A COMPARISON OF METHODS FOR RAPID CALCULATION OF FRICTIONAL PRESSURE DROP AND HEAT TRANSFER COEFFICIENT FOR FLOW OF NON-NEWTONIAN FLUIDS IN DUCTS WITH NON-CIRCULAR CROSS SECTIONS.

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ABSTRACT

The paper presents a comparison of exact and approximate methods for calculating the frictional pressure drop with flow of purely viscous non-Newtonian fluids in passages with non-circular cross-sectional geometry. While comparing existing approximate methods for calculating the fully developed friction factor for power-law fluids, special emphasis is given to the widely used methods of Kozicki, Chou and Tiu (1966), Miller (1972) and the recent procedure by Delplace and Leuliet (1995).

For some cross-sections consisting of doubly connected regions, values of geometrical parameters necessary to apply the methods cited above, are reported.

For narrow passages, similar to those, encountered e.g. in CHE, analytical approximations of fully developed friction factors, and Nusselt number are reported.

INTRODUCTION

For stabilised laminar flow of Newtonian fluids, the exact solution for the frictional pressure drop and heat transfer coefficients with a non-circular geometry of the cross-section consists essentially in solving the momentum and energy equations in the corresponding cross-sectional geometry. Most complete source of results in this respect is available e.g. in Shah and London (1978).

For laminar flow of purely viscous fluids, the momentum equation becomes non-linear and, therefore, integration is generally possible using numerical procedure only. Consequently, for most engineering purposes, rapid and reliable approximate procedures become important.

For laminar flow of a Newtonian fluid in a circular tube the frictional pressure drop is related to the Reynolds number with the familiar hydraulic characteristic relation

$$f = \frac{16}{\mathrm{Re}}, \tag{1}$$

where the Fanning friction factor and Reynolds number are given as

$$f = \frac{D_e \Delta p}{2\rho \bar{u}^2 L}, \tag{2}$$

and

$$\mathrm{Re} = \frac{\bar{u} D_e \rho}{\mu}. \tag{3}$$

As far as purely viscous non-Newtonian fluids are concerned, the power-law rheological model

$$\tau = K\dot{\gamma}^n \tag{4}$$

is very useful for most engineering applications. Three most frequently used approximate methods for calculating the frictional pressure drop for flow of power-law fluids in non-circular ducts are listed below.

APPROXIMATE METHODS FOR THE FRICTIONAL PRESSURE DROP

Kozicki et al (1966) proposed a method for predicting the hydraulic characteristic

$$f = \frac{16}{\mathrm{Re}^*} \tag{5}$$

where the generalised Reynolds number Re^* was defined as

$$\mathrm{Re}^* = \frac{\bar{u}^{2-n} D_e^{\,n} \rho}{K[\frac{a+bn}{n}]^n 2^{3(n-1)}}. \tag{6}$$

In Eq.(6), a and b are geometrical parameters related to the corresponding solution of the laminar Newtonian flow problem in the same cross-sectional geometry. Values of a and b for some cross-sectional geometries are given in the original paper cited above. Furthermore, it can be shown that,

$$a + b = \frac{f\,\mathrm{Re}}{16} = Po, \tag{7}$$

and

$$\frac{b}{a} = 2\frac{u_{max}}{\bar{u}} - 1, \qquad (8)$$

where the dimensionless group on the r.h.s. of Eq.(7) is sometimes denoted as the Poiseuille number Po. For flow in a circular tube, for which a=1/4, b=3/4, the Kozicki Re^* in Eq.(6) reduces to the Re' introduced by Metzner and Reed (1955). Although the Re^* concept was originally developed by Kozicki at al. to correlate data in the laminar flow region, in subsequent papers, see e.g. Irvine (1988), it has been proved that it is also applicable in the turbulent flow region. A comparison of the Kozicki at al. approximation with experimental data obtained in four non-circular geometries were published by Šesták and Žitný (1998).

Miller (1972) published an extremely simple method for calculating the hydraulic characteristic for power-law fluids in non-circular ducts. This author assumes that it is reasonable to write for any geometry,

$$\bar{\tau}_w = K\left(\frac{3n+1}{4n}\right)\bar{\dot{\gamma}}^n, \qquad (9)$$

where $\bar{\tau}_w$ is the average value of the wall shear stress along the periphery of the cross section,

$$\bar{\tau}_w \cong \frac{D_e \Delta p}{4L}, \qquad (10)$$

and $\bar{\dot{\gamma}}$ denotes the apparent mean wall shear rate corrected with the value of the Poiseuille number corresponding to the geometry of the particular duct under consideration,

$$\bar{\dot{\gamma}} = \frac{8\bar{u}}{D_e}.\frac{f\,\mathrm{Re}}{16} = \frac{8\bar{u}}{D_e}(a+b). \qquad (11)$$

Delplace and Leuliet (1995), after a thorough analysis of Kozicki et al (1966) method, postulated that Kozicki's a and b parameters may be related by the expression

$$\frac{b}{a} = \frac{24}{\xi}, \qquad (12)$$

where

$$\xi = 8(a+b) = \frac{1}{2} f\,\mathrm{Re}. \qquad (13)$$

Thereafter, these authors defined a new, generalised Reynolds number,

$$\mathrm{Re}_g = \frac{\bar{u}^{2-n} D_e^{\,n} \rho}{K\left[\frac{24n+\xi}{(24+\xi)n}\right]^n \xi^{n-1}}. \qquad (14)$$

Making use of this definition, their friction factors were expected to follow from the relation,

$$\frac{1}{2} f\,\mathrm{Re}_g = \xi. \qquad (15)$$

Delplace et al. (1977) compared friction factor values in Eq.(15) with experimental data obtained with flow of power-law fluids in four conduits with different non-circular cross sections. Agreement between theory and experiment was found to be very good. However, it is worth to note that, all conduits used in their experiments had cross-sectional geometries formed by singly connected regions only.

A COMPARISON OF THE APPROXIMATE METHODS

In order to compare the results of all three approximate methods outlined above, the following Reynolds number introduced by Bukovský (1981) has been chosen as a reference,

$$\mathrm{Re}_B = \frac{\bar{u}^{2-n} D_e^{\,n} \rho}{2^{3(n-1)} K} = \left(\frac{a+bn}{n}\right)^n \mathrm{Re}^* = \left(\frac{1+3n}{4n}\right)^n \mathrm{Re}. \qquad (16)$$

Making use of this dimensionless group, all three friction factor - Reynolds number correlations are transformed into the following expressions,

Kozicki et al. (1966),

$$f\,\mathrm{Re}_B = 16\left[\frac{a+bn}{n}\right]^n, \qquad (17)$$

Miller (1972)

$$f\,\mathrm{Re}_B = 16\left[(a+b)\frac{3n+1}{4n}\right]^n, \qquad (18)$$

and Delplace and Leuliet (1995)

$$f\,\mathrm{Re}_B = 16\left[(a+b)\frac{3n+a+b}{(3+a+b)n}\right]^n. \qquad (19)$$

Cross section formed by a simply connected region

As an example of a cross-section formed by a singly connected region, flow of power-law fluids in ducts of symmetrical L-shaped cross-sections was investigated. Results of this investigation are shown in Fig.1, where values of $f\mathrm{Re}_B$ are plotted against the aspect ratio B/A. Results of a numerical solution of the momentum equation for n=1 and n=0.5 were taken from Bukovský (1981).

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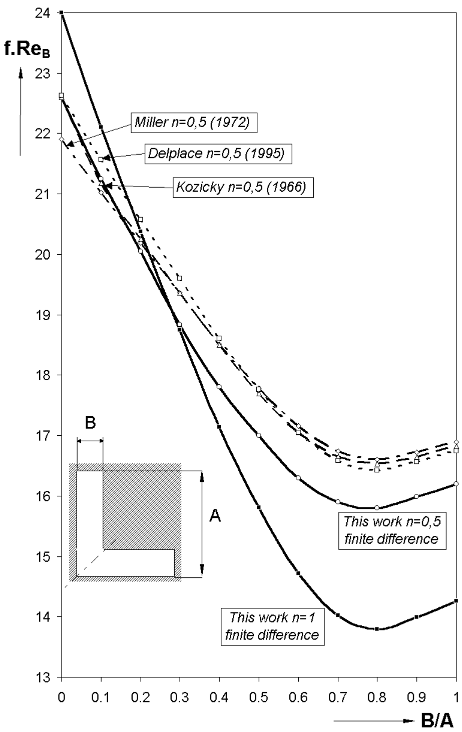

Fig.1 Comparison of approximate methods for L-shaped cross-section

All values which were needed to calculate fRe_B from Eqs.(17), (18) and (19) are listed in Table 1.

Table 1. Values for Newtonian and power-law flow in ducts of L-shaped cross sections. Bukovský (1981).

	fRe_B	Kozicki et al. parameters		
B/A	n=1	n=0.5	a	b
0.1	22.10	-	0.3713	1.0101
0.2	20.38	20.05	0.3299	0.9434
0.3	18.75	18.84	0.2941	0.8765
0.4	17.14	17.81	0.2628	0.8110
0.5	15.81	17.00	0.2359	0.7516
0.6	14.72	16.30	0.2168	0.7024
0.7	14.02	15.90	0.2067	0.6696
0.8	13.79	15.80	0.2049	0.6576
0.9	13.99	15.99	0.2086	0.6653
1	14.26	16.20	0.2130	0.6800

From Fig.1 one can see that Kozicki's method matches the numerical solution for n=0.5 very closely for values of B/A up to about 0.2. For higher values of the B/A aspect ratio, all the three methods yield essentially the same results with a maximum relative deviation from the numerical solution about +4% for B/A=1 (square). Thus, for this particular cross-sectional geometry, all three approximate methods yield fairly good engineering estimates.

Cross section formed by a doubly connected region

Ratkowsky and Epstein (1968) investigated flow of Newtonian fluids in regular polygonal shaped ducts with circular centered cores. In this work, the particular case of flow in a square duct with a centered cylindrical core was studied.

Results of a numerical solution for n=0.5 are compared with the predictions of approximate methods in Fig.3

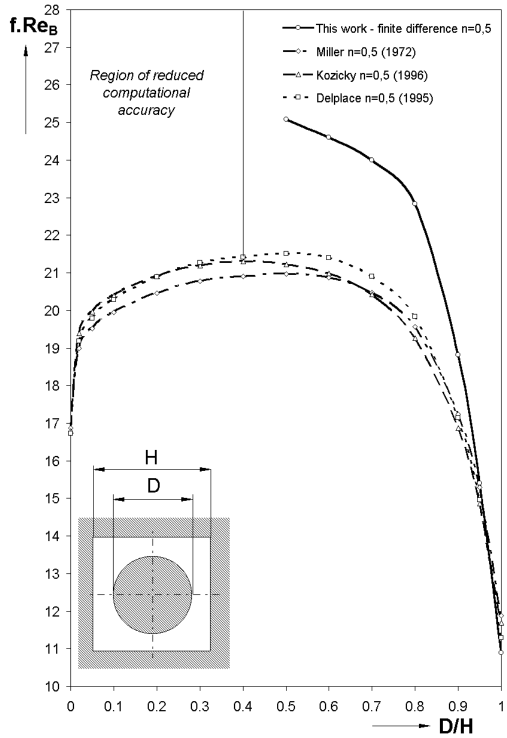

Fig.3 Comparison of approximate methods for a square duct with a cylindrical core.

All values necessary to calculate fRe_B values are listed in Table 2.

Table 2. Value of Newtonian and power-law flow in a gap between a square duct and cylindrical centered core. Bukovský (1981).

	fRe_B	**Kozicki et al. parameters**		
B/A	n=1	n=0.5	a	b
0	14.23	15.58	0.2121	0.6766
0.02	18.05		0.3313	0.8056
0.05	19.06	27.77	0.3574	0.8413
0.1	19.90	27.19	0.3780	0.8701
0.2	20.93		0.3987	0.9107
0.3	21.59	25.63	0.4050	0.9441
0.4	21.85	25.08	0.3996	0.9743
0.5	22.00		0.3817	0.9970
0.6	21.80	24.53	0.3524	1.0163
0.7	20.96		0.3091	1.0115
0.8	19.15	22.84	0.2511	0.9482

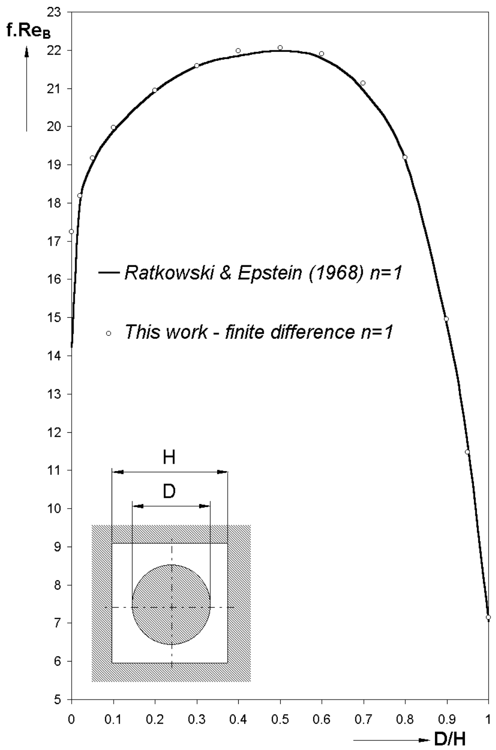

Fig.2 Friction factor for flow in a square duct with a centered cylindrical core - Newtonian fluid.

In Fig.2, analytically obtained values of Ratkowsky and Epstein for flow of Newtonian fluids are compared with results of a numerical analysis. The agreement was found to be very good.

0.9	14.85	18.81	0.1763	0.7587
0.95	11.70	15.40	0.1392	0.5846
1	7.06	10.89	0.0870	0.3599

From Fig.3 it is clear that all three approximate procedures yield values which are far below the predictions of the numerical analysis, Delplace's method yielding relatively best results. According to our belief, use of any of the previously mentioned approximate methods for calculation of the friction factor - Reynolds number relation for flow of power law fluids in ducts with cross sections formed by multiply connected regions remains therefore questionable.

APPROXIMATION FOR NARROW PASSAGES

For narrow passages, similar to those, encountered e.g. in compact heat exchangers, see Fig.4, Maclaine-Cross (1969) published an interesting method for rapid estimates of the frictional pressure drop, entrance region pressure drop and heat transfer coefficient. The Maclaine-Cross (1969) method for estimating the value of the heat transfer ceofficient is valid for the constant heat flux boundary condition. A similar method for the isothermal boundary condition was published by James (1970). Šesták and Žitný (1998) published recently a generalisation of these results for power-law fluids.

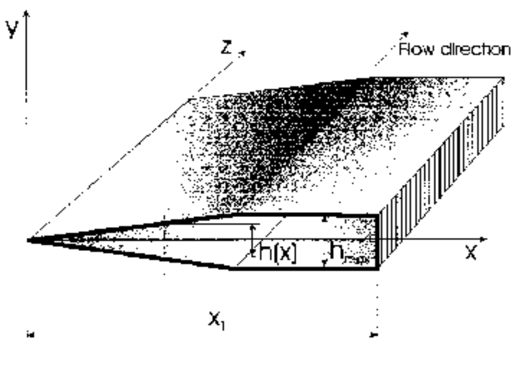

Fig.4 Cross-section of a narrow duct

The friction factor may be calculated from the expression

$$\lambda_f \ Re^{**} = (\lambda_f \ Re^{**})_{p.p.} \frac{\left(\int_0^{x_1} h dx\right)^{2n+1}}{x_1^{n+1}\left(\int_0^{x_1} h^{\frac{2n+1}{n}} dx\right)^{n}} . \tag{20}$$

where the relation for parallel plates,

$$(f \, \mathrm{Re}^{**})_{p.p} = 96 \tag{21}$$

and the generalised Reynolds number Re^{**}

$$Re^{**} = 3.2^{2(1-n)} \frac{\rho \bar{u}^{2-n} D_e^n}{k} \left[\frac{n}{2n+1}\right]^n . \tag{22}$$

For a constant heat flux w at the boundaries of the duct, the fully developed heat transfer coefficient α is

$$\alpha = \frac{q}{T_w - T_b} \tag{23}$$

The Nusselt number is

$$Nu = \frac{\alpha D_e}{\lambda} = \frac{q D_e}{\lambda (T_w - T_b)} \tag{24}$$

Nusselt number in narrow passages may then be obtained from the approximate expression,

$$Nu = (Nu)_{p.p.} \frac{\int_0^{x_1} h dx \left[\int_0^{x_1} h^{\frac{1}{n}+2} dx\right]^2}{x_1^2 \int_0^{x_1} h^{\frac{2}{n}+5} dx} , \tag{25}$$

where, for constant and equal heat fluxes at each wall,

$$(Nu)_{p.p} = \frac{12(n+1)^2 (3n+1)(4n+1)(5n+2)}{96 n^5 + 275 n^4 + 285 n^3 + 131 n^2 + 27n + 2} . \tag{26}$$

For flow of a Newtonian fluid, n=1, Eq.(26) yields,

$$(Nu)_{p.p} = 140/17 , \tag{27}$$

as expected, see Shah and London(1978).

CONCLUSIONS

While comparing existing approximate methods for calculating the fully developed friction factor for flow of power - law fluids in non-circular ducts with values obtained by numerical integration of the corresponding momentum equation, the following results are reported: in the cross- sectional geometry of the symmetric L-profile,as a representative of simply connected regions, all three approximate methods yield essentially the same results, predicting values about 4% higher than the numerical solution for a power-law fluid with the flow behaviour index value of n= 0.5. Since Kozicki´s method employs two independent parameters, it seems that the use of either Delplace´s, or Miller´s method, both employing a single parameter only, yields engineering estimates of essentially the same accuracy and less effort. As far as cross sections formed by multiply connected regions are concerned, for the particular case of flow of power law fluids, n=0.5, in a square duct with a centered cylindrical core, all three approximate methods yield values of questionable reliability and more accurate procedures must be developed.For calculating the heat transfer coefficient with flow of power-law fluids in narrow passages with the constant heat flux boundary condition, a generalization of the Maclaine-Cross(1969) approximate procedure, given in Eq.(25), may be used.

NOMENCLATURE

Latin symbols

a Kozicki geometric parameter eqs. (7), and (8), dimensionless
A dimension of the duct cross section, Fig.1, m
b Kozicki geometric parameter eqs. (7),and (8), -
B dimension of the duct cross section, Fig.1, m
D tube or cylinder diameter, m
$D_e=4S/O$ equivalent diameter, m
f Fanning friction factor, Eq.(2), dimensionless
h distance between parallel plates, m
H duct dimension, m
K power-law model consistency, Pa.s^n
L duct length in the flow direction, m
n flow behaviour index, dimensionless
O wetted perimeter, m
Δp pressure difference, Pa
S cross sectional area, m^2
q wall heat flux, W.m^{-2}
T temperature, K
$T_b=\int u_z T\,dA \,/ \int u_z\,dA$ fluid bulk temperature, K
T_w wall temperature, K
u_z velocity component in the z-direction, m.s^{-1}
u_{max} maximum velocity in the cross-section, m.s^{-1}
$\overline{u}$ volumetric mean velocity, m.s^{-1}
x,y,z cartesian coordinates, m
x_1 width of the narrow duct, m

Greek symbols

α heat transfer coefficient, W.m^{-2}.K^{-1}
$\dot{\gamma}$ shear rate, s^{-1}
$\overline{\dot{\gamma}}$ corrected mean value of the shear rate, s^{-1}
λ fluid thermal conductivity, W.m^{-1}.K^{-1}
μ dynamic viscosity, Pa.s
ξ defined in Eq.(15), -
ρ fluid density, kg.m^{-3}
τ shear stress component, Pa
$\overline{\tau}_w$ wall shear stress mean value, Pa

Dimensionless groups

Nu Nusselt number, defined in Eq. (24)
Po Poiseuille number, defined in Eq.(7)
Re Reynolds number for Newtonian fluids, Eq.(3)
Re^* Kozicki et al Reynolds number, Eq.(6)
Re' Metzner and Reed's Reynolds number, defined in Eq.(6) for a=1/4; b=3/4
Re_g Delplace and Leuliet's group defined in Eq.(14)
Re_B Reynolds number for power-law fluids, Eq.(16)
Re^{**} Reynolds number for the parallel plate approximation, Eq. (22)

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