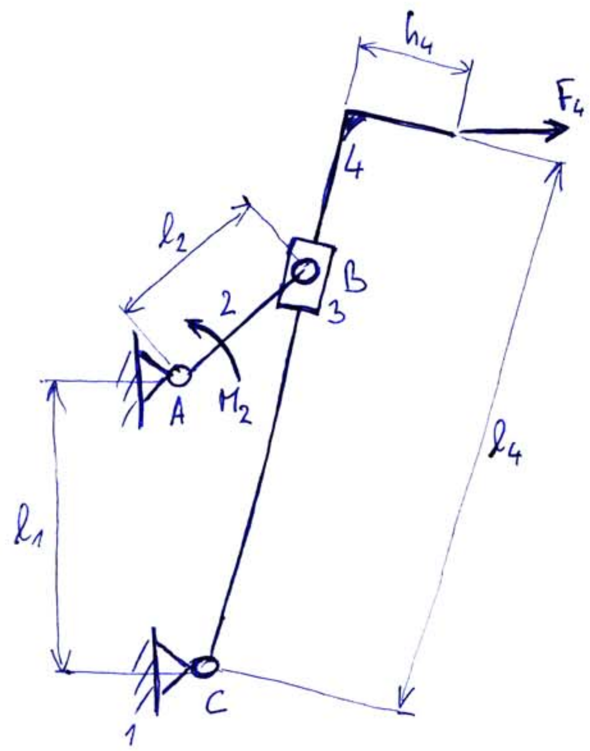
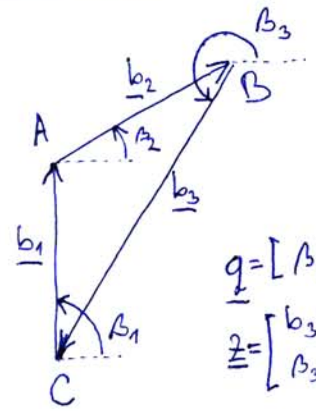


D: $l_1, l_2, l_4, h_4, m_2, m_3, m_4, I_{2S_2}, I_{3S_3}, I_{4S_4}, M_2, F_4$

$t = 0 \dots \varphi_{120}, \omega_{120}$

U: pohyb mechanismu a reakce ve vazbách

Řešení:



$$\underline{q} = [\beta_2] \qquad \underline{z} = \begin{bmatrix} b_3 \\ \beta_3 \end{bmatrix}$$

$$\underline{b_1} + \underline{b_2} + \underline{b_3} = \underline{\emptyset}$$

$$b_1 \cos\beta_1 + b_2 \cos\beta_2 + b_3 \cos\beta_3 = 0$$
$$b_1 \sin\beta_1 + b_2 \sin\beta_2 + b_3 \sin\beta_3 = 0$$

$$-b_2 \dot\beta_2 \sin\beta_2 + \dot b_3 \cos\beta_3 - b_3 \dot\beta_3 \sin\beta_3 = 0$$
$$b_2 \dot\beta_2 \cos\beta_2 + \dot b_3 \sin\beta_3 + b_3 \dot\beta_3 \cos\beta_3 = 0$$

$$-b_2 \ddot\beta_2 \sin\beta_2 + \ddot b_3 \cos\beta_3 - b_3 \ddot\beta_3 \sin\beta_3 - b_2 \dot\beta_2^2 \cos\beta_2 - 2\dot b_3 \dot\beta_3 \sin\beta_3 - b_3 \dot\beta_3^2 \cos\beta_3 = 0$$
$$b_2 \ddot\beta_2 \cos\beta_2 + \ddot b_3 \sin\beta_3 + b_3 \ddot\beta_3 \cos\beta_3 - b_2 \dot\beta_2^2 \sin\beta_2 + 2\dot b_3 \dot\beta_3 \cos\beta_3 - b_3 \dot\beta_3^2 \sin\beta_3 = 0$$

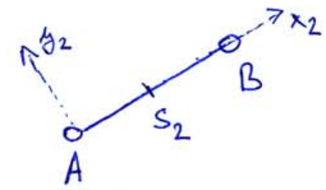
$$\underline{J_z}\, \underline{\ddot z} + \underline{J_q}\, \underline{\ddot q} + \underline{j_{qz}} = \underline{0}$$

$$\underline{J_z} = \begin{bmatrix} \cos\beta_3 & -b_3 \sin\beta_3 \\ \sin\beta_3 & b_3 \cos\beta_3 \end{bmatrix} \qquad \underline{J_q} = \begin{bmatrix} -b_2 \sin\beta_2 \\ b_2 \cos\beta_2 \end{bmatrix}$$

$$\underline{j_{qz}} = \begin{bmatrix} -b_2 \dot\beta_2^2 \cos\beta_2 - 2\dot b_3 \dot\beta_3 \sin\beta_3 - b_3 \dot\beta_3^2 \cos\beta_3 \\ -b_2 \dot\beta_2^2 \sin\beta_2 + 2\dot b_3 \dot\beta_3 \cos\beta_3 - b_3 \dot\beta_3^2 \sin\beta_3 \end{bmatrix}$$

Střediska hmotnosti

těleso 2

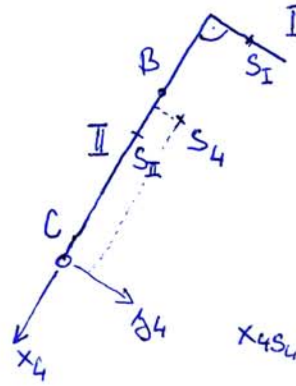


$$x_{2S_2} = \frac{l_2}{2}$$

$$y_{2S_2} = 0$$

těleso 3

těleso 4



$$x_{4S_I} = -l_4$$

$$y_{4S_I} = \frac{h_4}{2}$$

$$x_{4S_{II}} = -\frac{l_4}{2}$$

$$y_{4S_{II}} = 0$$

$$x_{4S_4} = \frac{m_I \cdot x_{4S_I} + m_{II} \cdot x_{4S_{II}}}{m_I + m_{II}} =$$

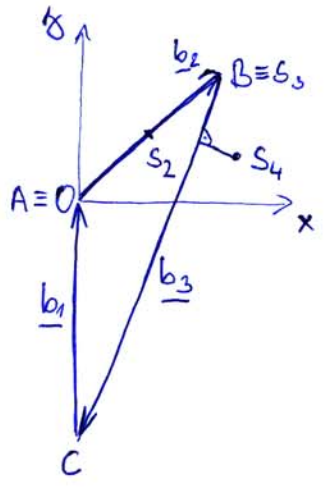
$$= \frac{\mu \cdot h_4 \cdot x_{4S_I} + \mu \cdot l_4 \cdot x_{4S_{II}}}{\mu \cdot h_4 + \mu \cdot l_4}$$

$$x_{4S_4} = \frac{h_4 \cdot x_{4S_I} + l_4 \cdot x_{4S_{II}}}{h_4 + l_4}$$

$(\mu [kg \cdot m^{-1}])$

$$y_{4S_4} = \frac{h_4 \cdot y_{4S_I} + l_4 \cdot y_{4S_{II}}}{h_4 + l_4}$$

V základním souř. s.



$$S_2 \ldots \underline{r}_{S_2} = \underline{x}_{2S_2}$$

$$S_3 \ldots \underline{r}_{S_3} = \underline{b}_2$$

$$S_4 \ldots \underline{r}_{S_4} = \underline{b}_2 + \underline{b}_3 + \underline{x}_{4S_4} + \underline{y}_{4S_4}$$

(Zjednodušení indexování .. $x_{1S_2} \rightarrow x_2$
$v_{1S_{2x}} \rightarrow v_{2x}$
⋮)

$$x_2 = x_{2S_2} \cdot \cos\beta_2$$

$$y_2 = x_{2S_2} \cdot \sin\beta_2$$

$$x_3 = b_2 \cdot \cos\beta_2$$

$$y_3 = b_2 \cdot \sin\beta_2$$

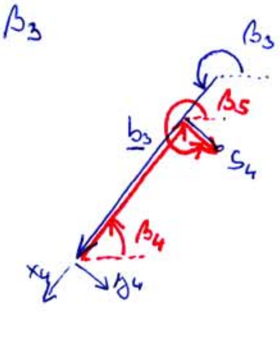
$$x_4 = b_2 \cdot \cos\beta_2 + b_3 \cdot \cos\beta_3 + x_{4S_4} \cdot \cos\beta_3 - y_{4S_4} \cdot \sin\beta_3$$

$$y_4 = b_2 \cdot \sin\beta_2 + b_3 \cdot \sin\beta_3 + x_{4S_4} \cdot \sin\beta_3 + y_{4S_4} \cdot \cos\beta_3$$

(Pozn.
lze např. i takto:

$$x_4 = b_2 \cdot \cos\beta_2 + b_3 \cdot \cos\beta_3 + |x_{4S_4}| \cdot \cos\beta_4 + |y_{4S_4}| \cdot \cos\beta_5$$

$$y_4 = b_2 \cdot \sin\beta_2 + b_3 \cdot \sin\beta_3 + |x_{4S_4}| \cdot \sin\beta_4 + |y_{4S_4}| \cdot \sin\beta_5$$



$$\beta_4 = \beta_3 - \pi$$

$$\beta_5 = \beta_3 + \frac{\pi}{2}$$

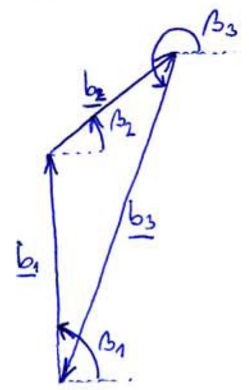
)

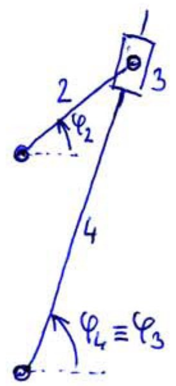
rychlosti a zrychlení

$$v_{2x} = -x_{2s_2}\dot{\beta}_2 \sin\beta_2$$
$$v_{2y} = x_{2s_2}\dot{\beta}_2 \cos\beta_2$$
$$v_{3x} = -b_2\dot{\beta}_2 \sin\beta_2$$
$$v_{3y} = b_2\dot{\beta}_2 \cos\beta_2$$
$$v_{4x} = -b_2\dot{\beta}_2\sin\beta_2 + \dot{b}_3\cos\beta_3 - b_3\dot{\beta}_3\sin\beta_3 - x_{4s_4}\dot{\beta}_3\sin\beta_3 - y_{4s_4}\dot{\beta}_3\cos\beta_3$$
$$v_{4y} = b_2\dot{\beta}_2\cos\beta_2 + \dot{b}_3\sin\beta_3 + b_3\dot{\beta}_3\cos\beta_3 + x_{4s_4}\dot{\beta}_3\cos\beta_3 - y_{4s_4}\dot{\beta}_3\sin\beta_3$$

$$a_{2x} = -x_{2s_2}\ddot{\beta}_2\sin\beta_2 - x_{2s_2}\dot{\beta}_2^2\cos\beta_2$$
$$a_{2y} = x_{2s_2}\ddot{\beta}_2\cos\beta_2 - x_{2s_2}\dot{\beta}_2^2\sin\beta_2$$
$$a_{3x} = -b_2\ddot{\beta}_2\sin\beta_2 - b_2\dot{\beta}_2^2\cos\beta_2$$
$$a_{3y} = b_2\ddot{\beta}_2\cos\beta_2 - b_2\dot{\beta}_2^2\sin\beta_2$$
$$a_{4x} = -b_2\ddot{\beta}_2\sin\beta_2 + \ddot{b}_3\cos\beta_3 - \left[(b_3+x_{4s_4})\sin\beta_3 + y_{4s_4}\cos\beta_3\right]\ddot{\beta}_3 - b_2\dot{\beta}_2^2\cos\beta_2 - 2\dot{b}_3\dot{\beta}_3\sin\beta_3 -$$
$$- b_3\dot{\beta}_3^2\cos\beta_3 - x_{4s_4}\dot{\beta}_3^2\cos\beta_3 + y_{4s_4}\dot{\beta}_3^2\sin\beta_3$$
$$a_{4y} = b_2\ddot{\beta}_2\cos\beta_2 + \ddot{b}_3\sin\beta_3 + \left[(b_3+x_{4s_4})\cos\beta_3 - y_{4s_4}\sin\beta_3\right]\ddot{\beta}_3 - b_2\dot{\beta}_2^2\sin\beta_2 + 2\dot{b}_3\dot{\beta}_3\cos\beta_3 -$$
$$- b_3\dot{\beta}_3^2\sin\beta_3 - x_{4s_4}\dot{\beta}_3^2\sin\beta_3 - y_{4s_4}\dot{\beta}_3^2\cos\beta_3$$

Úhly natočení těles, úhlové rychlosti, úhlová zrychlení





Volba φ_i v 1. kvadrantu je výhodná pro snazší sestavení rovnic

$$\varphi_2 = \beta_2$$
$$\varphi_3 = \beta_3 - \pi$$
$$\varphi_4 = \beta_3 - \pi$$

$$\omega_2 = \dot{\beta}_2$$
$$\omega_3 = \dot{\beta}_3$$
$$\omega_4 = \dot{\beta}_3$$

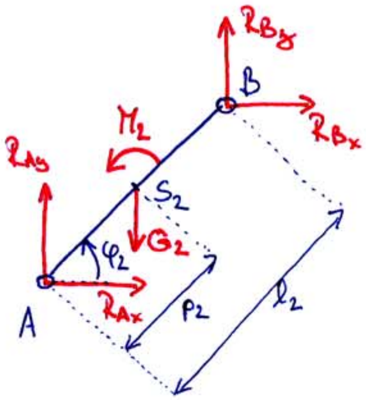
$$\alpha_2 = \ddot{\beta}_2$$
$$\alpha_3 = \ddot{\beta}_3$$
$$\alpha_4 = \ddot{\beta}_3$$

$$\underline{a} = \underline{V_z}\,\underline{\ddot{z}} + \underline{V_q}\,\underline{\ddot{q}} + \underline{a_{qz}}$$

$$\underline{V_z} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ \cos\beta_3 & -(b_3+x_{4s_4})\sin\beta_3 - y_{4s_4}\cos\beta_3 \\ \sin\beta_3 & (b_3+x_{4s_4})\cos\beta_3 - y_{4s_4}\sin\beta_3 \\ 0 & 1 \end{bmatrix} \qquad \underline{V_q} = \begin{bmatrix} -x_{2s_2}\sin\beta_2 \\ x_{2s_2}\cos\beta_2 \\ 1 \\ -b_2\sin\beta_2 \\ b_2\cos\beta_2 \\ 0 \\ -b_2\sin\beta_2 \\ b_2\cos\beta_2 \\ 0 \end{bmatrix}$$

$$\underline{a_{qz}} = \begin{bmatrix} -x_{2s_2}\dot{\beta}_2^2\cos\beta_2 \\ -x_{2s_2}\dot{\beta}_2^2\sin\beta_2 \\ 0 \\ -b_2\dot{\beta}_2^2\cos\beta_2 \\ -b_2\dot{\beta}_2^2\sin\beta_2 \\ 0 \\ -b_2\dot{\beta}_2^2\cos\beta_2 - 2\dot{b}_3\dot{\beta}_3\sin\beta_3 - [(b_3+x_{4s_4})\cos\beta_3 - y_{4s_4}\sin\beta_3]\dot{\beta}_3^2 \\ -b_2\dot{\beta}_2^2\sin\beta_2 + 2\dot{b}_3\dot{\beta}_3\cos\beta_3 - [(b_3+x_{4s_4})\sin\beta_3 + y_{4s_4}\cos\beta_3]\dot{\beta}_3^2 \\ 0 \end{bmatrix}$$

$$\underline{a} = \begin{bmatrix} a_{2x} \\ a_{2y} \\ \alpha_2 \\ a_{3x} \\ a_{3y} \\ \alpha_3 \\ a_{4x} \\ a_{4y} \\ \alpha_4 \end{bmatrix}$$

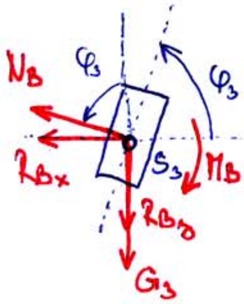


$$m_2 a_{2x} = R_{Ax} + R_{Bx}$$

$$m_2 a_{2y} = R_{Ay} + R_{By} - G_2$$

$$I_{2S_2}\alpha_2 = M_2 + R_{Ax}\cdot p_2 \cdot \sin\varphi_2 - R_{Ay}\cdot p_2 \cos\varphi_2 - R_{Bx}(l_2 - p_2)\sin\varphi_2 + R_{By}(l_2 - p_2)\cos\varphi_2$$

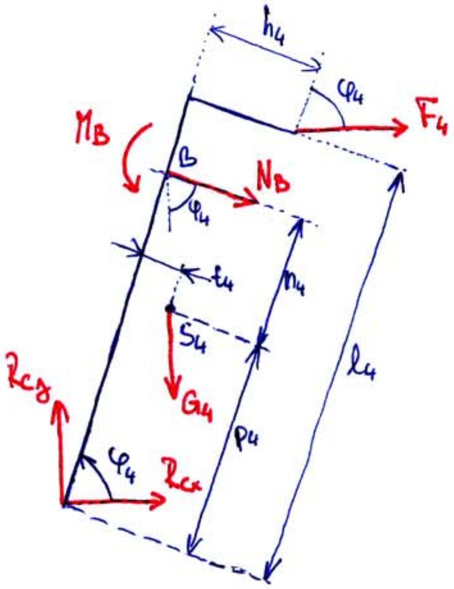
$(p_2 = x_{2S_2})$



$$m_3 a_{3x} = -R_{Bx} - N_B \sin\varphi_3$$

$$m_3 a_{3y} = N_B \cdot \cos\varphi_3 - R_{By} - G_3$$

$$I_{3S_3}\alpha_3 = -M_B$$



$$m_4 a_{4x} = R_{Cx} + F_4 + N_B \sin\varphi_4$$

$$m_4 a_{4y} = R_{Cy} - G_4 - N_B \cdot \cos\varphi_4$$

$$I_{4S_4}\alpha_4 = M_B - (R_{Cx}\cos\varphi_4 + R_{Cy}\sin\varphi_4)\, t_4 + (R_{Cx}\sin\varphi_4 - R_{Cy}\cos\varphi_4)\, p_4 - N_B n_4 + F_4\cos\varphi_4 (h_4 - t_4) - F_4 \sin\varphi_4 (l_4 - p_4)$$

$(p_4 = |x_{4S_4}|,\ t_4 = y_{4S_4},\ n_4 = b_3 - p_4)$

$$\underline{M}\,\underline{a} = \underline{D}\,\underline{R} + \underline{Q}$$

$$\underline{M} = \begin{bmatrix} m_2 & & & & & & & & \\ & m_2 & & & & & & & \\ & & I_{2S_2} & & & & & & \\ & & & m_3 & & & & & \\ & & & & m_3 & & & & \\ & & & & & I_{3S_3} & & & \\ & & & & & & m_4 & & \\ & & & & & & & m_4 & \\ & & & & & & & & I_{4S_4} \end{bmatrix}$$

$$\underline{M}\,\underline{a} = \underline{D}\,\underline{R} + \underline{Q}$$

$$\begin{bmatrix}
1 & & 1 & & & & & \\
 & 1 & & 1 & & & & \\
p_2 \sin\varphi_2 & -p_2 \cos\varphi_2 & -(l_2-p_2)\sin\varphi_2 & (l_2-p_2)\cos\varphi_2 & & & & \\
 & & -1 & & -\sin\varphi_3 & & & \\
 & & & -1 & \cos\varphi_3 & & & \\
 & & & & & -1 & & \\
 & & & & \sin\varphi_4 & & 1 & \\
 & & & & -\cos\varphi_4 & & & 1 \\
 & & & & -n_4 & 1 & -t_4\cos\varphi_4 + p_4 \sin\varphi_4 & -t_4 \sin\varphi_4 - p_4 \cos\varphi_4
\end{bmatrix}
\begin{bmatrix} R_{Ax} \\ R_{Ay} \\ R_{Bx} \\ R_{By} \\ N_B \\ M_B \\ R_{Cx} \\ R_{Cy} \end{bmatrix}$$

$$\underline{Q} = \begin{bmatrix} 0 \\ -G_2 \\ M_2 \\ 0 \\ -G_3 \\ 0 \\ F_4 \\ -G_4 \\ F_4[\cos\varphi_4 (h_4 - t_4) - \sin\varphi_4 (l_4 - p_4)] \end{bmatrix}$$

$$\begin{bmatrix} \underline{M} & -\underline{D} & \underline{O}_1 & \underline{O}_2 \\ \underline{I} & \underline{O}_3 & -\underline{V}_z & -\underline{V}_q \\ \underline{O}_4 & \underline{O}_5 & \underline{J}_z & \underline{J}_q \end{bmatrix}
\begin{bmatrix} \underline{a} \\ \underline{R} \\ \underline{\ddot{z}} \\ \underline{\ddot{q}} \end{bmatrix}
= \begin{bmatrix} \underline{Q} \\ \underline{a}_{qz} \\ -\underline{j}_{qz} \end{bmatrix}$$