R_G(v - s - t - u) + R_{Ey}(v - s - t) = 0 \end{aligned}$$

Member 3:

$$\sum F_{iy} : R_D - R_H = 0$$

Member 4:

$$\begin{aligned}\sum F_{iy} : & -R_G + R_K - Q = 0 \\ \sum M_{iK} : & -R_{Gy}(v - s - u) - Q(v - s - u - x) = 0\end{aligned}$$

Member 5:

$$\sum F_{iy} : R_C - R_K = 0$$

Member 6:

$$\begin{aligned}\sum F_{iy} : & -Z + R_B - R_C - R_D = 0 \\ \sum M_{iB} : & Zr - R_C s - R_D(s + t) = 0\end{aligned}$$

Excluding trivial scalar equations we have system of only 8 equilibrium equation. These are in matrix form:

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & v-s-t & v-s-t-u & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & v-s-u & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -s & -(s+t) & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} R_B \\ R_C \\ R_D \\ R_{Ey} \\ R_G \\ R_H \\ R_K \\ Z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ Q \\ Q(v-s-u-x) \\ 0 \\ Z \\ Zr \end{bmatrix}$$

Notice: See Matlab file S6111.m for numerical solution.

.

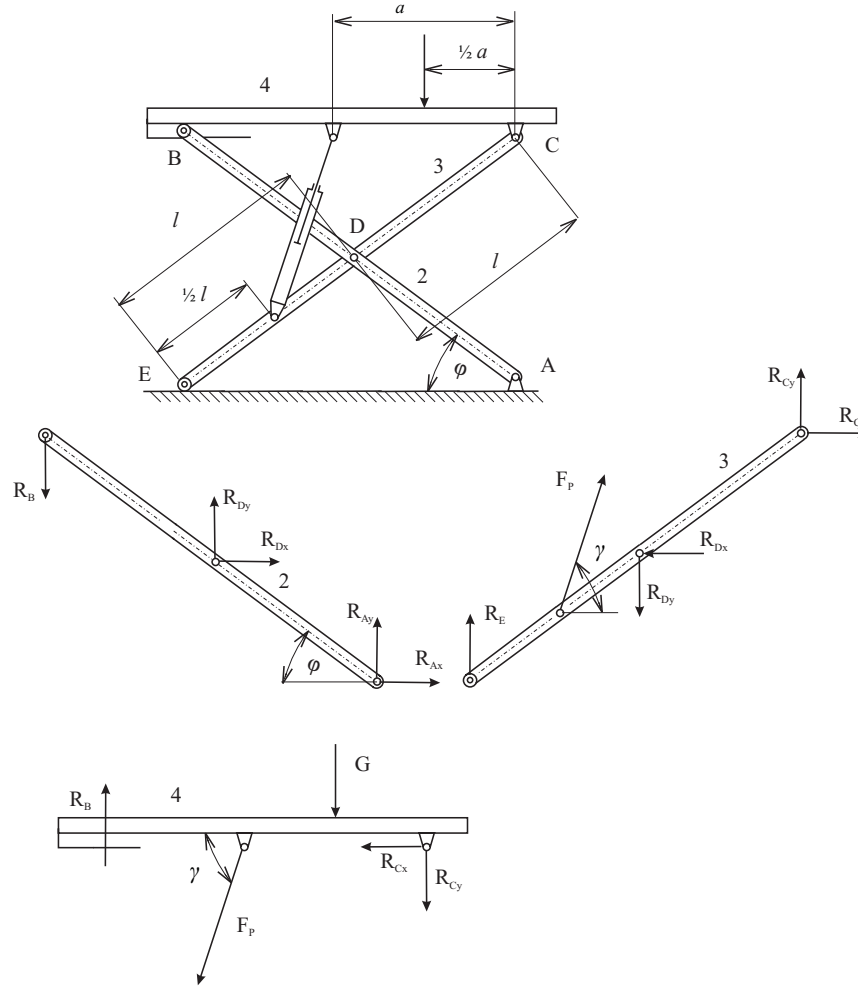
Solution of Exercise 3.6.6

Figure 3.104: Exercise 3.6.6. Equilibrium of the lifting platform

First we express the geometrical dependencies:

$$\eta = \sqrt{\left(l + \frac{l}{2}\right)^2 + a^2 - 2a\left(l + \frac{l}{2}\right)\cos\phi}$$

$$\gamma = \arccos\left(\frac{\left(l + \frac{l}{2}\right)\cos\phi - a}{\eta}\right)$$

$$\epsilon = \gamma - \psi$$

Next we free bodies 2, 3, 4, and write the particular equations of equilibrium.

Member 2:

$$\begin{aligned}\sum F_{ix} : & R_{Ax} + R_{Dx} = 0 \\ \sum F_{iy} : & R_{Ay} + R_{Dy} - R_B = 0 \\ \sum M_{iA} : & R_B 2l \cos \phi - R_{Dy} l \cos \phi - R_{Dx} l \sin \phi = 0\end{aligned}$$

Member 3:

$$\begin{aligned}\sum F_{ix} : & R_{Cx} - R_{Dx} + F_p \cos \gamma = 0 \\ \sum F_{iy} : & R_{Cy} - R_{Dy} + R_E + F_p \sin \gamma = 0 \\ \sum M_{iC} : & R_{Dy} l \cos \phi - R_{Dx} l \sin \phi - R_E 2l \cos \phi - F_p \sin(\gamma - \phi) \frac{l}{2} = 0\end{aligned}$$

Member 4:

$$\begin{aligned}\sum F_{ix} : & -R_{Cx} - F_p \cos \gamma = 0 \\ \sum F_{iy} : & -R_{Cy} - R_B - F_p \sin \gamma = 0 \\ \sum M_{iC} : & -R_B 2l \cos \phi + F_p \sin \gamma a = 0\end{aligned}$$

We have system of only 9 equilibrium equation.

If we introduce auxiliary variables:

$$cp = \cos \phi, \quad sp = \sin \phi, \quad cg = \cos \gamma, \quad sg = \sin \gamma, \quad se = \sin \epsilon$$

Can be equations of equilibrium written in matrix form:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2lcp & 0 & 0 & -lsp & -lcp & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & cg \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & 1 & sg \\ 0 & 0 & 0 & 0 & 0 & -lsp & lcp & -2lcp & -se(l + \frac{l}{2}) \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & -cg \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & -sg \\ 0 & 0 & -2lcp & 0 & 0 & 0 & 0 & 0 & sg a \end{bmatrix} \begin{bmatrix} R_{Ax} \\ R_{Ay} \\ R_B \\ R_{Cx} \\ R_{Cy} \\ R_{Dx} \\ R_{Dy} \\ R_E \\ F_p \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ G \\ -Q_{\frac{a}{2}} \end{bmatrix}$$

Notice: See Matlab file S6120.m for numerical solution.

Solution of Exercise 3.6.7

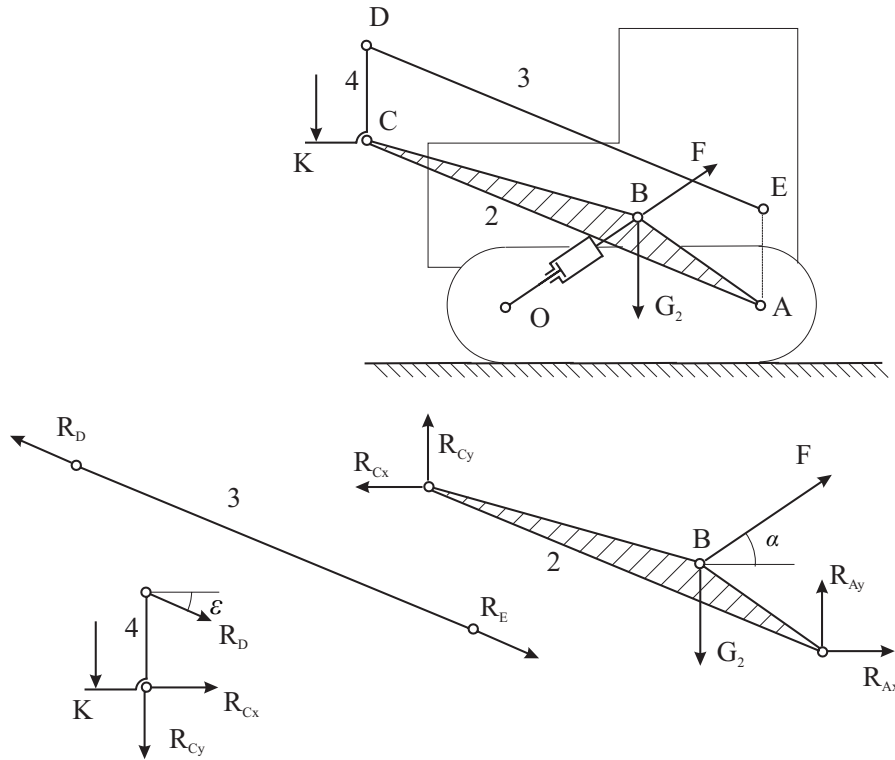


Figure 3.105: Exercise 3.6.7. Equilibrium of a hub lifting mechanism

First we express the geometrical dependencies:

$$\alpha = \arccos\left(\frac{(OB^2 + OA^2 - AB^2)}{2 OB OA}\right)$$

$$\delta = \arccos\left(\frac{(AC^2 + AB^2 - BC^2)}{2 AC AB}\right)$$

$$\beta = \arcsin\left(\frac{OB}{AB \sin \alpha}\right)$$

$$\epsilon = \beta - \delta$$

Next we free bodies 2, 4, and write the particular equations of equilibrium.

Member 2:

$$\begin{aligned} \sum F_{ix} : & R_{Ax} - R_{Cx} + F \cos \alpha = 0 \\ \sum F_{iy} : & R_{Ay} + R_{Cy} + F \sin \alpha - G_2 = 0 \\ \sum M_{iA} : & R_{Cx} AC \cos \epsilon - R_{Cy} AC \sin \epsilon - F \cos \alpha AB \sin \beta - F \sin \alpha AB \cos \beta = 0 \end{aligned}$$

Member 4:

$$\begin{aligned}\sum F_{ix} : \quad R_{Cx} + R_D \cos \epsilon &= 0 \\ \sum F_{iy} : \quad -R_{Cy} - R_D \sin \epsilon - Z_4 &= 0 \\ \sum M_{iC} : \quad Z_4 CK - R_D \cos \epsilon CD &= 0\end{aligned}$$

We have system of only 6 equilibrium equation.

If we introduce auxiliary variables:

$$mf = -\cos \alpha AB \sin \beta - \sin \alpha AB \cos \beta$$

Can be equations of equilibrium written in matrix form:

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & \cos \alpha \\ 0 & 1 & 0 & 1 & 0 & \sin \alpha \\ 0 & 0 & AC \sin \epsilon & -AC \cos \epsilon & 0 & mf \\ 0 & 0 & 1 & 0 & \cos \epsilon & 0 \\ 0 & 0 & 0 & -1 & -\sin \epsilon & 0 \\ 0 & 0 & 0 & 0 & -\cos \epsilon CD & 0 \end{bmatrix} \begin{bmatrix} R_{Ax} \\ R_{Ay} \\ R_{Cx} \\ R_{Cy} \\ R_D \\ F \end{bmatrix} = \begin{bmatrix} 0 \\ G_2 \\ -G_2 AB \cos \beta \\ 0 \\ Z_4 \\ -Z_4 CK \end{bmatrix}$$

Notice: See Matlab file S6122.m for numerical solution.

Solution of Exercise 3.8.2

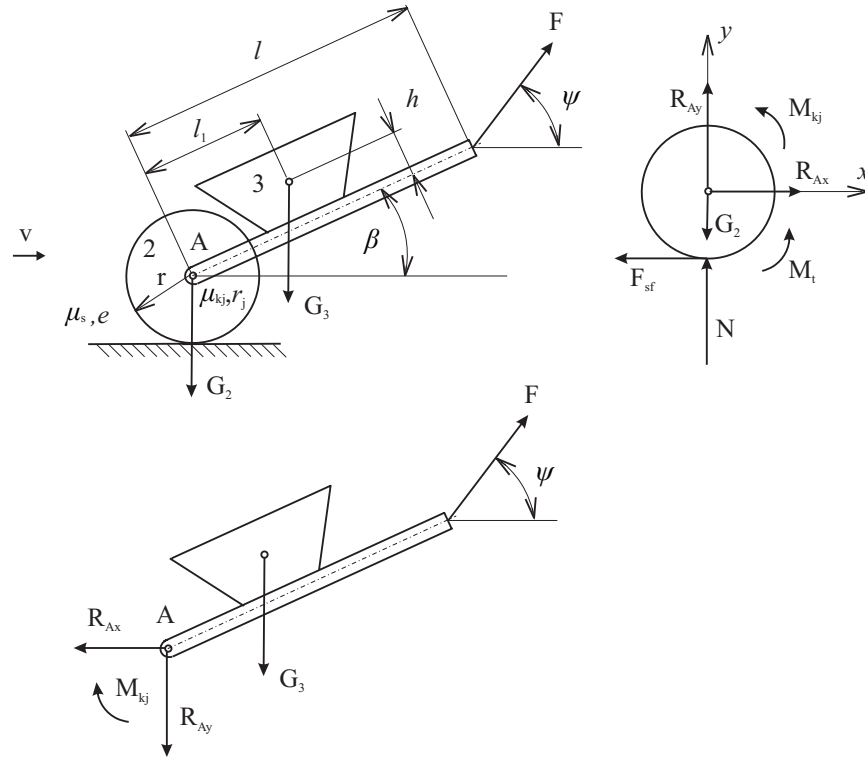


Figure 3.106: Exercise 3.8.2. Equilibrium of a hand-barrow

First we free particular bodies and write down the equations of equilibrium.

Body 2:

$$\begin{aligned} \sum F_{ix} : \quad R_{Ax} - F_{sf} &= 0 \\ \sum F_{iy} : \quad R_{Ay} - G_2 + N &= 0 \\ \sum M_{iA} : \quad M_{kj} + M_t - F_{sf} r &= 0 \end{aligned}$$

Body 3:

$$\begin{aligned} \sum F_{ix} : \quad F \cos \psi - R_{Ax} &= 0 \\ \sum F_{iy} : \quad -R_{Ay} - G_3 + F \sin \psi &= 0 \\ \sum M_{iA} : \quad F \sin \psi l \cos \beta - F \cos \psi l \sin \beta - G_3 (l_1 \cos \beta - h \sin \beta) - M_{kj} &= 0 \end{aligned}$$

Then we express the friction forces using their definitions

$$M_{kj} = r_j \mu_{kj} \sqrt{R_{Ax}^2 + R_{Ay}^2} \quad , \quad M_t = |N| e$$

and we substitute them into the equations of equilibrium. We get a system of 6 nonlinear algebraic equations containing 6 unknowns R_{Ax} , R_{Ay} , F_{sf} , N , F , ψ . We solve the system using Matlab. The result is $F = 154.7 \text{ N}$, $\psi = 84.7^\circ$.

After the solution we check the condition of rolling using the formula

$$|F_{sf}| \leq |N| \mu_s \quad .$$

The condition is valid in our case because $14.34 < 144.36$.

In case we have no solver for system of nonlinear algebraic equations we can use the linearized expression

$$M_{kj} = r_j \mu_{kj} (0.96 |R_{Ay}| + 0.4 |R_{Ax}|)$$

for the moment of friction. Supposing

$$|R_{Ay}| > |R_{Ax}|$$

we get a system of 6 linear algebraic equations after linearizing. These can be written and solved using familiar approach.

Solution of linearizing equations:

$$F = 149.53 \text{ N}, \quad \psi = 88.88^\circ$$

Solution of nonlinearizing equations:

$$F = 158.18 \text{ N}, \quad \psi = 82.33^\circ$$

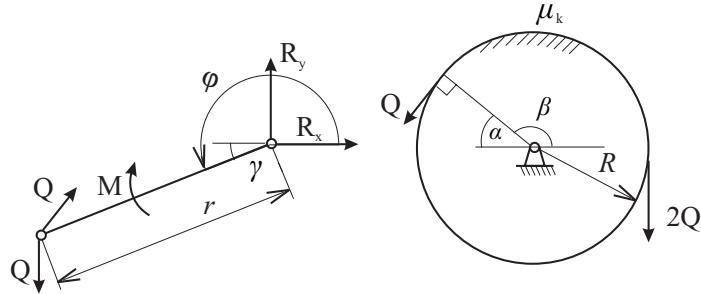
Solution of Exercise 3.8.3

Figure 3.107: Exercise 3.8.3. Free-body diagram

First we use the standard expression for belt friction:

$$\frac{2Q}{Q} = e^{\beta \mu_k}$$

This yields

$$\ln 2 = \beta \mu_k$$

$$\beta = \frac{1}{\mu_k} \ln 2 = 2.310 \text{ rad} = 132.353^\circ$$

From geometry we have

$$\cos \alpha = \frac{R}{r}$$

$$\alpha = \arccos \frac{R}{r} = 60^\circ$$

$$\varphi = \beta + \alpha = 192.353^\circ$$

$$\gamma = \varphi - 180^\circ = 12.353^\circ$$

Equilibrium equation is

$$M + Q R - Q r \cos \gamma = 0$$

This yields

$$M = Q (r \cos \gamma - R) = 9.54 \text{ Nm}$$

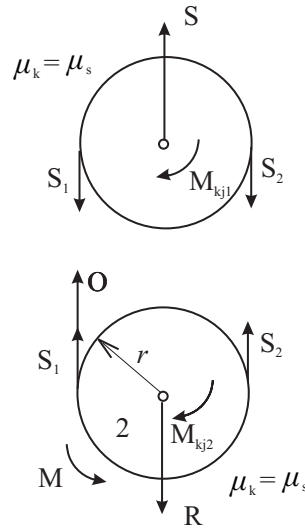
Solution of Exercise 3.8.4

Figure 3.108: Exercise 3.8.4. Free-body diagram

Equations of equilibrium for the upper wheel:

$$\begin{aligned} S &= S_1 + S_2 \\ r_j \mu_{kj} (S_1 + S_2) &= (S_1 - S_2) r \end{aligned}$$

Equations of equilibrium for the lower wheel:

$$\begin{aligned} R &= S_1 + S_2 + O \\ M &= (S_1 + O - S_2) r + r_j \mu_{kj} (S_1 + S_2 + O) \end{aligned}$$

Expression for the belt friction:

$$\frac{S_1 + O}{S_2} = e^{\pi \mu_k} \rightarrow \min$$

Altogether we have 5 equations for 5 unknowns, namely S, S_1, S_2, R, M . After some manipulations we have

$$\begin{aligned} S_2 &= \frac{r_j \mu_{kj} - r}{r_c \mu_{kj} (e^{\pi \mu_k} + 1) - r (e^{\pi \mu_k} - 1)} O = 123.1 \text{ N} \\ S_1 &= S_2 e^{\pi \mu_k} - O = 130.74 \text{ N} \\ S &= S_1 + S_2 = 253.846 \text{ N} \\ M &= 23.652 \text{ Nm} \\ R &= S_1 + S_2 + O = 353.84 \text{ N} \end{aligned}$$

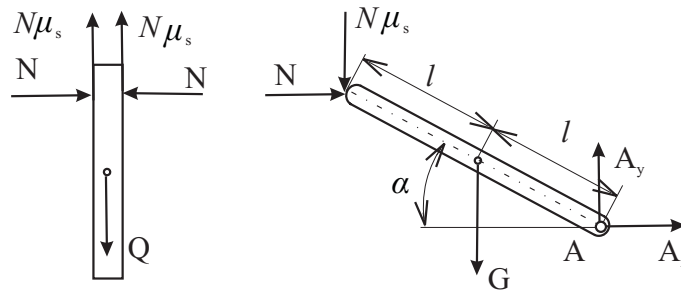
Solution of Exercise 3.8.5

Figure 3.109: Exercise 3.8.5. Free-body diagram

Equation of equilibrium for the plate:

$$Q = 2 N \mu_s$$

Equation of equilibrium for the rod:

$$G l \cos \alpha + N \mu_s 2 l \cos \alpha = N 2 l \sin \alpha$$

Solution:

$$N = \frac{G \cos \alpha}{2 (\sin \alpha - \mu_s \cos \alpha)} = 116.839 \text{ N}$$

$$Q = 2 N \mu_s = 35.052 \text{ N}$$

The condition for $\alpha_{\max}$ is

$$N \rightarrow \infty$$

Hence:

$$\tan \alpha_{\max} = \mu_s$$

and

$$\alpha_{\max} = 8.53^\circ$$

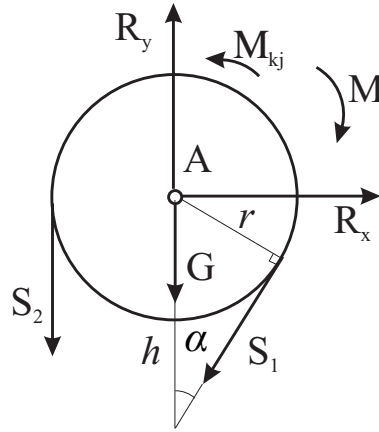
Solution of Exercise 3.8.6

Figure 3.110: Exercise 3.8.6. Free-body diagram

Equations of equilibrium :

$$\begin{aligned} R_x - S_1 \sin \alpha &= 0 \\ R_y - G - S_2 - S_1 \cos \alpha &= 0 \\ -M + M_{kj} + (S_2 - S_1) r &= 0 \end{aligned}$$

The expressions for friction forces are:

$$M_{kj} = r_j \mu_{kj} R$$

$$\frac{S_2}{S_1} = e^{(\pi + \alpha) \mu_k}$$

Using Poncelet expression for linearization of friction moment we write

$$M_{kj} = r_j \mu_{kj} (0.96 (G + S_2 + S_1 \cos \alpha) + 0.4 S_1 \sin \alpha)$$

Geometry yields

$$\sin \alpha = \frac{r}{h} = \frac{1}{2} \Rightarrow \alpha = \frac{\pi}{6}$$

hence

$$S_2 = S_1 e^{\frac{7\pi}{6} 0.3} = 3.003 S_1$$

After substitution we have

$$M_{kj} = r_f \mu_{kj} (0.96 (G + S_1 (e^{\frac{7\pi}{6} 0.3} + \cos \alpha)) + 0.4 S_1 \sin \alpha)$$

and

$$M_{kj} + S_1 (e^{\frac{7\pi}{6} 0.3} - 1) r = M$$

$$S_1 [0.96 r_j \mu_{kj} (e^{\frac{7\pi}{6} 0.3} + \cos \alpha)) + r_j \mu_{kj} 0.4 \sin \alpha + (e^{\frac{7\pi}{6} 0.3} - 1) r = M - 0.96 r_j \mu_{kj} G$$

$$S_1 = \frac{M - 0.96 r_j \mu_{kj} G}{0.96 r_j \mu_{kj} (e^{\frac{7\pi}{6} 0.3} + \cos \alpha)) + r_j \mu_{kj} 0.4 \sin \alpha + (e^{\frac{7\pi}{6} 0.3} - 1) r} = 792.385 \text{ N}$$

At the end

$$P l = S_1 r \cos 30^\circ$$

$$P = \frac{r}{l} S_1 \cos 30^\circ = 214.44 \text{ N}$$

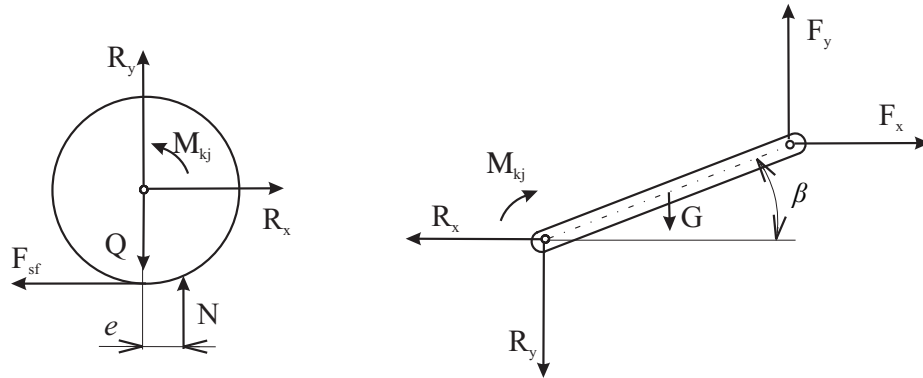
Solution of Exercise 3.8.7

Figure 3.111: Exercise 3.8.7. Free-body diagram

Equations of equilibrium of the roller:

$$\begin{aligned} R_x - F_{sf} &= 0 \\ R_y + N - Q &= 0 \\ M_{kj} + N e - F_{sf} r &= 0 \end{aligned}$$

Equations of equilibrium of the tow bar:

$$\begin{aligned} F_x - R_x &= 0 \\ F_y - G - R_y &= 0 \\ -M_{kj} + F_y 2l \cos \beta - F_x 2l \sin \beta + G l \cos \beta &= 0 \end{aligned}$$

Poncelet expression for the friction moment:

$$M_{kj} = r_j \mu_{kj} [0.96 R_y + 0.4 R_x]$$

Equations of equilibrium in matrix form:

$$\begin{bmatrix} -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ -r & e & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & -2l \sin \beta & 2l \cos \beta & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -r_j \mu_{kj} 0.4 & -r_j \mu_{kj} 0.96 & 1 \end{bmatrix} \begin{bmatrix} T_{sf} \\ N \\ F_x \\ F_y \\ R_x \\ R_y \\ M_{kj} \end{bmatrix} = \begin{bmatrix} 0 \\ Q \\ 0 \\ 0 \\ G \\ -G l \cos \beta \\ 0 \end{bmatrix}$$

Notice: See Matlab file S736.m for numerical solution.

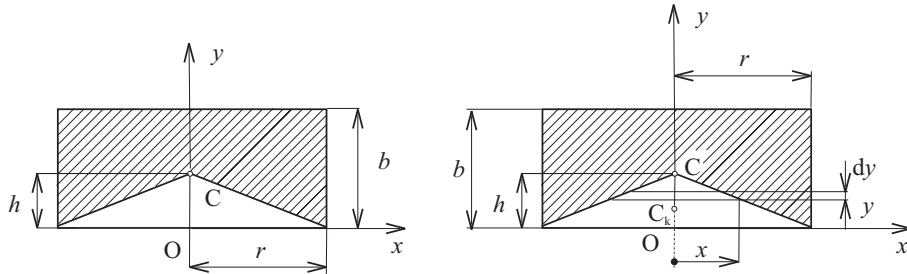
Solution of Exercise 3.9.2

Figure 3.112: Exercise 3.9.2. The centroid of a flywheel

The flywheel is composed from a cylinder from which a cone is extracted. The centroid is located on the axis y due to symmetry and the following is valid

$$y_C = \frac{y_{C1} V_1 - y_{C2} V_2}{V_1 - V_2} = h \quad (3.54)$$

where the subscript 1 denotes the cylinder and the subscript 2 denotes the cone. According to Fig. 3.112 we have

$$y_{C2} = \frac{\int y dV}{V_2} = \frac{\int_0^h y \pi \frac{r^2}{h^2} (h-y)^2 dy}{\frac{1}{3} \pi r^2 h} = \frac{1}{4} h \quad (3.55)$$

The substitution 3.55 to 3.54 yields

$$h = \frac{\frac{b}{2} \pi r^2 b - \frac{h}{4} \cdot \frac{1}{3} \pi r^2 h}{\pi r^2 b - \frac{1}{3} \pi r^2 h}$$

and after some manipulation we have

$$h^2 - 4bh + 2b^2 = 0$$

The root $h = b(2 - \sqrt{2}) = 0.586 b$ is acceptable.

Notice: See Matlab file SCG102.m for numerical solution.

Solution of Exercise 3.9.3

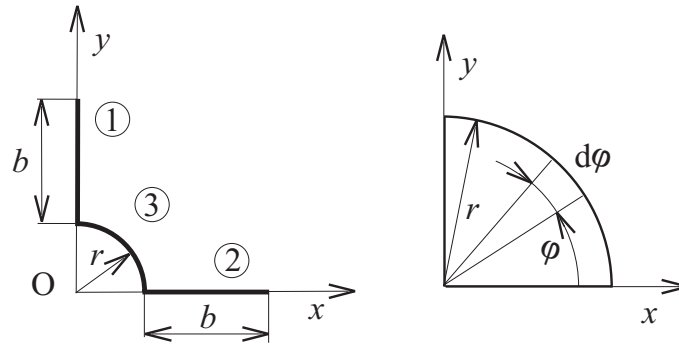


Figure 3.113: Exercise 3.9.3. Division of the wire

We split the wire into three parts:

$$\text{Part1 : } x_{C1} = 0, \quad y_{C1} = r + \frac{b}{2}$$

$$\text{Part2 : } x_{C2} = r + \frac{b}{2}, \quad y_{C2} = 0$$

$$\text{Part3 : } x_{C3} = \frac{2r}{\pi}, \quad y_{C3} = \frac{2r}{\pi}$$

To compute x_{C3} we can write

$$x_{C3} \frac{\pi}{2} r = \int_0^{\frac{\pi}{2}} r \cos \varphi r d\varphi = r^2 [\sin \varphi]_0^{\frac{\pi}{2}} = r^2$$

and hence

$$x_{C3} = \frac{2r}{\pi}$$

Due to symmetry

$$y_{C3} = x_{C3}$$

.

To compute x_C we write

$$\begin{aligned} x_C l &= x_{C1} l_1 + x_{C2} l_2 + x_{C3} l_3 \\ x_C (2b + \frac{\pi r}{2}) &= 0 + (r + \frac{b}{2}) b + \frac{2r}{\pi} \frac{\pi r}{2} \\ x_C &= 0.0334 \text{ m} \end{aligned}$$

Due to symmetry

$$y_C = x_C$$

.

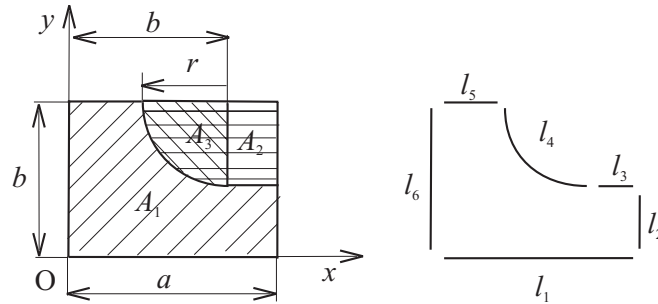
Solution of Exercise 3.9.4

Figure 3.114: Exercise 3.9.4. Division of the area

We solve the exercise for the centre of gravity of the area first.

We split the area into three parts. Areas of the particular parts are:

$$A_1 = a b; \quad A_2 = (a - b) r; \quad A_3 = \frac{\pi r^2}{4}$$

and the interesting area A is

$$A = A_1 - A_2 - A_3$$

Coordinates of centers of gravity are

$$\begin{aligned} \text{Part 1 : } x_{C1} &= \frac{a}{2}, & y_{C1} &= \frac{b}{2} \\ \text{Part 2 : } x_{C2} &= \frac{a+b}{2}, & y_{C2} &= b - \frac{r}{2} \\ \text{Part 3 : } x_{C3} &= b - \frac{4r}{3\pi}, & y_{C3} &= b - \frac{4r}{3\pi} \end{aligned}$$

To compute x_C we write

$$x_{C_{\text{area}}} A = x_{C1} A_1 - x_{C2} A_2 - x_{C3} A_3$$

To compute y_C we write

$$y_{C_{\text{area}}} A = y_{C1} A_1 - y_{C2} A_2 - y_{C3} A_3$$

After substitution of numerical values we have the result

$$x_{C_{\text{area}}} = 0.02714 \text{ m} \quad y_{C_{\text{area}}} = 0.01343 \text{ m}$$

Second we solve the problem of centres of gravity of circumference. We split the area into six parts. Data necessary for computation are in table:

Part No.	l_i	x_{Ci}	y_{Ci}
1	a	$\frac{a}{2}$	0
2	$b - r$	a	$\frac{b-r}{2}$
3	$a - b$	$\frac{a+b}{2}$	$b - r$
4	$\frac{\pi r}{2}$	$b - \frac{2r}{\pi}$	$b - \frac{2r}{\pi}$
5	$b - r$	$\frac{b-r}{2}$	b
6	b	0	$\frac{b}{2}$

To compute $x_{C_{\text{circu}}}$ and $y_{C_{\text{circu}}}$ we write

$$x_{C_{\text{circu}}} l = \sum_1^6 x_{Ci} l_i; \quad y_{C_{\text{circu}}} l = \sum_1^6 y_{Ci} l_i$$

After substitution of numerical values we have the result

$$x_{C_{\text{circu}}} = 0.0296 \text{ m} \quad y_{C_{\text{circu}}} = 0.0138 \text{ m}$$

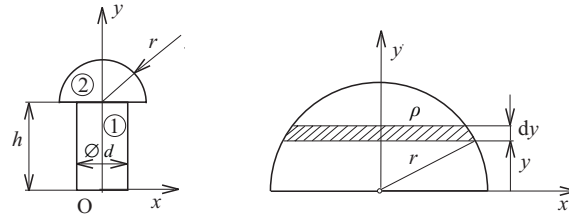
Solution of Exercise 3.9.5

Figure 3.115: Exercise 3.9.5. Decomposition of a rivet

We decompose the rivet into two parts. Part 1 is the cylinder, part 2 is the semisphere.

$$\begin{aligned} \text{Part 1 : } V_1 &= \frac{\pi d^2}{4} h; & y_{C1} &= \frac{h}{2} \\ \text{Part 2 : } V_2 &= \frac{2}{3} \pi r^3; & y_{C2} &= \frac{3r}{8} \end{aligned}$$

The centroid of the semisphere we compute as follows:

$$\begin{aligned} y_{C2} V_2 &= \int y \, dV = \int y \pi \rho^2 \, dy = \int y \pi (r^2 - y^2) \, dy = \int_0^r \pi r^2 y \, dy - \int_0^r \pi y^3 \, dy = \\ &= \frac{1}{2} \pi r^4 - \frac{1}{4} \pi r^4 = \frac{1}{4} \pi r^4 \end{aligned}$$

Hence

$$y_{C2} = \frac{\frac{1}{4} \pi r^4}{\frac{2}{3} \pi r^3} = \frac{3r}{8}$$

The coordiante y_C of the centroid of the whole rivet we compute from the equation

$$y_C V = y_{C1} V_1 + y_{C2} V_2$$

After substitutions we have

$$y_C \left(\frac{\pi d^2}{4} h + \frac{\pi d^3}{12} \right) = \frac{h}{2} \frac{\pi d^2}{4} h + \left(h + \frac{3r}{8} \right) \frac{2}{3} \pi r^3$$

The result is

$$y_C = 0.0476m$$

Solution of Exercise 3.9.6

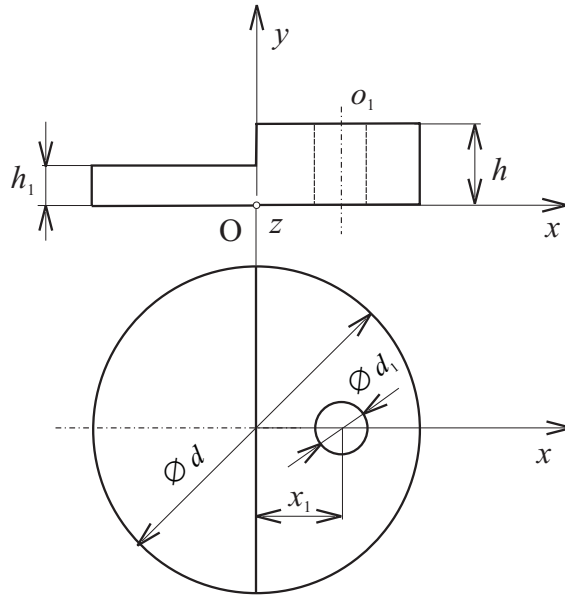


Figure 3.116: Exercise 3.9.6. The particular volumes

We first split the circular plate into three parts:

$$\begin{aligned} \text{Part1 : } V_1 &= \frac{\pi d^2}{4} h; & x_{C1} &= 0 \\ \text{Part2 : } V_2 &= -\frac{\pi d^2}{8} (h - h_1); & x_{C2} &= -\frac{2d}{3\pi} \\ \text{Part3 : } V_3 &= -\frac{\pi d_1^2}{4} h; & x_{C3} &= x_1 \end{aligned}$$

For x_C we have

$$x_C V = x_{C1} V_1 + x_{C2} V_2 + x_{C3} V_3$$

Using the condition

$$x_C = 0$$

we find that

$$\frac{2}{3} \frac{d}{\pi} \frac{\pi d^2}{8} (h - h_1) - x_1 \frac{\pi d_1^2}{4} h = 0$$

From the last equation we compute

$$d_1 = \sqrt{\frac{(h - h_1) d^3}{3 \pi x_1 h}} = 0.0583 \text{ m}$$

Notice: No Matlab file is necessary.

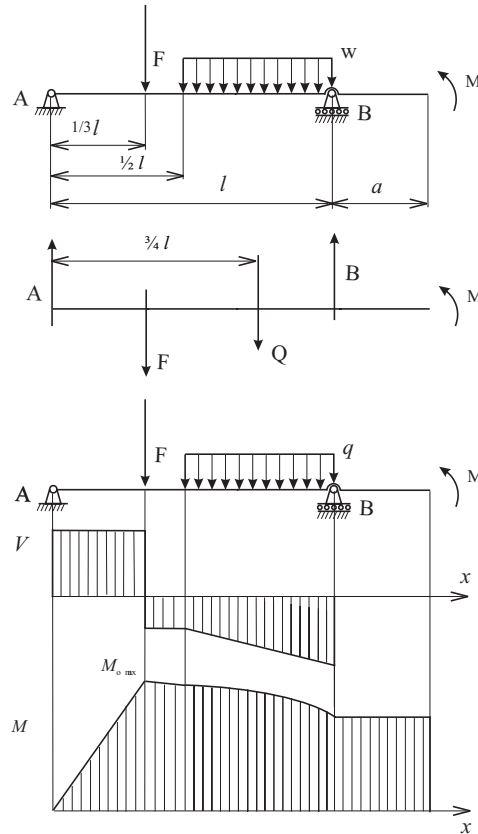
Solution of Exercise 3.10.2

Figure 3.117: Exercise 3.10.2. Internal forces in a beam

Solution

We compute reaction forces R_A, R_B first. We replace the distributed load by a force $W = w \frac{l}{2} = 30 \text{ N}$ the point of action of which is in the centroid of the rectangle (see Fig. 3.68). Using moment equilibrium equation we have

$$\begin{aligned} \sum M_{iA} : \quad & M + R_B l - W \frac{3}{4} l - F \frac{l}{3} = 0 \\ \sum M_{iB} : \quad & R_A l - F \frac{2}{3} l - W \frac{1}{4} l - M = 0 \end{aligned}$$

The result is $R_A = 174.16 \text{ N}$, $R_B = 55.83 \text{ N}$.

We compute the particular internal forces in intervals where no change of load type occurs using definitions.

Interval $0 < x < \frac{l}{3}$:

$$\begin{aligned} N &= 0 \\ V &= R_A = 174.16 \text{ N} \\ M_b &= R_A x = (174.16 x) \text{ Nm} \end{aligned}$$

Interval $\frac{l}{3} < x < \frac{l}{2}$:

$$\begin{aligned} N &= 0 \\ V &= R_A - F = -25.840 \text{ N} \\ M_b &= R_A x - (x - \frac{l}{3}) F = (-25.84 x + 40) \text{ Nm} \end{aligned}$$

Interval $\frac{l}{2} < x < l$:

$$\begin{aligned} N &= 0 \\ V &= R_A - F - w(x - \frac{l}{2}) = (4, 16 - x) \text{ N} \\ M_b &= R_A x - (x - \frac{l}{3}) F - w(x - \frac{l}{2})\frac{1}{2}(x - \frac{l}{2}) \end{aligned}$$

Interval $l < x < l + a$:

$$\begin{aligned} N &= 0 \\ V &= R_A - F - w\frac{l}{2} + R_B = 0 \text{ N} \\ M_b &= R_A x - F(x - \frac{l}{3}) - w\frac{1}{2}(x - \frac{3}{4}l) + R_B(x - l) \end{aligned}$$

The plot of results is shown in Fig.3.68. You can see that maximum bending moment $M_{b\max}$ is in position

$$x = \frac{l}{3} = 0.2 \text{ m}$$

where $V = 0$ occurs. Its value is $M_{o\max} = 34.832 \text{ Nm}$.

Notice: See Matlab file `beam2D.m` for numerical solution.

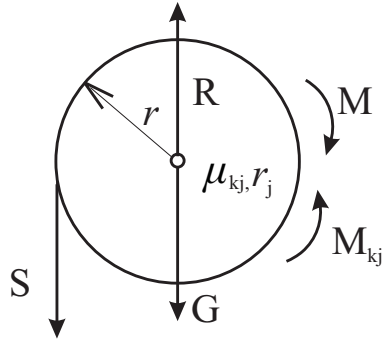
Solution of Exercise 3.11.2

Figure 3.118: Exercise 3.11.2. Free-body diagram

Equation of equilibrium:

$$M = S r + M_{kj}$$

The force in the spring:

$$S = k x$$

The moment of friction:

$$M_{kj} = r_j \mu_{kj} R = r_j \mu_{kj} (S + G)$$

The moment M as a function of x :

$$M = k x r + r_j \mu_{jk} (k x + G)$$

Mechanical work of M :

$$W = \int M d\varphi$$

Geometry:

$$x = r \varphi, \quad dx = r d\varphi$$

Computation:

$$W = \int M \frac{dx}{r} = \int_0^h \left[\frac{r_j \mu_{kj}}{r} G + k \left(1 + \frac{r_j \mu_{jk}}{r} \right) x \right] dx$$

$$W = \frac{r_j \mu_{jk}}{r} G h + \frac{1}{2} k \left(1 + \frac{r_j \mu_{kj}}{r} \right) h^2$$

Result:

$$W = 15.3 \text{ Nm}$$

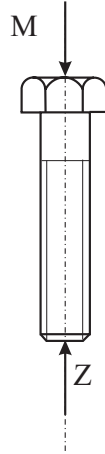
Solution of Exercise 3.11.3

Figure 3.119: Exercise 3.11.3. Free-body diagram

Moment:

$$M = Z r \tan(\alpha + \varphi)$$

The force in spring:

$$Z = k x$$

The friction angle:

$$\varphi = \arctan \mu_k = 2.862^\circ$$

Geometry:

$$\tan \alpha = \frac{x}{r}, \quad \varphi = \frac{x}{r \tan \alpha}, \quad d\varphi = \frac{1}{r \tan \alpha} dx$$

After substitution we have:

$$M = k x r \tan(\alpha + \varphi)$$

Mechanical work of M :

$$W = \int M d\varphi = \int \frac{M}{r \tan \alpha} dx$$

$$W = \int_0^h k x \frac{\tan(\alpha + \varphi)}{\tan \alpha} dx = \frac{1}{2} k h^2 \frac{\tan(\alpha + \varphi)}{\tan \alpha}$$

Result:

$$W = 11.6 \text{ Nm}$$

Solution of Exercise 3.11.4

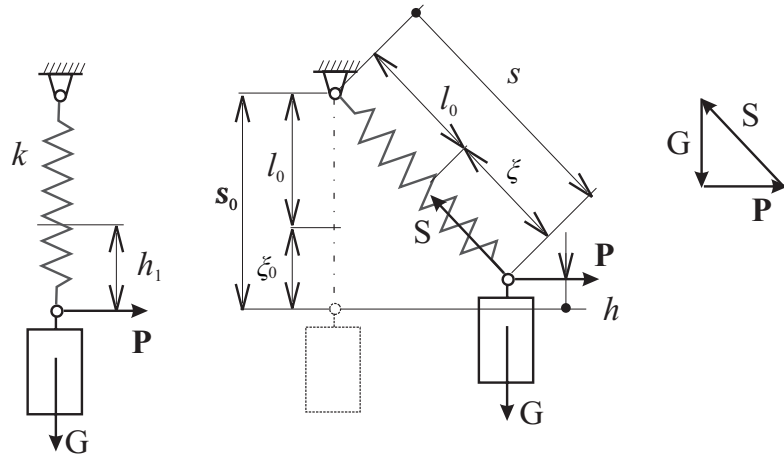


Figure 3.120: Exercise 3.11.4. Free-body diagram

First we redraw the body into the current position and we sketch all relevant forces, namely G , P , S . Mechanical work of the force P is the sum of mechanical work W_1 needed for lifting the body, and mechanical work W_2 needed for stretching of the spring.

Mechanical work W_1 :

$$W_1 = \int_0^{h_1} G \, dh = G h_1 = 10 \cdot 0,2 = 2 \text{ Nm}$$

Mechanical work W_2 :

$$W_2 = \int_{s_0}^{s_1} S \, ds = \int_{\xi_0}^{\xi_1} k \xi \, d\xi = \frac{1}{2} k (\xi_1^2 - \xi_0^2)$$

where ξ , ξ_0 , ξ_1 denote deformations of the spring in current, original, and end positions.

Geometry:

$$\frac{l_0 + \xi}{l_0 + \xi_0 - h} = \frac{S}{G}$$

As $S = k \xi$, $G = k \xi_0$ the following is valid

$$\xi = \frac{G l_0}{k (l_0 - h)}$$

and for $h = h_1$ we have

$$\xi_1 = \frac{G l_0}{k (l_0 - h_1)}$$

$$W_2 = \frac{1}{2} k (\xi_1^2 - \xi_0^2) = \frac{G^2}{2k} \left[\frac{l_0^2}{(l_0 - h_1)^2} - 1 \right] = \frac{10^2}{2 \cdot 100} \left[\frac{0.3^2}{(0.3 - 0.2)^2} - 1 \right] = 4 \text{ Nm}$$

Mechanical work W_P of the force P :

$$W_P = W_1 + W_2 = 6 \text{ Nm}$$

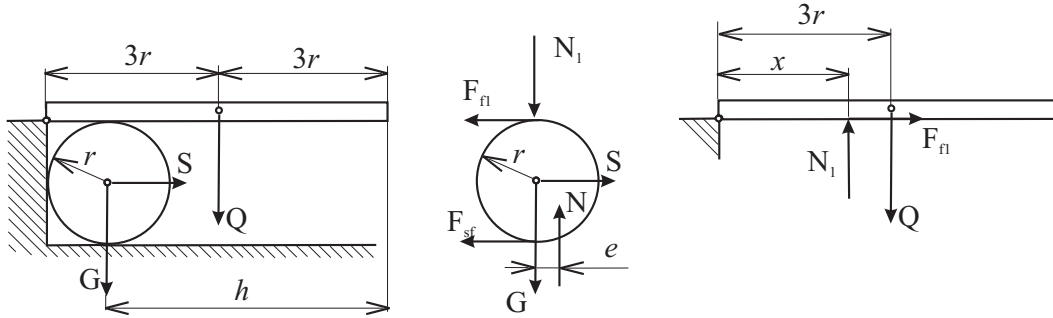
Solution of Exercise 3.11.5

Figure 3.121: Exercise 3.11.5. Free-body diagram

Mechanical work of the force S done along the path h is

$$W_S = \int_0^h S \, dx$$

First we have to express the magnitude of the force S as a function of x . We free the system of bodies in the current position for the purpose. We suppose rolling of the cylinder without slipping on the ground.

Equations of equilibrium of the cylinder:

$$\begin{aligned} S - F_{sf} - F_{fl} &= 0 \\ N - G - N_1 &= 0 \\ F_{fl} r + N e - F_{sf} r &= 0 \end{aligned}$$

Equation of equilibrium of the plate:

$$N_1 x - Q 3 r = 0$$

Friction force:

$$F_{fl} = N_1 \mu_k$$

After substitution and some manipulations we have:

$$S = Q 3 r \left(2 \mu_k + \frac{e}{r} \right) \frac{1}{x} + G \frac{e}{r}$$

hence

$$\begin{aligned}
 W_S &= \int_r^{6r} S \, dx = Q \, 3r \left(2\mu_k + \frac{\epsilon}{r} \right) \int_r^{6r} \frac{dx}{x} + G \frac{\xi}{r} \int_r^{6r} dx \\
 W_S &= Q \, 3r \left(2\mu_k + \frac{\xi}{r} \right) \ln 6 + G \xi \, 5 \\
 W_S &= 500 \cdot 3 \cdot 0.1 \left(2 \cdot 0.3 + \frac{0.01}{0.1} \right) \ln 6 + 300 \cdot 0.01 \cdot 5 \\
 W_S &= 203.13 \text{ Nm}
 \end{aligned}$$

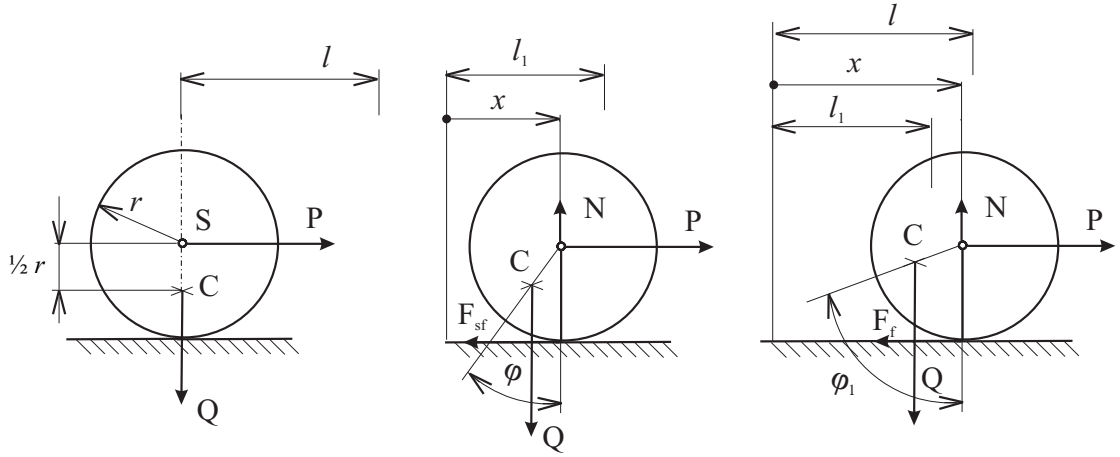
Solution of Exercise 3.11.6

Figure 3.122: Exercise 3.11.6. Free-body diagram

The cylinder will roll without slipping at the beginning of its motion due to the magnitude of μ_s given. Hence the equations of equilibrium of the cylinder are

$$P - F_{sf} = 0, \quad N - Q = 0, \quad Q \frac{r}{2} \sin \varphi - F_s r = 0$$

and

$$P = \frac{Q}{2} \sin \varphi$$

Simultaneously the condition for rolling have to be fulfilled:

$$F_s \leq N \mu_s, \quad F_s \leq Q \mu_{sf}$$

The maximum angle φ_1 follows from the maximum value of F_{sf} which is

$$F_{sf\max} = Q \mu_{sf} = \frac{Q}{2} \sin \varphi_1$$

$$2 \mu_{sf} = \sin \varphi_1$$

For $\mu_s = 0.25$, $\sin \varphi_1 = 0.5$, $\varphi_1 = 30^\circ$. It follows that in the first stage of motion the coordinate x changes from 0 to $l_1 = r \varphi_1$.

Geometry gives

$$x = r \varphi, \quad dx = r d\varphi$$

Mechanical work of the force P during the first stage of motion is

$$\begin{aligned} W_{P1} &= \int_0^{l_1} P \, dx = \frac{Qr}{2} \int_0^{\varphi_1} \sin \varphi \, d\varphi \\ W_{P1} &= Q \frac{r}{2} (1 - \cos \varphi_1) = 80 \frac{0.3}{2} (1 - \cos 30^\circ) \quad \text{Nm} \\ W_{P1} &= 1.607 \text{ Nm} \end{aligned}$$

During the second stage of motion cylinder slips on the ground along the path $l_1 \rightarrow l$ due to force $P = Q \mu_k$ magnitude of which is constant.

Mechanical work of the force P during the second stage of motion is

$$\begin{aligned} W_{P2} &= P \int_{l_1}^l dx = Q \mu_k (l - r \varphi_1) \\ W_{P2} &= 80 \cdot 0.25 \left(1 - 0.3 \frac{30\pi}{180}\right) \quad \text{Nm} \\ W_{P2} &= 16.858 \text{ Nm} \end{aligned}$$

Mechanical work of the force P along the whole path l is

$$W_P = W_{P1} + W_{P2} = (1.607 + 16.858) = 18.466 \text{ Nm}$$

Solution of Exercise 3.12.2

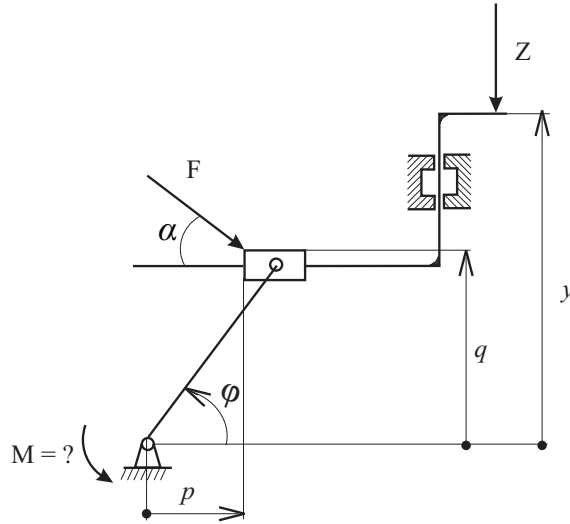


Figure 3.123: Exercise 3.12.2. Designation

The basic equation of pvw:

$$M \delta\varphi + F \cos \alpha \delta p - F \sin \alpha \delta q - Z \delta y = 0$$

Geometry:

$$\begin{aligned} p &= \text{konst.} + r \cos \varphi; & \delta p &= -r \sin \varphi \delta \varphi \\ q &= \text{konst.} + r \sin \varphi; & \delta q &= r \cos \varphi \delta \varphi \\ y &= \text{konst.} + r \sin \varphi; & \delta y &= r \cos \varphi \delta \varphi \end{aligned}$$

After substitution we have

$$M \delta\varphi - F \cos \alpha r \sin \varphi \delta\varphi - F \sin \alpha r \cos \varphi \delta\varphi - Z r \cos \varphi \delta\varphi = 0$$

Hence

$$M = r [F (\cos \alpha \sin \varphi + \sin \alpha \cos \varphi) + Z \cos \varphi]$$

or

$$M = r [F \sin(\alpha + \varphi) + Z \cos \varphi]$$

Result

$$M = 42.8 \text{ Nm}$$

Solution of Exercise 3.12.3

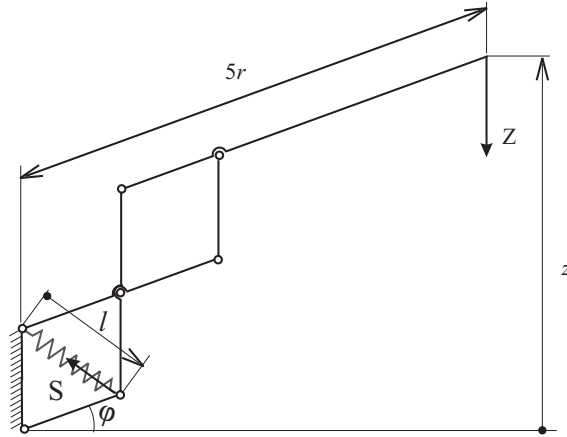


Figure 3.124: Exercise 3.12.3. Designation

The basic equation of pvw:

$$-S \delta l - Z \delta z = 0$$

Geometry:

$$\begin{aligned} l &= 2r \sin \frac{\pi - \varphi}{2}; & \delta l &= -r \cos \left(\frac{\pi}{4} - \frac{\varphi}{2} \right) \delta \varphi \\ z &= \text{konst.} + 5r \sin \varphi; & \delta z &= 5r \cos \varphi \delta \varphi \end{aligned}$$

After substitution we have:

$$S r \cos \left(\frac{\pi}{4} - \frac{\varphi}{2} \right) \delta \varphi - Z 5r \cos \varphi \delta \varphi = 0$$

Hence

$$S = \frac{5 \cos \varphi}{\cos \left(\frac{\pi}{4} - \frac{\varphi}{2} \right)} Z = \frac{5 \cos 306^\circ}{\cos 30^\circ} 50 = 250 \text{ N}$$

The force in the spring:

$$S = k (2r \sin 30^\circ - l_0)$$

The stiffness:

$$k = \frac{250}{20.1 \sin 30^\circ - 0.07} = 8333.3 \text{ Nm}^{-1}$$

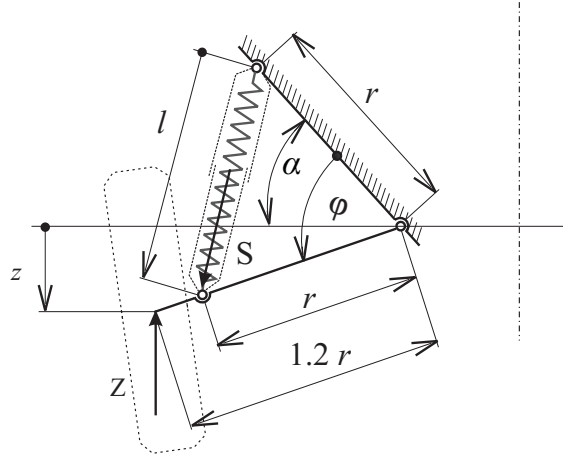
Solution of Exercise 3.12.4

Figure 3.125: Exercise 3.12.4. Designation

The basic equation of pvw:

$$-Z \delta z + S \delta l = 0$$

Geometry:

$$\begin{aligned} z &= 1.2 r \sin(\varphi - \alpha); & \delta z &= 1.2 r \cos(\varphi - \alpha) \delta \varphi \\ l &= 2 r \sin \frac{\varphi}{2}; & \delta l &= 2 r \cos \frac{\varphi}{2} \frac{1}{2} \delta \varphi \end{aligned}$$

After substitution we have

$$S = \frac{\delta z}{\delta l} Z = \frac{1.2 r \cos(\varphi - \alpha) \delta \varphi}{2 r \cos \frac{\varphi}{2} \frac{1}{2} \delta \varphi} Z$$

Hence

$$\begin{aligned} S &= \frac{1.2 \cos(\varphi - \alpha)}{\cos \frac{\varphi}{2}} Z \\ S &= \frac{1.2 \cos 5^\circ}{\cos 30^\circ} 2500 = 3450.92 \text{ N} \end{aligned}$$

The force in the spring is

$$S = k \xi = k 0.1$$

Result:

$$k = 10 S = 34509.2 \text{ Nm}^{-1}$$

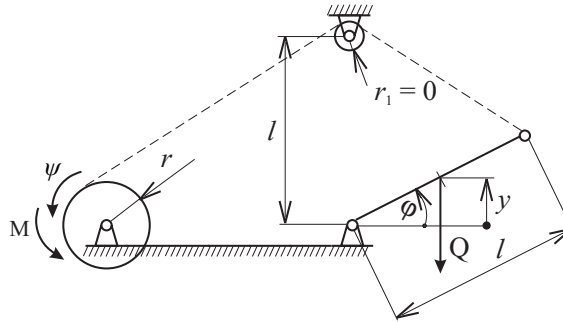
Solution of Exercise 3.12.5

Figure 3.126: Exercise 3.12.5. Designation

The basic equation of pvw:

$$-Q \delta y + M \delta \psi = 0$$

Geometry:

$$\begin{aligned} y &= \frac{l}{2} \sin \varphi; & \delta y &= \frac{l}{2} \cos \varphi \delta \varphi \\ r \psi &= l \sqrt{2} - 2l \sin \frac{\pi - \varphi}{2}; & \delta \psi &= \frac{l}{r} \cos\left(\frac{\pi}{4} - \frac{\varphi}{2}\right) \delta \varphi \end{aligned}$$

After substitution we have

$$-Q \frac{l}{2} \cos \varphi \delta \varphi + M \frac{l}{r} \cos\left(\frac{\pi}{4} - \frac{\varphi}{2}\right) \delta \varphi = 0$$

Result:

$$M = \frac{r}{2} \frac{\cos \varphi}{\cos\left(\frac{\pi}{4} - \frac{\varphi}{2}\right)} Q = \frac{0.1 \cos 30^\circ}{2 \cos 30^\circ} 5000 = 250 \text{ Nm}$$

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- [3] V. Stejskal, J. Březina, and J. Knězu. *Mechanika I. Řešené příklady*. Vydavatelství ČVUT, Praha, 1st edition, 1999. (in Czech).