PARALLEL FLOW ASYMMETRIES IN HEAT EXCHANGERS

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Abstract. The effect of flow asymmetry was observed experimentally in lateral parallel channels of continuous direct ohmic heater. While the flow is uniform at isothermal conditions, flow-rate in parallel channels differs in case of heating. The effect can be explained by buoyancy and theoretical analysis predicts existence of two solutions, symmetric and asymmetric distribution of flow-rates, which can be stable only within a certain range of temperatures and flow-rates. Three different configurations of parallel flows (with and without internal heating in channels) are analysed, giving critical values of dimensionless criteria $Z$, $Z_G$, $Z_H$ ensuring stable symmetric distribution of flow into parallel channels.

Experimental verification was based upon a) flow visualisation (injection of a coloured tracer and monitoring the tracer by Canon MV-100 camera), b) measurement of temperature profiles (11 thermometers Pt100), and on c) stimulus response experiments using KCl as a tracer for conductivity methods (2 Pt conductivity probes) and Tc99 as a radioisotope tracer (collimated scintillation detectors). Results confirm predicted influence of operational parameters and geometry on the stability of flow.

INTRODUCTION

Parallel flows are typical for many important apparatuses of process industries, e.g. flows in shell&tube or plate heat exchangers, heaters, reactors. Sometimes flow irregularities, instabilities or just only non-uniform distribution of flow in parallel channels occur. These undesirable phenomena can be caused by natural convection if the apparatus operates at non-isothermal conditions, which is typical for heat exchangers or heaters.

The effect of flow asymmetry was observed experimentally in lateral parallel channels of continuous direct ohmic heater, with two planar electrodes (electrical current flows directly through the heated liquid), see Fig. 1. Principle of operation is as follows: Liquid enters the top of heater and flows downwards through two rectangular channels where liquid is preheated only by warm electrodes. At the bottom of heater the two parallel streams join and liquid flows upward in a nearly uniform electrical field between electrodes (distance 0.036 m,
voltage 220 V, 50 Hz). In order to suppress the electrode fouling, a perforation of electrodes was suggested, assuming that the cold cross-flow could displace overheated substance moving slowly along the electrode surface, see Žitný et al (1999), (2000) for more details.

While the flow in parallel channels is uniform at isothermal conditions, a nonuniform distribution of flowrate and different temperature profiles are developed in parallel channels at heating and also the cross flow is changed. These phenomena can be explained by the effect of buoyancy in the buoyancy opposed flow, e.g. in a downward flow into a heated section.

The combined forced and natural convection in vertical channels has been analysed by many authors, e.g. Dutta (1999), or Joyce (1996) present relationships between Nusselt number, Reynolds number Re and Grashof number Gr for buoyancy assisted and buoyancy opposed convection in either laminar and turbulent flow regime. Flow instabilities and flow reversal due to buoyancy induced recirculation bubbles have been observed and criteria for flow reversal to occur have been suggested by Evans (1996) (Gr/Re²), Cheng (2000) (Gr/Re). On the other hand the problem of buoyancy opposed flow in parallel channels and the flow reversal onset is only scarcely mentioned. A little bit similar case of mixed convection in two vertical flow channels with heat sources between them (the application concerns nuclear reactors and liquid metals as a working liquid) has been analysed by Kim (2001) using a one-dimensional model.

STABILITY OF PARALLEL FLOWS-CONSTANT WALL TEMPERATURE

Parallel laminar flows in lateral channels of continuous ohmic heater can lose symmetry in case of heating: one stream is delayed and even stopped or reversed if the temperature increase is too high. A similar situation occurs in a simpler and probably more frequent case when two vertical parallel streams are separated by wall having a constant temperature \( T_e \), see Fig.2. It is assumed that

- the liquid having temperature \( T_0 \) enters two identical rectangular channels (cross section \( H \times B \), length \( L \)) where is heated from the wall; it does not matter whether all four or just only one side of channel are hold at temperature \( T_e \) – the only difference is heat transfer surface. Heat transfer coefficient \( \alpha \) [W.m⁻².K⁻¹] is considered constant. Pressures at inlet of both channels equal, \( p_l(0)=p_r(0) \), and the same holds at outlet into an infinite reservoir \( p_l(L)=p_r(L) \).
- Flow in parallel channels is laminar, fully developed and internal recirculation due to nonuniform transversal temperature profile is negligible.
- Heat exchanger is perfectly insulated.

![Figure 2. Parallel flows heated by wall at constant temperature \( T_e \).](image-url)
Distribution of flow-rate will be expressed in terms of relative flow-rate in the left channel $\psi$

$$Q_l = \psi Q, \quad Q_r = (1 - \psi)Q. \quad (1)$$

The value $\psi = 0.5$ corresponds to the symmetric distribution of flow-rates, while $\psi = 0, \psi = 1$ corresponds to the completely stopped flow in the left or in the right channel.

The following analysis is based upon the fact that the overall pressure difference $p(L) - p(0)$ must be the same in the left and in the right channel at a steady state. We shall consider only two contributions to the pressure difference: the first represents viscous forces (written for flowrate $Q_l = \psi Q [m^3.s^{-1}]$ in the left channel)

$$p_f(0) - p_f(L) = \frac{12\mu f}{BH^3} Q_l \quad (2)$$

where $B, H [m]$ are dimensions of rectangular cross-section (depth and width of channel respectively) and $\mu [Pa.s]$ is dynamic viscosity. The coefficient $f$ equals 1 for fully developed laminar flow between infinite plates (in this case $f$ represents only the effect of friction) or

$$f = \frac{1}{1 - 192 \frac{H}{B}} \sum_{n=1,3,5,...} \frac{1}{n^4} \tgh \frac{n\pi B}{2H} + \frac{H\rho Q_l}{24BL\mu} \quad (3)$$

for laminar flow in a rectangular channel $B \times H$. The first term in Eq.(3) represents correction to the finite ratio of $B/H$ (rectangular channel) and the second term is a correction for dynamic pressure ($\Delta p \approx 0.5 \rho u^2$).

While the component of pressure $p_f(x)$ accordant with friction forces decreases in the direction of flow, the hydrostatic pressure $p_b(x)$ increases depending upon the axial temperature profile. For a constant value of the heat transfer coefficient $\alpha [W.m^{-2}.K^{-1}]$, constant heat capacity $c_p [J.\text{kg}^{-1}.K^{-1}]$ and density $\rho_0$ the axial temperature profiles are given by

$$\frac{T_l - T_e}{T_0 - T_e} = e^{-\frac{x}{W_L}}, \quad \frac{T_r - T_e}{T_0 - T_e} = e^{-\frac{x}{W_R}}, \quad (4)$$

where $W_l = \psi W$, $W_r = (1 - \psi)W$, and $W = \frac{\rho_0 c_p}{\alpha BL}$.

The dimensionless parameter $W$ is reciprocal value Stanton criterion.

Assuming linear dependence of density upon temperature (characterised by proportionality coefficient $\beta [K^{-1}]$) and the exponential temperature profiles in parallel channels Eq.(4), the contribution of hydrostatic pressure can be expressed as

$$p_b(0) - p_b(L) = -\rho_0 g \int_0^L [1 + \frac{\beta (T_l - T_0)]dx = -\rho_0 g L \{1 + \beta (T_0 - T_e)[1 - W_l(1 - e^{-\frac{x}{W_L}})]\} \quad (6)$$
Summing pressure differences corresponding to the friction forces (2) and buoyant forces (6) we can express equilibrium of forces in the left (lower index \(l\)) and right (index \(r\)) channel by equation

\[
\frac{12 \mu Lf}{BH^3} (Q_l - Q_r) = \rho_0 g L \beta (T_e - T_0) [W_l (1 - e^{-1/W_l}) - W_r (1 - e^{-1/W_r})].
\] (7)

Eq.(7) can be rearranged to a dimensionless form by introducing Grashoff and Reynolds numbers \(Gr\), \(Re\) and the criterion \(Z\) characterising relative influence of buoyancy with respect to forced flow

\[
Z = (T_e - T_0) \frac{g BH^3 \beta \rho_0}{12 \mu^2 Q} = \frac{Gr}{96 Re}, \quad Gr = \frac{\rho_0^2 g \beta (T_e - T_0) (2H)^3}{\mu^2}, \quad Re = \frac{Q \rho_0}{B \mu}.
\] (8)

The balance of forces expressed in terms of \(Z\) and \(W\) criteria has the following form

\[
2 \psi - 1 = ZW[(1 - e^{-1/\psi}) - (1 - \psi)(1 - e^{-1/(1-\psi)W})]
\] (9)

The balance of forces Eq.(9) is satisfied by the symmetric solution \(\psi=0.5\) for any combination of parameters \(Z\), and \(W\), however an asymmetric solution can also exist for sufficiently high values of \(Z\), see Fig.3

![Figure 3. Z(W) for asymmetric distributions of flow-rate \(\psi=0.001,0.005,0.01\), Eq.(9).](image)

The curve corresponding to \(\psi=1\) or 0 (all liquid flows in the one channel only) is described by equation following immediately from Eq.(9)

\[
1 = ZW(1 - e^{-1/W}).
\] (10)

For the values of \(Z\) bellow this curve (for example for any \(Z<1\)) the symmetric solution must be stable because it is the only solution satisfying the balance (7). Nevertheless, the symmetric solution could be possibly stable even for higher value of \(Z\). To analyse this, let us
assume a small disturbance of flow-rate $Q_l + \delta Q$, $Q_r - \delta Q$, i.e. slightly increased flow-rate in the left channel and properly decreased flow-rate in the right channel. Assuming unchanged pressures at outlet (infinite reservoir) the pressure at the left channel inlet will be changed by increment (see Eq. (7)),

$$\delta p_l = \delta p_l(0) = \delta Q \left( \frac{12 \mu L_f}{BH^3} - (T_c - T_0) \frac{p_0^2 c_p g \beta}{\alpha B} \left(1 - \frac{W_i + 1}{W_i} e^{-1/W_r}\right) \right) \tag{11a}$$

and similarly in the right channel

$$\delta p_r = \delta p_r(0) = -\delta Q \left( \frac{12 \mu L_f}{BH^3} - (T_c - T_0) \frac{p_0^2 c_p g \beta}{\alpha B} \left(1 - \frac{W_r + 1}{W_r} e^{-1/W_r}\right) \right). \tag{11b}$$

In the case that $\delta p_l > \delta p_r$, the pressure at the inlet to the left channel will be slightly higher than the pressure in the right channel and this difference induces transversal flow towards the right channel. This redistribution of flow acts against the disturbance $\delta Q$, which means that the flowrates $Q_l, Q_r$ will be stable. The stability condition $\delta p_l - \delta p_r > 0$ for $\delta Q > 0$ can be rewritten using Eqs. (11a) and (11b) into dimensionless form

$$1 > ZW \left(1 - \frac{W_r + 1}{2W_r} e^{-1/W_r} - \frac{W_i + 1}{2W_i} e^{-1/W_i}\right) \tag{12}$$

This general inequality (12) can be applied to the symmetric solution when $W_r = W_i = W/2$, giving:

$$1 > ZW \left(1 - \frac{W + 2}{W} e^{-2/W}\right). \tag{13}$$

This inequality, together with Eq. (10) is presented in Fig. 4

Figure 4. Regions of unconditionally and conditionally stable symmetric solution

This Figure demonstrates that there exists a rather wide range of $Z$ where a symmetric flow distribution could, but need not exist (stability of symmetric distribution depends upon magnitude of flow-rate disturbance).
\[
\frac{1}{W(1-e^{-\frac{1}{W}})} < Z < \frac{1}{W-(W+2)e^{-\frac{2}{W}}}.
\]

(14)

For a small temperature increase, i.e. for small values of Stanton number (W>5) this region can be characterised by a simple inequality

\[
\frac{4}{2W-1} < \frac{2Z}{W} = \frac{(T_e-T_0)g\alpha\beta H^3B^2L}{6\mu f c_p Q^2} < 1.
\]

(15)

A similar stability analysis can be performed also for the case of asymmetric solution, e.g. for the case of very small \(\psi<<0.1\). In this case the Eq.(9), balance of forces, reduces to

\[
1 = ZW[1 - \frac{1-\psi}{1-2\psi}e^{-\frac{1}{(1-\psi)W}}]
\]

and the inequality (12) to

\[
1 > ZW[1 - \frac{W(1-\psi)}{2W(1-\psi)}e^{-\frac{1}{(1-\psi)W}}].
\]

(17)

Combining (16) and (17)

\[
(1-\psi)^2 < (1-\psi)^2 - \frac{\psi}{2}
\]

(18)

we arrive to the conclusion that the asymmetric solution cannot be stable for any positive \(\psi\). This conclusion casts a new light to the physical meaning of the asymmetric solution: It represents a magnitude of the flow-rate disturbance which is necessary to make the symmetric solution unstable in the range of \(Z\) given by inequalities (14).

STABILITY OF PARALLEL FLOWS-VOLUMETRIC HEAT SOURCE IN CENTRAL CHANNEL

A similar procedure can be applied for the case of ohmic heater shown in Fig.1, not considering cross flow through perforated wall. The only principal difference is in temperature profiles in parallel channels, because the wall temperature \(T_e\) is no longer a constant. The axial temperature profiles follow from the following assumptions:

- Temperature depends only on axial coordinate \(x\) (or dimensionless coordinate \(\xi=x/L\)).
- Heat transfer coefficient \(\alpha\ [W.m^2.K^{-1}]\) comprises thermal resistances of liquid layers in the lateral and central channels and also thermal resistance of wall (of electrode separating lateral and central channel). This coefficient is the same in the both lateral channels.
- The heater is perfectly insulated.
- Two streams flowing out of the lateral channels are ideally mixed at the bottom of heater and flow upwards between electrodes, see Fig.5. There is a uniform volumetric source of heat in the central channel characterised by intensity of heat generation \(G\ [W/m^3]\).
Temperature profiles in lateral channels (cross-section $H \times B$) are described by equations

$$Q_r \rho c_p \frac{dT_l}{dx} = B \alpha (T - T_l) \tag{19}$$

$$Q_r \rho c_p \frac{dT_r}{dx} = B \alpha (T - T_r) \tag{20}$$

while the temperature in the central channel (cross-section $H_c \times B$) is governed by equation

$$-(Q_l + Q_r) \rho c_p \frac{dT}{dx} = B \alpha (T_l + T_r - 2T) + GH_i B \tag{21}$$

This system of equations can be solved analytically giving temperature profiles in lateral channels in the form

$$T_l - T_0 = \frac{T_G}{F} [(W_r - M(1 - W_l) - M^2) (1 - e^{-\xi/M}) + (1 + M - W_r) \xi - \frac{\xi^2}{2} ] \tag{22}$$

$$T_r - T_0 = \frac{T_G}{F} [(W_l - M(1 - W_r) - M^2) (1 - e^{-\xi/M}) + (1 + M - W_l) \xi - \frac{\xi^2}{2} ] \tag{23}$$

where

$$T_G = \frac{GH_i}{\alpha}, \quad F = W_l^2 + W_r^2, \quad M = \frac{W_l W_r (W_l + W_r)}{W_l^2 + W_r^2}, \quad \xi = \frac{x}{L}, \tag{24}$$

and the meaning of $W_l, W_r$ is the same as previously, see Eq.(5). $T_G$ is a reference temperature proportional to the intensity of heating in the central channel $G$ [W.m$^{-3}$]. The ratio $T_G/F$ is proportional to both $G$ and the overall heat transfer coefficient $\alpha$ [W.m$^{-2}$.K$^{-1}$] between central and lateral channels.

These temperature profiles enable to express pressure differences corresponding to buoyancy (using similar procedure and assumptions as in Eq.(6))
\[ p_b(0) - p_b(L) = -\rho_0 g L \left( 1 - \beta \frac{T_G}{F} \right) \left[ (W_r - M(1 + M - W_r))(1 - M(1 - \frac{1}{M})) + \frac{1 + M - W_r}{2} - \frac{1}{6} \right] \] (25)

Friction forces are the same as previously so that we can immediately write balance of forces in the left and right lateral channel (we need not take into account the central channel)

\[ \frac{12 \mu L_f}{B H^3} (Q_i - Q_r) = \rho_0 g L \beta \frac{T_G}{F} (W_i - W_r)((1 + M)(1 - M(1 - e^{-1/M})) - \frac{1}{2}) \] (26)

This equation can be rewritten into dimensionless form

\[ 1 = Z_G \frac{(1 + M)(1 - M(1 - e^{-1/M})) - \frac{1}{2}}{\psi^2 + (1 - \psi)^2} \] (27)

where \( M \) is a function of \( W \) and \( \psi \)

\[ M = \frac{W_r(W_i + W_r)}{W_i^2 + W_r^2} = W_r \frac{\psi (1 - \psi)}{\psi^2 + (1 - \psi)^2} \] (28)

and the dimensionless number \( Z_G \)

\[ Z_G = \frac{g \alpha \beta T_G B^2 H^3 L}{12 \mu c_p Q^2} \] (29)

reminds \( Z/W \) in Eq.(15), the only difference is that the temperature \( T_G \) substitutes the temperature difference \( T_e - T_0 \). For practical calculations it is possible to use specific power \( G \) [W.m\(^{-3}\)], total power \( P \) [W] or corresponding adiabatic temperature increase \( T_{\text{max}} - T_0 \) in the definition of \( Z_G \)

\[ Z_G = \frac{g \alpha \beta T_G B^2 H^3 L}{12 \mu c_p Q^2} = \frac{g \beta G H}{12 \mu c_p Q^2} = \frac{g \beta P B H^3}{12 \mu c_p Q^2} = \frac{g \beta P B H^3}{12 \mu c_p Q^2} = \frac{96 \mu B}{Q_0} = \frac{96 J Z_G}{\text{Re}} \] (30)

This criterion can be expressed in terms of Richardson number \( Ri \), which allows to evaluate relative importance of free and forced convection in case of volumetric heat source

\[ Ri = \frac{Gr}{Re^2} = \frac{g \beta P}{\rho_0 c_p u^4 B} = \frac{g \beta P}{\rho_0 c_p (Q / (2BH))^3 B} = \frac{g \beta P B H^3}{12 \mu c_p Q^2} = \frac{96 \mu B}{Q_0} = \frac{96 J Z_G}{\text{Re}} \]

The Eq.(27) and the region of \( W, T_G \) where the asymmetric solution can exist is represented in graphical form in Fig.6
The solutions of Eq.(27) for $\psi=0$, and $\psi=0.5$ give conditional stability limits of symmetric solution

$$2 < Z_G < \frac{1}{(2+W)[1-\frac{W}{2}(1-e^{-2/W})]-1} \approx \frac{W}{2}. \quad (31)$$

This statement, inequality (31), can be proved rigorously repeating the stability analysis applied for derivation Eq.(12). Thus we arrive to the general stability constraint

$$2 > \frac{Z_G}{\psi^2 + (1-\psi)^2} [2(1+m)[1-M(1-e^{-\frac{1}{M}})]-1-$$

$$\frac{(1-2\psi)^2}{\psi^2 + (1-\psi)^2} \left[ \frac{(1+2M+2M^2)e^{-\frac{1}{M}}-2M^2}{\psi(1-\psi)} + 2(1+M)[1-M(1-e^{-\frac{1}{M}})]-1 \right]}$$

which reduces to (31) for $\psi=0.5$, i.e. for the case of symmetric flow.

**STABILITY OF PARALLEL FLOWS - OTHER FLOW CONFIGURATIONS**

There are several other possibilities of the parallel flows arrangement – volumetric heating in parallel channels (with and without heat transfer between parallel streams), co-current flow in the central channel and so on. We confine oneself to the summary of the two previous cases and the case of volumetric heating in parallel channels in the following table.
Tab.1 Criteria ensuring uniform distribution of flow between parallel channels

<table>
<thead>
<tr>
<th>Case</th>
<th>Criterion</th>
<th>Stability limits of symmetric flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant temperature of wall $T_e$</td>
<td>$Z = (T_e - T_0) \frac{g\beta H^3 B \rho_0}{12\mu Q}$</td>
<td>$Z &lt; \frac{1}{W - (W + 2)e^{-2/W}}$</td>
</tr>
<tr>
<td></td>
<td>$Q$ – sum of flowrates in parallel channels</td>
<td>$W = \frac{1}{St} = \frac{Q e}{aBL}$</td>
</tr>
<tr>
<td>Volumetric heat source in central channel</td>
<td>$Z_G = \frac{g\beta GH B^2 H^3 L}{12\mu c_p Q^2}$</td>
<td>$Z_G &lt; \frac{1}{(2 + W)[1 - \frac{W}{2}(1 - e^{-2/W})] - 1}$</td>
</tr>
<tr>
<td>Volumetric heat source in lateral channels (insulated wall)</td>
<td>$Z_H = \frac{g\beta GB^2 H^4 L}{12\mu c_p Q^2}$ (33)</td>
<td>$Z_H &lt; \frac{1}{2}$ (34)</td>
</tr>
</tbody>
</table>

This table offers a very simple method of parallel flows assessment. However, these simple criteria result from the idea, that recirculations or flowrate asymmetries appear initially between channels and not inside a channel alone. This question is shortly (and by no way definitely) discussed in the following section.

**NATURAL CONVECTION IN LATERAL CHANNELS**

The previous analysis of parallel flows was based on assumption, that the effect of internal recirculation in lateral channels can be neglected, however, this restriction is to be quantified. The following estimates are rather speculative and crude, because they are based upon assumption of only one dimensional velocity profile, characterising internal recirculation in a slim vertical channel with one side held at a constant temperature $T_e$ and other sides insulated, see Fig.7:

![Fig.7 Transversal temperature and velocity profiles](image)

Assuming linear temperature profile near the heat transfer surface we can solve the Navier Stokes equation
\[
0 = -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial y^2} + \rho g [1 - \beta (T - T_0)]
\] (35)

giving the cubic velocity profile, shown in Fig.7 (this velocity profile represents a slight disturbance of the main flow parabolic velocity profile). Pressure gradient \(dp/dx\) in Eq.(35) is adjusted so that the net flow-rate is zero, because the velocity profile should characterise only internal recirculation flow in the lateral channel. Now we want to compare this velocity profile with the velocity profile corresponding to forced axial flow \(Q/2\). As a measure of comparison the velocity gradients at wall can be used. For the gradient of recirculating flow follows from the cubic velocity profile

\[
\frac{\partial u}{\partial y}{|_{y=H}} = -\rho g \beta (T_e - T_0) \frac{\delta^2 (4H + 9\delta)}{24H^2 \mu} \approx -\rho g \beta (T_e - T_0) \frac{2\lambda LB}{3\mu c_p Q}
\] (36)

assuming sufficiently thin thermal boundary layer

\[
\delta^2 \approx \frac{4\lambda LBH}{\rho c_p Q} \ll H^2.
\] (37)

The gradient (36) can be related to the velocity gradient of fully developed axial flow (parabolic velocity profile in laminar flow between infinite plates)

\[
\frac{\partial u}{\partial y}{|_{\text{circulation}}} = \frac{\partial u}{\partial y}{|_{\text{axial}}} = \frac{g \beta (T_e - T_0) \frac{2\lambda LB}{3\mu c_p Q}}{\frac{3Q}{BH^2}} = \frac{g \beta (T_e - T_0) \frac{2\lambda LB^2 H^2}{9\mu c_p Q^2}}{R \ll 1}
\] (38)

and this is the criterion \((R \ll 1)\) ensuring validity of simplified analysis of parallel flows. Let us assume that the stability criterion (15) predicts the upper bound of stability of parallel flows for the case with constant wall temperature,

\[
(T_e - T_0) \frac{g \alpha H^3 B^2 L}{6 \mu c_p Q^2} = 1.
\] (39)

Substituting Eq.(39) to the inequality (38) follows

\[
\frac{4\lambda}{3\alpha H} = \frac{4}{3Nu} \ll 1, \quad Nu = \frac{\alpha H}{\lambda},
\] (40)

and this inequality can be satisfied only for thermally developing flows when \(Nu\) is sufficiently large.

**EXPERIMENTS AND RESULTS**

Experiments were performed with different thickness of lateral channels \((H_l=H_r=18, 11\) and \(7.8\) mm). Geometry of heating tank, and position of thermometers is presented in Fig.8.
Experiments were carried out with water, with well known thermophysical properties. Based upon data Kast (1974) we have evaluated temperature dependence of $\beta$

$$\beta = (-2.5 + 0.9T - 0.004T^2) \cdot 10^{-5} \quad (41)$$

and temperature dependence of $\frac{\beta}{\mu c_p}$ as

$$\frac{\beta}{\mu c_p} = (0.04T^2 + 2T - 8) \cdot 10^{-6} \quad (42)$$

see Figs.9,10
Results of experiments are shown in Figs. 11a, 11b, 11c and comprise data for a wide gap ($H=18\text{mm}$) different heating power (up to 6 kW) and different flow rates (20 to 80 ml/s). As an independent variable $Z_G-Z_{G_{\text{crit}}}$ according to Eq.(30-31) is used. Experiments carried out at $Z_G-Z_{G_{\text{crit}}}>0$ should exhibit non-uniform flow rates:

*Fig. 11a (difference of velocities), 11b (difference of mean residence times), 11c (difference of temperatures)*
Fig. 11a presents results obtained by direct measurement of velocities in lateral channels by visual observation of color tracer (water solution of potassium permanganate) injected as a short pulse at inlet. Tracer pattern was recorded by digital camera. Symmetric and stable distribution of flow is characterized by same velocities in both channels \( \frac{u_{\text{min}}}{u_{\text{max}}}=1 \), while values \( \frac{u_{\text{min}}}{u_{\text{max}}}=0 \) correspond to the case when flow in one channel stops or even reverses.

Fig. 11.b is derived from results of stimulus response experiments carried out by using KCl and Tc-99 as a tracer, Žitný (2001). Results obtained with these tracers are very similar, however the radioisotope Tc-99 is a better tracer at heating because its solution has no effect upon direct ohmic heating (this is not true for solution of KCl because electric power and temperature increases when the conductive tracer passes between electrodes). Residence time distribution (RTD) and mean residence time evaluated from these experiments enables to calculate active volume of apparatus knowing flowrate (active volume is simply mean residence time times flowrate). In the case that the both parallel channels are active (when water flows in both channels) the active volume equals the actual inner volume of apparatus \( V_{\text{theor}} \). When the flow in one channel stops, the active volume \( V \) is reduced which is manifested by decrease of \( \frac{V}{V_{\text{theor}}} \).

Fig. 11c represents results obtained by measurement of temperatures recorded by two Pt100 probes and optical fibre probes Nortech TP-21-M02 inserted into lateral channels.

**CONCLUSION**

Non-uniform distribution of flow into parallel channels has been observed experimentally in a direct ohmic heater and this effect can be explained by buoyancy (buoyancy opposed convection). Simplified theory suggests dimensionless criteria suitable for assessment of three different parallel flow configuration: two parallel flows separated by wall held at constant temperature (for example parallel flows in a plate heat exchanger), two parallel streams with internal heating, and three streams with internal heating in the central channel. For each criterion a theoretical critical value is derived above which a non-uniformity of flowrates can be expected. This analysis need not be correct, because it is based only upon a model of one dimensional flow and this model should be confirmed (or denied or improved) by numerical and laboratory experiments. However, the first tentative experiments (even if not very accurate and reliable) indicate that the suggested results and first of all the suggested way of analysis can be applicable.

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