AXIAL FLOW OF PURELY VISCOUS FLUIDS IN ECCENTRIC ANNULI: GEOMETRIC PARAMETERS FOR MOST FREQUENTLY USED APPROXIMATE PROCEDURES

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ABSTRACT

The paper presents values of the a and b geometrical parameters, necessary to apply the well known Kozicki, Chou and Tiu (1966) [1] approximate procedure for calculating the relation between the flowrate and frictional pressure drop in laminar, axial flow of power-law fluids in ducts with a cross section formed by an eccentric annulus. Values of the parameters were calculated from the simplest exact expressions for the velocity profile and flowrate given by Piercy et al (1933) [8]. Theoretical predictions of the approximate method were compared with experimental data obtained in flow of pseudoplastic, n=0.5-0.6, aqueous solutions of polyacrylamide. Since Šesták et al (2000) [7] have shown that geometric parameters used in most of the later published approximate methods are related to the a and b values by simple expressions, the present data may also be utilized while applying these methods for the cross-section of an eccentric annulus as well.

KEYWORDS: DUCT FLOW, POWER LAW FLUID,FRICTION FACTOR, ECCENTRIC ANNULUS CROSS SECTION

INTRODUCTION

For rapid engineering estimates of the relation between the frictional pressure drop and flowrate with axial flow of purely viscous non-Newtonian fluids in straight, constant cross section noncircular ducts, several approximate procedures had been developed in the past, the most frequently used being those, published by Kozicki et al [1] , Miller [2], Delplace and Leuliet [3] and Liu and Masliyah [4]. Practical use of any of the above methods in a particular cross-sectional geometry requires the knowledge of one or more geometrical parameters, obtainable mostly from the corresponding solution for flow of a Newtonian fluid. Kozicki et al, [1] developed their generalized Reynolds number Re^* as

$$\mathrm{Re}^* = \frac{\bar{u}^{2-n} D_e^n \rho}{k\left[\dfrac{a+bn}{n}\right]^n 2^{3(n-1)}} \quad . \qquad (1)$$

In Eq.(1), $\bar{u}$ and ρ denote the volumetric mean velocity and fluid density, D_e stands for the equivalent diameter D_e= $4S/O$, where S and O denote the cross sectional area and wetted perimeter and K and n denote the consistency and flow behavior index in the constitutive equation of the power/law fluid,

$$\tau = k\dot{\gamma}^n , \qquad (2)$$

relating the shear stress τ with the shear rate $\dot{\gamma}$. Using the Fanning friction factor f,

$$f = \frac{D_e \Delta p}{2\rho\bar{u}^2 L} , \qquad (3)$$

where Δp is the pressure drop measured on a tube length L; the relation

$$f\,\mathrm{Re}^* = 16 \qquad (4)$$

serves for the determination of the pressure drop – flowrate relation regardless of the geometry of the duct cross section. The influence of the cross-sectional geometry is reflected in the values of the geometrical parameters a and b. Šesták, [5] demonstrated that for the geometrical parameters the following relations hold:

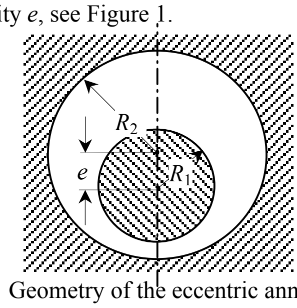
$$a + b = \frac{f\,\mathrm{Re}}{16} = Po \text{ , and} \qquad (5)$$

$$\frac{b}{a} = 2\frac{u_{\max}}{\bar{u}} - 1 , \qquad (6)$$

where the dimensionless group on the r.h.s. of Eq.(5) is usually called as the Poiseuille group Po. For flow in a circular tube, for which a=1/4, b=3/4 and n=1; k=μ and Re^* reduces to the conventional $\mathrm{Re} = \bar{u}D_e\rho/\mu$. Although the Re^* concept was originally developed by Kozicky et al to predict the hydraulic characteristic in the laminar flow region, in subsequent papers, see e.g. Irvine [6], it has been proved that it is applicable in the turbulent region as well.

VALUES OF a AND b FOR THE ECCENTRIC ANNULUS

The geometry of the eccentric annulus is completely defined by the radii R_1, R_2 and dimensional eccentricity e, see Figure 1.



$e^*=e/(R_2-R_1)$

$\kappa=R_1/R_2$

Figure.1: Geometry of the eccentric annulus.

The values of the hydraulic characteristic $f\mathrm{Re}(\kappa,e^*)$ and of the velocity ratio $u_{\max}/\bar{u}(\kappa,e^*)$ were determined using the exact expression for the velocity distribution and flowrate in bipolar coordinates published by Pierey et al [8] and checked with the most reliable numerical data of Tiedt [9] and Filip [11]. Values $f\mathrm{Re}(\kappa,e^*=1)$ were calculated from Stevenson's [10] exact formula as adapted by Tiedt [9],

$$f\,\mathrm{Re} = \frac{16(1-\kappa^2)(1-\kappa)^2}{1-\kappa^4 - 4\kappa^2 \sum\limits_{n=0}^{\infty}\{\dfrac{1}{n+[1/(1-\kappa)]}\}^2} \qquad (7)$$

Values of the $u_{max}/\bar{u}(\kappa, e^*)$ ratio were calculated by Filip [11]. Partial results in terms of numerical values of fRe, $u_{max}/\bar{u}$, a and b are given in Table 1, a graphical representation is shown in Figure 2.

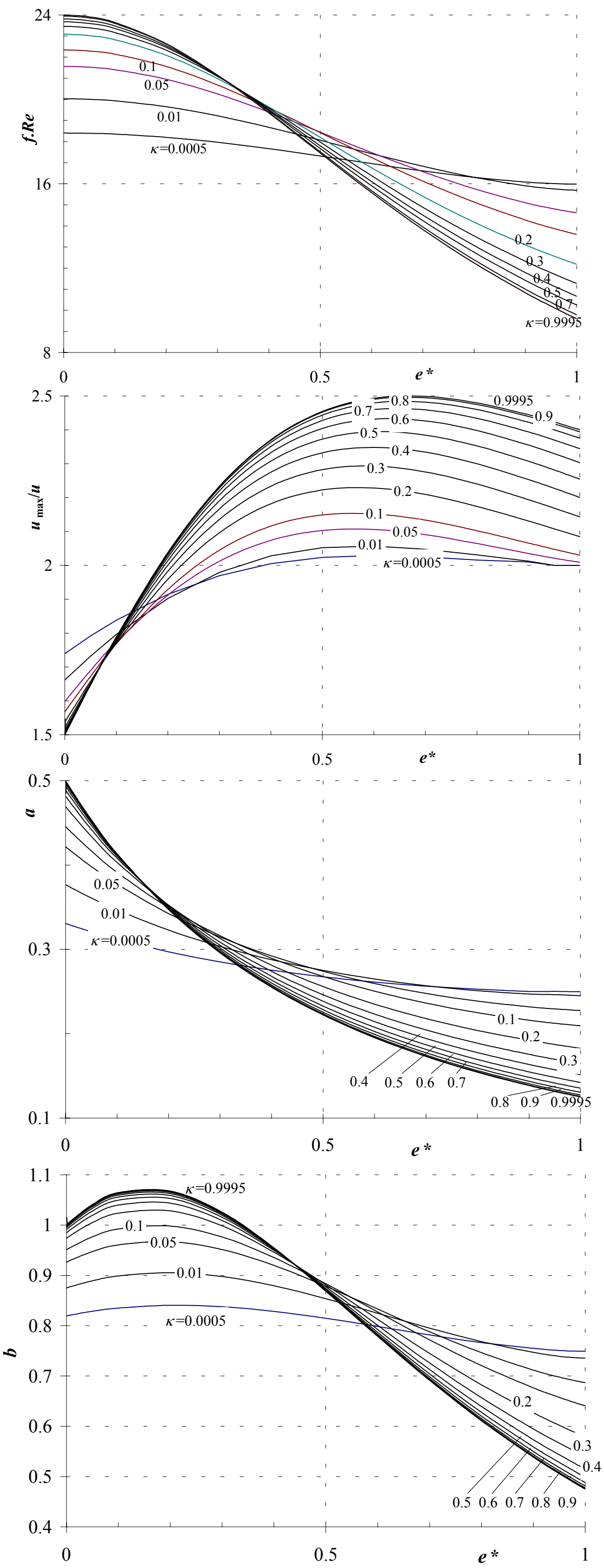


Figure 2: Graphical representation of fRe, $u_{max}/\bar{u}$, a and b as a function of $\kappa=R_1/R_2$ and dimensionless eccentricity e^*

Table 1. Numerical values of fRe, $u_{max}/\bar{u}$, a and b as a function of $\kappa=R_1/R_2$ and dimensionless eccentricity e^*

f.Re

	e^*=0.1	0.3	0.5	0.7	0.9	0.999
κ=0.1	22.141	20.680	18.423	16.129	14.280	13.606
0.3	23.168	21.086	17.975	14.887	12.325	11.286
0.5	23.481	21.139	17.671	14.256	11.422	10.265
0.7	23.601	21.146	17.518	13.955	11.005	9.800
0.9	23.641	21.145	17.460	13.844	10.851	9.629
0.999	23.645	21.145	17.455	13.833	10.835	9.612

$u_{max}/\bar{u}$

	e^*=0.1	0.3	0.5	0.7	0.9	0.999
κ=0.1	1.768	2.045	2.149	2.134	2.066	2.030
0.3	1.774	2.136	2.283	2.276	2.194	2.144
0.5	1.782	2.190	2.373	2.384	2.307	2.255
0.7	1.786	2.220	2.428	2.459	2.394	2.345
0.9	1.788	2.232	2.452	2.495	2.439	2.395
0.999	1.788	2.234	2.455	2.499	2.445	2.401

a

	e^*=0.1	0.3	0.5	0.7	0.9	0.999
κ=0.1	0.391	0.316	0.268	0.236	0.216	0.209
0.3	0.408	0.309	0.246	0.204	0.176	0.165
0.5	0.412	0.302	0.233	0.187	0.155	0.142
0.7	0.413	0.298	0.225	0.177	0.144	0.131
0.9	0.413	0.296	0.223	0.173	0.139	0.126
0.999	0.413	0.296	0.222	0.173	0.139	0.125

b

	e^*=0.1	0.3	0.5	0.7	0.9	0.999
κ=0.1	0.992	0.976	0.884	0.772	0.677	0.641
0.3	1.040	1.009	0.877	0.726	0.595	0.541
0.5	1.056	1.020	0.872	0.704	0.559	0.499
0.7	1.062	1.024	0.869	0.695	0.544	0.482
0.9	1.064	1.026	0.869	0.692	0.539	0.476
0.999	1.065	1.026	0.869	0.692	0.539	0.476

EXPERIMENTAL

Experiments were made on an apparatus with κ=0.538 permitting continuous change of the eccentricity e^*. As a test fluid, aqueous solutions of polyacrylamide, n=0.5÷0.6, were used. Experimental data were obtained making test runs for eight values of the dimensionless eccentricity in the range $0.02 < Re^* < 10$, $0.0242 \le e^* \le 1$. More details about experimental apparatus and about results may be found in Ondrušova's thesis, [12]. On comparing experimental data with Kozicki's et al prediction, good agreement was found, see Figure 3.

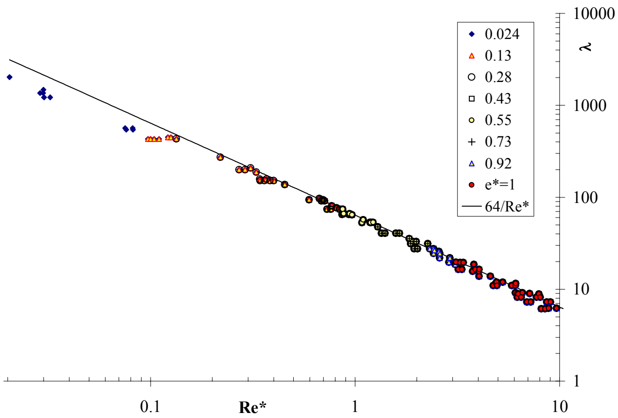


Figure 3: Hydraulic characteristic $\lambda=4f=64/Re^*$ and experimental data for polyacrylamide (n=0.5÷0.6), κ=0.538.

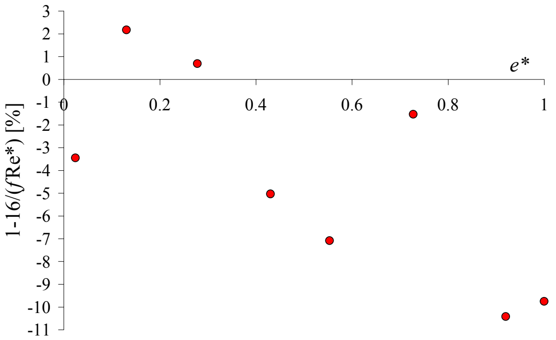


Figure 4: Relative deviation of measured and calculated pressure drop as a function of eccentricity. Mean values of experimental data for $Re^{*}>1$ are used.

CONCLUSION

Reliable numerical values of the Kozicki et al [1] geometrical parameters a and b enabling approximate prediction of the frictional pressure drop – flowrate relation with flow of power-law fluids in straight ducts are reported for the cross-sectional geometry of the eccentric annulus. Experimental data for flow of power-law polyacrylamide obtained in [12], together with results published in [7] seem to justify the conclusion that, predictions error of the Kozicki et al approximate procedure for eccentric annuli does not exceed 10% for flow behavior index $n>0.5$. Since it has been proved in [7] that later developed approximate methods [2,3,4] are closely related to [1], the range of applicability of the present numerical values of a and b is extended.

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