Př: 1/4

Úhlové zrychlení dobíhajícího rotoru ventilátoru je dáno fcí
$\alpha = \alpha(\omega) = -\alpha_0 - k\omega$; $\alpha_0 = 0{,}5\, s^{-2}$ (konst) ; $k = 0{,}01\, s^{-1}$ (konst)
Vypočítejte $\omega = \omega(t)$; $\varphi = \varphi(t)$; $\varphi = \varphi(\omega)$ pro p.p. $t=0, \omega(0) = \omega_0$, $\varphi(0) = 0$. Počáteční otáčky $n_0 = 300\ min^{-1}$. Určete čas T, za který klesnou otáčky na $\frac{n_0}{3}$ a příslušný úhel pootočení ϕ.

Řešení:

$$\alpha = \frac{d\omega}{dt} \rightarrow dt = \frac{d\omega}{\alpha}$$

$$\int_0^t dt = -\int_{\omega_0}^{\omega} \frac{d\omega}{\alpha_0 + k\omega} = \left| \begin{matrix} x = \alpha_0 + k\omega \\ dx = d\omega \cdot k \end{matrix} \right| = -\frac{1}{k}\int_{x_0}^{x} \frac{1}{x}dx = -\frac{1}{k}\left[\ln(\alpha_0 + k\omega)\right]_{\omega_0}^{\omega} =$$

$$= -\frac{1}{k}\ln\frac{\alpha_0 + k\omega}{\alpha_0 + k\omega_0} = \frac{1}{k}\ln\frac{\alpha_0 + k\omega_0}{\alpha_0 + k\omega}$$

$$t = \frac{1}{k}\ln\frac{\alpha_0 + k\omega_0}{\alpha_0 + k\omega} \qquad \left(k \cdot t = \ln\frac{\alpha_0 + k\omega_0}{\alpha_0 + k\omega} \rightarrow e^{kt} = \frac{\alpha_0 + k\omega_0}{\alpha_0 + k\omega} \rightarrow e^{-kt} = \frac{\alpha_0 + k\omega}{\alpha_0 + k\omega_0}\right)$$

$$\underline{\underline{\omega = \left(\frac{\alpha_0}{k} + \omega_0\right)e^{-kt} - \frac{\alpha_0}{k}}}$$

$\varphi = \varphi(t)$: $\quad \omega = \frac{d\varphi}{dt} \rightarrow d\varphi = \omega\, dt$

$$\int_0^{\varphi} d\varphi = \int_0^t \left[\left(\frac{\alpha_0}{k} + \omega_0\right)e^{-kt} - \frac{\alpha_0}{k}\right]dt = \left[\left(\frac{\alpha_0}{k} + \omega_0\right)\left(-\frac{1}{k}\right)e^{-kt} - \frac{\alpha_0}{k}t\right]_0^t$$

$$\underline{\underline{\varphi}} = \left(\frac{\alpha_0}{k} + \omega_0\right)\left(-\frac{1}{k}\right)e^{-kt} - \frac{\alpha_0}{k}t - \left(\frac{\alpha_0}{k} + \omega_0\right)\left(-\frac{1}{k}\right) = \underline{\underline{\frac{1}{k}\left(\frac{\alpha_0}{k} + \omega_0\right)\left(1 - e^{-kt}\right) - \frac{\alpha_0}{k}t}}$$

$\varphi = \varphi(\omega)$ - buď vyjádřením času ze vztahů $\omega = \omega(t)$ a $\varphi = \varphi(t)$
nebo (lépe) pomocí $\alpha = \frac{\omega\, d\omega}{d\varphi}$

$$\alpha = \frac{d\omega}{dt} = \frac{d\omega}{d\varphi}\frac{d\varphi}{dt} = \frac{\omega\, d\omega}{d\varphi}$$

$$\int_0^{\varphi} d\varphi = \int_{\omega_0}^{\omega} \frac{\omega\, d\omega}{\alpha} = -\int_{\omega_0}^{\omega} \frac{\omega\, d\omega}{\alpha_0 + k\omega}$$

$$\left| \begin{array}{l} \alpha_0 + k\omega = x \;\rightarrow\; \omega = \frac{x}{k} - \frac{\alpha_0}{k} \\ d\omega = \frac{1}{k} dx \end{array} \right|$$

$$= -\int_{x_0}^{x} \frac{\frac{x}{k} - \frac{\alpha_0}{k}}{x} \cdot \frac{1}{k} dx = -\frac{1}{k}\int_{x_0}^{x} \left(\frac{1}{k} - \frac{\alpha_0}{k}\frac{1}{x}\right) dx = -\frac{1}{k^2}\int_{x_0}^{x}\left(1 - \frac{\alpha_0}{x}\right) dx =$$

$$= -\frac{1}{k^2}\Big[\alpha_0 + k\omega - \alpha_0 \ln(\alpha_0 + k\omega)\Big]_{\omega_0}^{\omega} =$$

$$= -\frac{1}{k^2}\Big(\alpha_0 + k\omega - \alpha_0 \ln(\alpha_0 + k\omega) - \alpha_0 - k\omega_0 + \alpha_0 \ln(\alpha_0 + k\omega_0)\Big) =$$

$$\varphi = -\frac{1}{k^2}\left(k(\omega - \omega_0) + \alpha_0 \ln\frac{\alpha_0 + k\omega_0}{\alpha_0 + k\omega}\right) = \frac{1}{k}\left(\omega_0 - \omega - \frac{\alpha_0}{k}\ln\frac{\alpha_0 + k\omega_0}{\alpha_0 + k\omega}\right)$$