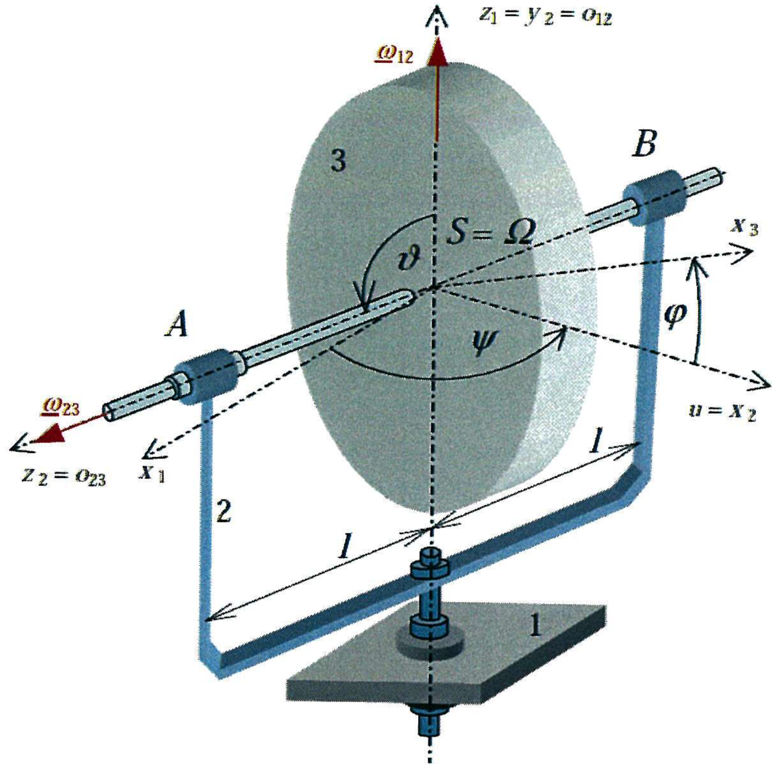
Sestavte vlastní pohybové rovnice setrvačníku 3, který rotuje kolem osy o_{23} konstantní úhlovou rychlostí ω_{23} a spolu s unášečem 2 rotuje kolem osy o_{12} konstantní úhlovou rychlostí ω_{12}. Pro zadané číselné hodnoty vypočítejte velikost reakcí v ložiskách A a B. Pasivní odpory neuvažujte.

Dáno:
$I_{3\,o_{23}} = 0{,}004\ \text{kgm}^2$,
$l = 0{,}1\ \text{m}$,
$m = 1{,}25\ \text{kg}$,
$\omega_{12} = 50\ \text{s}^{-1}$,
$\omega_{23} = 600\ \text{s}^{-1}$,
$\vartheta = \pi/2$.



Euler. kinem. rovnice:

$$^3\omega_{13x} = \dot\psi \sin\vartheta \sin\varphi + \dot\vartheta \cos\varphi$$
$$^3\omega_{13y} = \dot\psi \sin\vartheta \cos\varphi - \dot\vartheta \sin\varphi$$
$$^3\omega_{13z} = \dot\psi \cos\vartheta + \dot\varphi$$

→ tento příklad $\vartheta = \text{konst}$, $\dot\psi = \omega_{12} = \text{konst.}$, $\dot\varphi = \omega_{23} = \text{konst.}$

$$\Rightarrow \left.\begin{aligned} ^3\omega_{13x} &= \dot\psi \sin\vartheta \sin\varphi \\ ^3\omega_{13y} &= \dot\psi \sin\vartheta \cos\varphi \\ ^3\omega_{13z} &= \dot\psi \cos\vartheta + \dot\varphi \end{aligned}\right\} \Rightarrow \begin{aligned} ^3\alpha_{13x} &= \dot\psi\dot\varphi \sin\vartheta \cos\varphi \\ ^3\alpha_{13y} &= \dot\psi\dot\varphi \sin\vartheta \sin\varphi \\ ^3\alpha_{13z} &= \emptyset \end{aligned}$$

Eulerovy dynamické rovnice:

$$\frac{d\vec{L}}{dt} = \sum M \qquad \left[\frac{d\vec{L}}{dt}\right]_1 = \left[\frac{d\vec{L}}{dt}\right]_3 + {}^3\omega_{13} \times {}^3L \qquad {}^3L = {}^3I \cdot {}^3\omega_{13}$$

$$\left({}^3I\,{}^3\alpha_{13} + {}^3\omega_{13} \times {}^3I \cdot {}^3\omega_{13} = \sum_i M_i\right)$$

Pro ${}^3 I = \begin{bmatrix} I_x & & \\ & I_y & \\ & & I_z \end{bmatrix}$

$$I_{x_3} \cdot {}^3\alpha_{13x} + {}^3\omega_{13y} \cdot {}^3\omega_{13z} (I_{z_3} - I_{y_3}) = M_{x_3}$$
$$I_{y_3} \cdot {}^3\alpha_{13y} + {}^3\omega_{13x} \cdot {}^3\omega_{13z} (I_{x_3} - I_{z_3}) = M_{y_3}$$
$$I_{z_3} \cdot {}^3\alpha_{13z} + {}^3\omega_{13x} \cdot {}^3\omega_{13y} (I_{y_3} - I_{x_3}) = M_{z_3}$$

$$\Rightarrow I_{x_3} = I_{y_3} = I \quad ; \; I_{z_3} = I_0 \Rightarrow$$

$$I \cdot \dot\psi \dot\varphi \sin\vartheta \cos\varphi + \dot\psi \sin\vartheta \cos\varphi (\dot\psi \cos\vartheta + \dot\varphi) \cdot (I_0 - I) = M_{x_3}$$
$$I \cdot \dot\psi \dot\varphi \sin\vartheta \sin\varphi + \dot\psi \sin\vartheta \sin\varphi (\dot\psi \cos\vartheta + \dot\varphi) \cdot (I - I_0) = M_{y_3}$$
$$\emptyset = M_{z_3}$$

Musí platit pro všechny polohy dané úhlem φ

$\rightarrow$ tedy i pro polohu (tj. $\varphi = 0$) $\left.\begin{matrix} \sin\varphi = \emptyset \\ \cos\varphi = 1 \end{matrix}\right\}$ (osy x_3', y_3', z_3')

$$\Rightarrow I \cdot \dot\psi \dot\varphi \sin\vartheta + \dot\psi \sin\vartheta (\dot\psi \cos\vartheta + \dot\varphi)(I_0 - I) = M_{x_3}$$
$$\emptyset = M_{y_3}$$
$$\emptyset = M_{z_3}$$

$$\Rightarrow I \dot\psi \dot\varphi \sin\vartheta + I_0 \dot\psi^2 \sin\vartheta \cos\vartheta + \dot\psi \sin\vartheta \, \dot\varphi I_0 - \dot\psi^2 \sin\vartheta \cos\vartheta \, I - \dot\psi \sin\vartheta \dot\varphi I = M_{x_3}$$

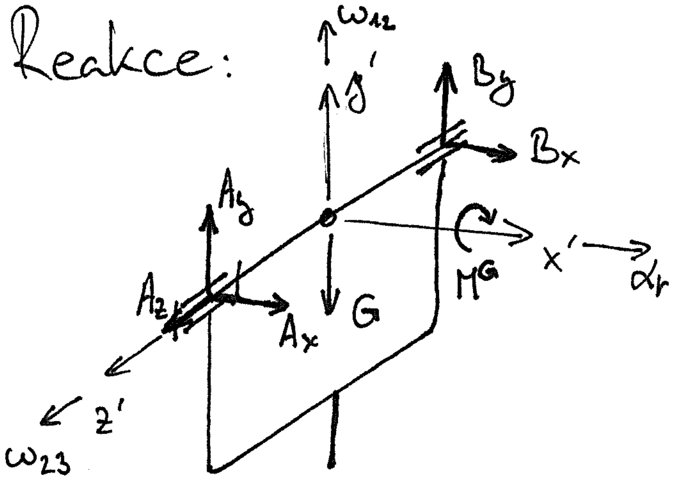
$$I_0 \dot\psi \dot\varphi \sin\vartheta + \dot\psi^2 \sin\vartheta \cos\vartheta (I_0 - I) = M_{x_3}$$

$$\dot\psi \dot\varphi \sin\vartheta \left[I_0 + \frac{\dot\psi}{\dot\varphi} \cos\vartheta (I_0 - I) \right] = M_{x_3} \qquad (\vec a \times \vec b = |a| \cdot |b| \cdot \sin\gamma)$$

je-li $\underline{\dot\varphi \gg \dot\psi}$ $\rightarrow$ $\emptyset = M_{x_3} + M^G$, kde $M^G = -I_0 \cdot \underline{\dot\psi} \times \underline{\dot\varphi} = -I_0 \cdot \underline{\alpha_r}$

$$m \, \underline{a_S} = \sum \underline{F_i}$$
$$0 = M_{x_3} + M^G$$
$$0 = M_{y_3}$$
$$0 = M_{z_3}$$

Reakce:



$$0 = A_x + B_x$$

$$0 = A_y + B_y - G$$

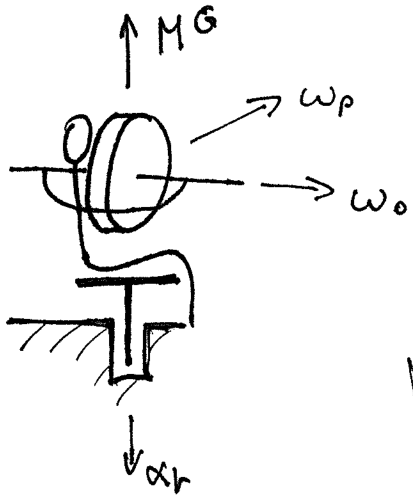
$$0 = A_z$$

$$0 = B_y \cdot l - A_y \cdot l - \overbrace{I_0 \omega_{12} \omega_{23}}^{M^G}$$

$$0 = A_x \cdot l - B_x \cdot l$$

$$|\alpha_r| = \omega_{12}\, \omega_{23} \sin\frac{\pi}{2} = \omega_{12}\omega_{23}$$

Otočná stolička



$$M^G = -I_o \cdot \omega_p \times \omega_o = -I_o \cdot \alpha_r$$