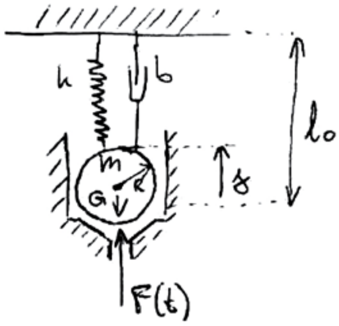
13/1

(Doporučuji pročíst kapitolu 4 ve skriptech Mechanika B)

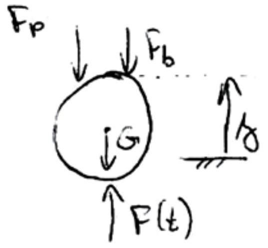


D: R, m, k, b, l_0

$F(t) = F_0 + F_1 \cos(\omega t + \varphi_F)$

p.p. $t=0 \,..\, y(0) = y_0$

$\dot{y}(0) = \emptyset$



$$m \cdot a = F(t) - F_b - F_P - G \qquad F_P = k \cdot y$$

$$F_b = b \cdot \dot{y}$$

$$m\ddot{y} + b \cdot \dot{y} + k y = F(t) - G$$

$$\ddot{y} + 2 b_r \Omega \dot{y} + \Omega^2 y = \frac{F_0}{m} - g + \frac{F_1}{m} \cos(\omega t + \varphi_F)$$

$\rightarrow \Omega = \sqrt{\frac{k}{m}},\ b_r = \frac{b}{2m\Omega},\ \Omega_b = \Omega\sqrt{1-b_r^2}$

$\eta = \frac{\omega}{\Omega}$ ... činitel naladění

$\eta = 1$ ... rezonance

Řešení: $y = y_h + y_{p1} + y_{p2}$

$$y_h = e^{-b_r \Omega t}(A \cos \Omega_b t + B \sin \Omega_b t)$$

$$y_{p1} = C \quad (\dot{y}_{p1} = \ddot{y}_{p1} = 0 \rightarrow \Omega^2 \cdot \underset{y_{p1}}{\underset{\parallel}{C}} = \frac{F_0}{m} - g \rightarrow y_{p1})$$

$$y_{p2} = r \cdot \cos(\omega t + \varphi_F - \varphi)$$

$$\left(\left. \begin{array}{l} \dot{y}_{p2} = \dots \\ \ddot{y}_{p2} = \dots \end{array} \right\} \Rightarrow \text{dosadit} \rightarrow r, \varphi \right)$$

$$y = e^{-b_r \Omega t}(A \cos \Omega_b t + B \sin \Omega_b t) + y_{p1} + r \cdot \cos(\omega t + \varphi_F - \varphi)$$

$$(\dot{y} = \dots + \text{p.p}(t=0) \rightarrow A = \dots, B = \dots)$$

Rozepsáno na další stránce

$$y_{p1} = C$$

$$\dot{y}_{p1} = \ddot{y}_{p1} = 0$$

$$\Rightarrow \quad \Omega^2 \cdot \underset{y_{p1}}{\underset{\shortparallel}{C}} = \frac{F_0}{m} - g \;\rightarrow\; y_{p1} = \frac{\frac{F_0}{m} - g}{\Omega^2} = \frac{F_0 - G}{k}$$

$$y_{p2} = r \cdot \cos\left(\omega t + \varphi_F - \varphi\right)$$

$$\dot{y}_{p2} = -r \cdot \omega \cdot \sin\left(\omega t + \varphi_F - \varphi\right)$$

$$\ddot{y}_{p2} = -r \cdot \omega^2 \cos\left(\omega t + \varphi_F - \varphi\right)$$

„finta" přičtení Ø $(\varphi - \varphi)$

$$\Rightarrow -r\omega^2 \cos(\omega t + \varphi_F - \varphi) - \underline{\underline{2 b_r \Omega r \omega}} \sin(\omega t + \varphi_F - \varphi) + \Omega^2 r \cos(\omega t + \varphi_F - \varphi) = \frac{F_1}{m} \cos(\omega t + \varphi_F)$$

$$= \frac{F_1}{m} \cos\left[(\omega t + \varphi_F - \varphi) + \varphi\right]$$

$$= \frac{F_1}{m}\left[\cos(\omega t + \varphi_F - \varphi)\cos\varphi - \sin(\omega t + \varphi_F - \varphi)\sin\varphi\right]$$

$$(\sim) \quad r\left(\Omega^2 - \omega^2\right) = \frac{F_1}{m} \cos\varphi$$

$$(=) \quad 2 b_r \Omega r \omega = \frac{F_1}{m} \sin\varphi$$

$$\Rightarrow \quad \operatorname{tg}\varphi = \frac{2 b_r \Omega r \omega}{r\left(\Omega^2 - \omega^2\right)} = \frac{2 b_r \Omega \omega}{\Omega^2 - \omega^2} = \frac{2 b_r \eta}{1 - \eta^2}$$

$$r^2\left[\left(\Omega^2 - \omega^2\right)^2 + 4 b_r^2 \Omega^2 \omega^2\right] = \left(\frac{F_1}{m}\right)^2 \left(\cos^2\varphi + \sin^2\varphi\right)$$

$$\rightarrow r^2 = \frac{\left(\frac{F_1}{m}\right)^2}{\left(\Omega^2 - \omega^2\right)^2 + 4 b_r^2 \Omega^2 \omega^2} = \frac{\frac{F_1^2}{k^2}}{\left(1 - \eta^2\right)^2 + 4 b_r^2 \eta^2}$$

$$\rightarrow r = \frac{F_1/k}{\sqrt{\left(1 - \eta^2\right)^2 + 4 b_r^2 \eta^2}}$$

$$\boxed{y = e^{-b_r \Omega t}\left(A \cos \Omega_b t + B \sin \Omega_b t\right) + y_{p1} + r \cdot \cos\left(\omega t + \varphi_F - \varphi\right)}$$

$$\dot{y} = e^{-b_r \Omega t}\left[\left(-A b_r \Omega + B \Omega_b\right)\cos \Omega_b t - \left(B b_r \Omega + A \Omega_b\right)\sin \Omega_b t\right] - r\omega \sin\left(\omega t + \varphi_F - \varphi\right)$$

$t = 0 \ldots$ $\quad y_0 = A + y_{p1} + r\cos(\varphi_F - \varphi) \quad \Rightarrow \quad A = y_0 - y_{p1} - r\cos(\varphi_F - \varphi)$

(p.p) $\quad 0 = -A b_r \Omega + B \Omega_b - r\omega \sin(\varphi_F - \varphi) \Rightarrow B = \frac{1}{\Omega_b}\left[A b_r \Omega + r\omega \sin(\varphi_F - \varphi)\right]$