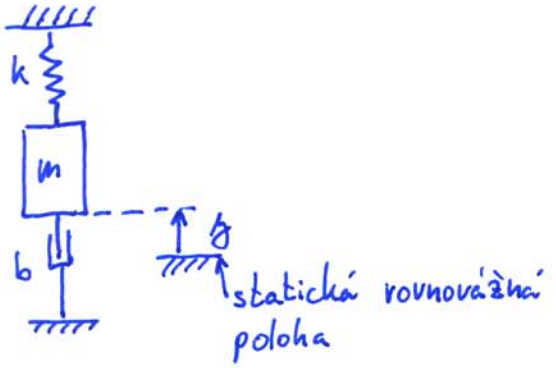
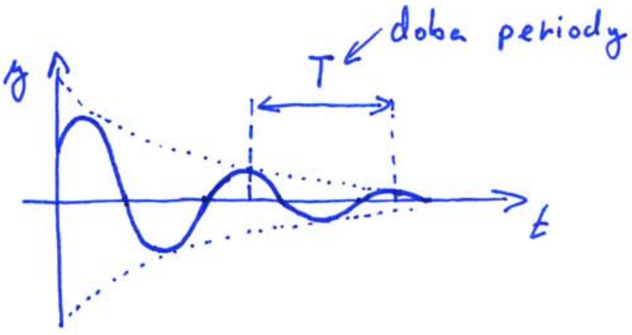
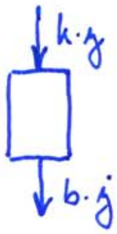
Z časového průběhu výchylky kmitající soustavy a ze známé hmotnosti určete parametry modelu soustavy – tuhost k a konstantu tlumení b.





Volné tlumené kmitání



$$m\ddot{y} = -b\dot{y} - ky$$
$$m\ddot{y} + b\dot{y} + ky = 0$$
$$\ddot{y} + \frac{b}{m}\dot{y} + \frac{k}{m}y = 0$$
$$\ddot{y} + 2b_r\Omega\dot{y} + \Omega^2 y = 0$$

$\Omega = \sqrt{\frac{k}{m}}$ ... vlastní (úhlová) frekvence netlumené soustavy

$2b_r\Omega = \frac{b}{m}$

$b_r = \frac{b}{2\Omega m} = \frac{b}{2\sqrt{km}}$

↑ poměrný útlum

charakteristická rovnice

$$\lambda^2 + 2b_r\Omega\lambda + \Omega^2 = 0$$
$$\lambda_{1,2} = -b_r\Omega \pm \Omega\sqrt{b_r^2 - 1}$$

$0 \leq b_r < 1$ .... podkritické tlumení

$\rightarrow \lambda_{1,2} = -b_r\Omega \pm i\,\Omega_b$ $\qquad$ $\Omega_b = \Omega\sqrt{1-b_r^2}$ ... vlastní (úhlová) frekvence tlumené soustavy

Řešení: $\underline{y = e^{-b_r\Omega t}(A\cdot\cos\Omega_b t + B\cdot\sin\Omega_b t)}$

výchylka v čase $t+nT$

(*)
$$\frac{y(t+nT)}{y(t)} = \frac{e^{-b_r\Omega(t+nT)}\left[A\cos(\Omega_b t + n\overbrace{\Omega_b T}^{2\pi}) + B\sin(\Omega_b t + n\overbrace{\Omega_b T}^{2\pi})\right]}{e^{-b_r\Omega t}\left[A\cos(\Omega_b t) + B\sin(\Omega_b t)\right]} = e^{-b_r\Omega nT}$$

výchylka v čase t

pro $n=1 \rightarrow \ln\frac{y(t)}{y(t+T)} = b_r\Omega T = \vartheta = \text{konst.}$ ....logaritmický dekrement

$\boxed{\text{(*) doba periody } T = \frac{2\pi}{\Omega_b}}$

Ze záznamu odečteme $y(t)$, $y(t+T)$, T; hmotnost m známe

$$\ln \frac{y(t)}{y(t+T)} = \vartheta$$

$$\vartheta = b_r \Omega T = b_r \Omega \frac{2\pi}{\Omega_b} = b_r \Omega \frac{2\pi}{\Omega\sqrt{1-b_r^2}} = \frac{2\pi \cdot b_r}{\sqrt{1-b_r^2}}$$

$$\rightarrow 1 - b_r^2 = \frac{4\pi^2 b_r^2}{\vartheta^2} \rightarrow 1 = b_r^2\left(\frac{4\pi^2+\vartheta^2}{\vartheta^2}\right) \rightarrow b_r^2 = \frac{\vartheta}{4\pi^2+\vartheta^2}$$

$$\Omega^2 = \frac{\Omega_b^2}{1-b_r^2} = \frac{\Omega_b^2}{\frac{4\pi^2+\vartheta^2-\vartheta^2}{4\pi^2+\vartheta^2}} = \frac{\Omega_b^2(4\pi^2+\vartheta^2)}{4\pi^2} = \frac{\frac{4\pi^2}{T^2}(4\pi^2+\vartheta^2)}{4\pi^2} = \frac{4\pi^2+\vartheta^2}{T^2}$$

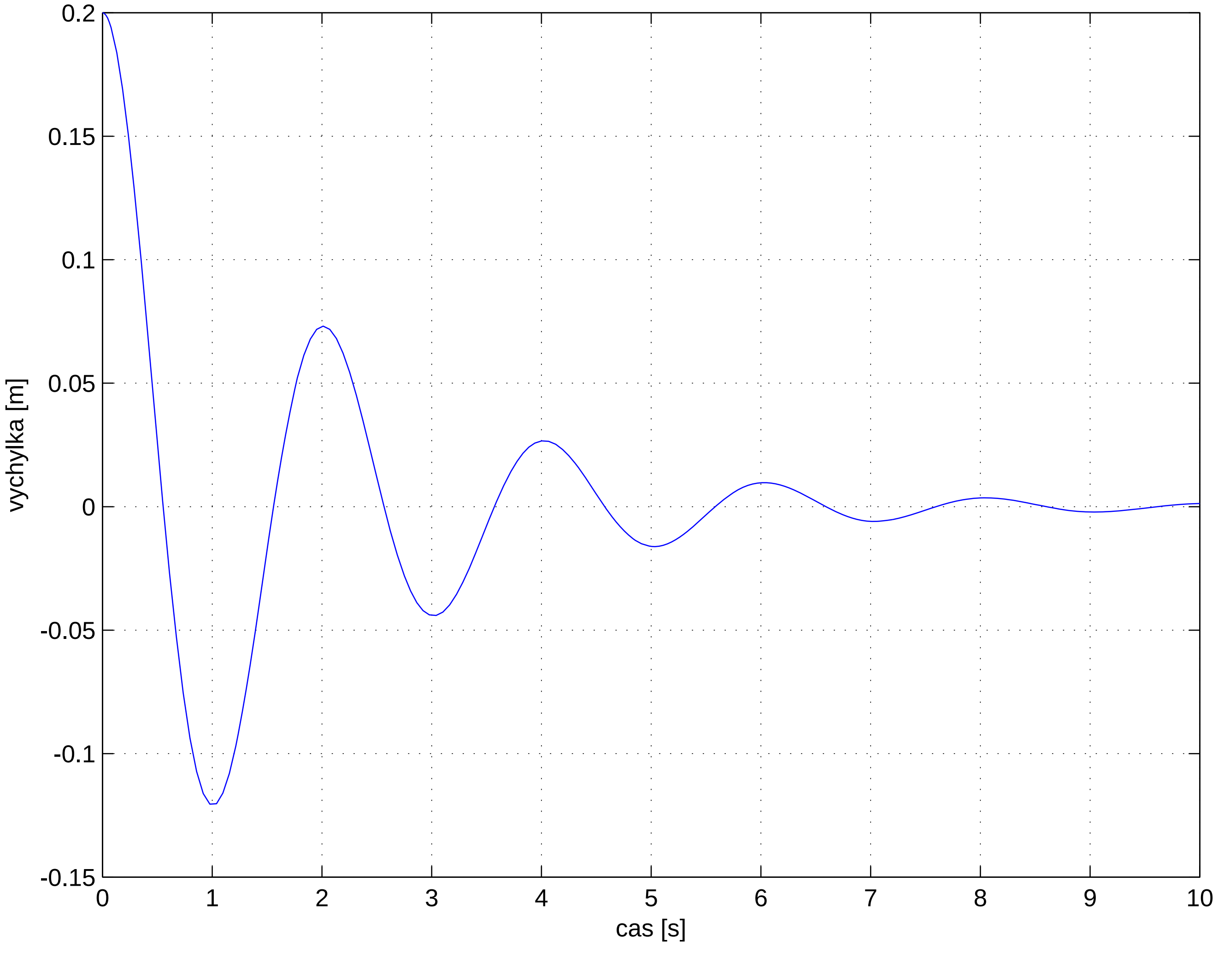
$$\Omega^2 = \frac{k}{m} \Longrightarrow \underline{\underline{k = m\frac{4\pi^2+\vartheta^2}{T^2}}}$$

$$b = 2 b_r \Omega m = 2\frac{\vartheta}{\Omega T}\Omega m = \underline{\underline{2\frac{\vartheta}{T}m}}$$

Matlab

$$\ddot{y} + 2b_r\Omega\dot{y} + \Omega^2 y = 0$$

$$\left.\begin{aligned} y &= z_1 \\ \dot{y} &= z_2 \end{aligned}\right\} \rightarrow \begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= -2b_r\Omega z_2 - \Omega^2 z_1 \end{aligned}$$


0.2
0.15
0.1
0.05
0
-0.05
-0.1
-0.15
vychylka [m]
0
1
2
3
4
5
6
7
8
9
10
cas [s]

```
% Tlumene kmitani
global br om

% parametry
m = 50      % hmotnost
k = 500     % tuhost pruziny
b = 50      % tlumeni

om=sqrt(k/m)        % vl. frekvence
br=b/(2*sqrt(k*m))  % pomerny utlum

y0=0.2      % pocatecni vychylka
y0dot=0     % pocatecni rychlost

t_konec=10  % delka simulace [s]

[t,y]=ode45(@model,[0 t_konec],[y0 y0dot]);
plot(t,y(:,1));
xlabel('cas [s]');
ylabel('vychylka [m]');
grid on

% hodnoty odectene z grafu
% y(2.01)=0.073
% y(4)=0.0266
% T = 4 - 2.01
% theta = log(0.073/0.0266)
% b = 2*theta/T*m = 50.73

-------------------------------------------------------------------------

function dz=model(t,z)

global br om

dz=zeros(2,1);
dz(1)=z(2);
dz(2)=-2*br*om*z(2)-om^2*z(1);
```