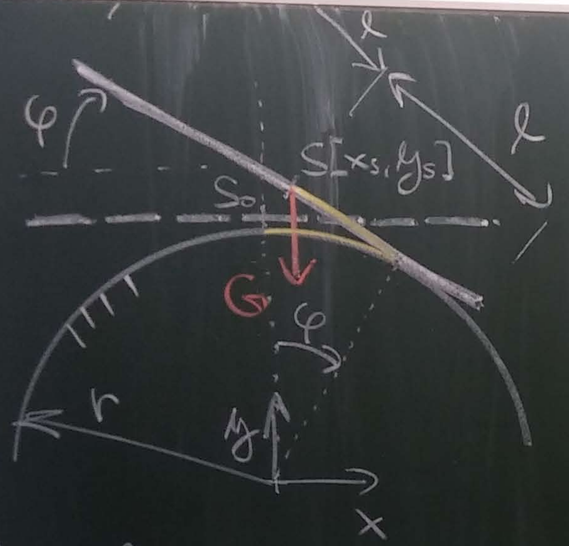

D: r, l, m, I_S

U: VPR (LR II. druhu)

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_i}\right) - \frac{\partial E_k}{\partial q_i} = Q_i$$

$$E_k = \frac{1}{2} m \dot{x}_S^2 + \frac{1}{2} m \dot{y}_S^2 + \frac{1}{2} I_S \dot{\varphi}^2$$

$q_r = \varphi$ (1 st. volnosti)

$x_S = r \sin\varphi - r\varphi\cos\varphi$

$y_S = r\cos\varphi + r\varphi\sin\varphi$

$\dot{x}_S = r\cos\varphi\,\dot{\varphi} - r\dot{\varphi}\cos\varphi + r\varphi\sin\varphi\,\dot{\varphi}$

$\dot{y}_S = -r\sin\varphi\,\dot{\varphi} + r\dot{\varphi}\sin\varphi + r\varphi\cos\varphi\,\dot{\varphi}$

$\mathrm{d}y_S = r\varphi\cos\varphi\,\mathrm{d}\varphi$

$$\dot{x}_S^2 + \dot{y}_S^2 = r^2\varphi^2\dot{\varphi}^2\sin^2\varphi + r^2\varphi^2\dot{\varphi}^2\cos^2\varphi = (r\varphi)^2\dot{\varphi}^2$$

$$E_k = \frac{1}{2} m (r\varphi)^2 \dot{\varphi}^2 + \frac{1}{2} I_S \dot{\varphi}^2 = \frac{1}{2}\left[m(r\varphi)^2 + I_S\right]\dot{\varphi}^2$$

$$\frac{\partial E_k}{\partial \dot{\varphi}} = \frac{1}{2}\left[m(r\varphi)^2 + I_S\right] 2\dot{\varphi}$$

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\varphi}}\right) = \left[mr^2 2\varphi\dot{\varphi}\right]\dot{\varphi} + \left[m(r\varphi)^2 + I_S\right]\ddot{\varphi}$$

$$= \left[m(r\varphi)^2 + I_S\right]\ddot{\varphi} + 2mr^2\varphi\dot{\varphi}^2$$

$$\frac{\partial E_k}{\partial \varphi} = \frac{1}{2}\left[mr^2 2\varphi\right]\dot{\varphi}^2 = mr^2\varphi\dot{\varphi}^2$$

$$Q\,\mathrm{d}q_r = Q\,\mathrm{d}\varphi = -G\,\mathrm{d}y_S \Rightarrow Q\,\mathrm{d}\varphi = -Gr\varphi\cos\varphi\,\mathrm{d}\varphi$$

$$\boxed{\left[m(r\varphi)^2 + I_S\right]\ddot{\varphi} + 2mr^2\varphi\dot{\varphi}^2 - mr^2\varphi\dot{\varphi}^2 = -Gr\varphi\cos\varphi}$$ VPR