# Flow inversion for RTD and heat transfer improvement in closed ducts

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Abstract

Inversion of streamlines between centerline and wall region of a pipe improves residence time distribution characteristics (RTD) and heat transfer in laminar flows. Suggested oneparametrical models of inversion predict RTD and Nusselt number values for flow of a Newtonian liquid in a pipe with one or more flow invertors. These models are used for description of invertor based upon principle of Deans vortices in a local bend of tube. Results obtained by CFD programs reveal optimum inversion effect at Re. $\Phi \approx 110$ , where  $\Phi$  is the angle of bend. Identification of model using first appearance time and heat transfer enhancement is discussed.

### Nomenclature

| a                       | temperature diffusivity $a = \lambda/(\rho.c_p)$   | $(m^2.s^{-1})$       |
|-------------------------|--|----------------------|
| c <sub>p</sub>          | specific heat of fluid   | $(J.kg^{-1}.K^{-1})$ |
| C <sub>kn</sub>         | eigenfunction factors, k=1 section before and k=2 after inverter   | (-)                  |
| D                       | internal diameter of circular tube   | (m)                  |
| Gz                      | Graetz number $G_z = P e \frac{D}{L} = \text{Re} \cdot \text{Pr} \cdot \frac{D}{L} = \frac{\overline{u}D^2}{aL}$ | (-)                  |
| L                       | length of pipe (length of heat transfer surface)   | (m)                  |
| Nu                      | Nusselt number $N_u = \frac{\alpha D}{\lambda}$  | (-)                  |
| Pé                      | Péclet number $P e = \text{Re} \cdot \text{Pr} = \frac{\overline{u}D}{a}$  | (-)                  |
| R                       | inner radius of circular tube  | (m)                  |
| R <sub>c</sub>          | radius of curvature  | (m)                  |
| r*                      | dimensionless radial coordinate r <sup>*</sup> =r/R  | (-)                  |
| Re                      | Reynolds number for Newtonian fluids Re = $\frac{\overline{u}D\rho}{\mu}$  | (-)                  |
| $T_k$                   | temperature (k=1 before, k=2 behind inverter)  | (K)                  |
| $\overline{\mathrm{T}}$ | mean-cup mixing temperature  | (K)                  |
| $T^*$                   | dimensionless temperature $T^* = \frac{T_W - T}{T_W - T_0}$  | (-)                  |
| $\overline{u}$          | mean velocity of fluid   | $(m s^{-1})$         |
| <br>V                   | volumetric flowrate  | $(m^3 s^{-1})$       |
| *<br>*                  | dimensionless axial coordinate $*$ ax $x/D$ 1  | (III .3 )            |
| л                       | $x^{+} = \frac{1}{\overline{u}D^{2}} = \frac{1}{Pe} = \frac{1}{Gz}$  | (-)                  |
| $Y_n(r^*$               | ) eigenfunction  | (-)                  |
| Gree                    | k letters  |                      |
| α                       | heat transfer coefficient  | $(W.m^{-2}.K^{-1})$  |
| Φ                       | angle of bend  | (rad)                |
| φ                       | inversion efficiency, see Eq.(1)   | (-)                  |

 $\lambda$  thermal conductivity of fluid

| •           | •       | 1    |
|-------------|---------|------|
| $\lambda_n$ | eigenva | lues |

- μ dynamic viscosity
- $\theta_{\min}$  first appearance time
- ρ density

### Subscripts

- o tube entry
- w tube wall
- s straight empty tube

## Introduction

The flow inverter is a unit, installed in a pipe, which moves the portion of slowly moving fluid near by the wall into the regions of faster moving fluid around the tube axis (and vice versa). This inversion improves the residence time distribution (RTD) and increases heat transfer in laminar flows.

A number of principles how to narrow residence time distributions (RTD) mostly by affecting flow along the whole pipe has been suggested. Examples of this approach are tube internals, like twisted tape, oscillating baffles, series of rotational mixers along tube axis, see Zhang [11]. Another interesting possibility is to employ secondary (transversal) flows induced by centrifugal forces in coiled pipes, see Saxena, Nigam [8]. They assume several coils in different geometrical arrangements, e.g. as two perpendicularly oriented coils which exhibits remarkably good efficiency. Similar principle makes use of a sequence of rectangular bends, see Crookes [2], Cassaday [1]. All these ways can be classified as *distributed* flow inverters. Such devices are effective in improving RTD but have high pressure drop compared to the empty pipe.

There is analogous situation in heat transfer. The heat transfer coefficient of the liquid flowing through the tubes can be increased by inserting suitable shaped elements into the pipe, e.g. heat transfer coefficient in pipe with tightly distributed static mixers Kenics is up to 3 times, Sulzer mixers is 5 times greater than the corresponding coefficient in an empty pipe. But the presence of the inserts causes an increase of the pressure drop.

This disadvantage of distributed inverters and mixers can be overcome by using one or more localized flow inverters separated by relatively long lengths of empty pipe. Examples are flow inverters designed by Nauman [6], and the flow inversion by a locally bent tube or by two parallel inclined ribs in a straight pipe,  $\check{Z}itn\check{y}$  [12] – similar arrangement appears in motionless mixers Sulzer Chemtech. These *local* inverters ensure the flow inversion only locally therefore flow field and pressure drop is affected only little, while the RTD characteristics and heat transfer are still improved significantly even not so much as by using distributed flow inverters. This holds only for highly viscous liquids characterized by much longer thermal development length in comparison with the hydrodynamics stabilization length (Prandtl number >>1) and in the laminar flow regime.

# 1. Mathematical model of flow inverters

It was just Nauman [7], who pioneered theory of flow inverters and introduced a class of flow inverters based upon division of incoming flow into two inverted streams. Characteristic parameter of this model is ratio of flow-rates. This paper is aimed at a slightly modified class of models, where the inlet is divided into three streams, two of them being inverted and parameter of the model is inversion efficiency  $\varphi$ :

(W.m<sup>-1</sup>.K<sup>-1</sup>) (-) (Pa.s) (-) (kg.m<sup>-3</sup>)

$$\varphi = \frac{\dot{V}_{inverted}}{\dot{V}}.$$
(1)

## 1.1. Convective model (CM)

This model is shown in Fig.1. Stream 1 at the tube core is transferred to the annular wall region in such a way that e.g. the streamline at r=0 is transferred to the wall r=R. Intermediate stream 2 flows straight, without changes. This model reduces to the model of perfect inverter (according to Nauman [7]) for  $\varphi=1$ .

#### 1.2. Mixing model (MM)

This model (Fig.2) is similar to the convective model with the only difference that each stream is ideally mixed when passing through the inverter. This model reduces to the ideally mixed unit for  $\varphi=0$ .

#### **1.3. Model with wall layer (WM)**

The model was recommended by Střasák [9] for description of heat transfer in a tube bend. In this case (Fig.3) the stream at the pipe wall remains without changes, the centerline and the middle streams are mixed and interchanged.



Fig. 1 Convective model CM,  $\varphi = (\dot{V}_1 + \dot{V}_3)/\dot{V}$ ,  $\dot{V}_1 = \dot{V}_3$ 



Fig. 2 Mixing model MM,  $\varphi = (\dot{V}_1 + \dot{V}_3) / \dot{V}$ ,  $\dot{V}_1 = \dot{V}_3$ 



Fig.3 Model with wall layer WM,  $\varphi = (\dot{V_1} + \dot{V_2})/\dot{V}$ ,  $\dot{V_1} = \dot{V_2}$ 

#### 2. Residence time distribution (RTD)

Residence times of liquid flowing through a straight empty pipe in laminar regime are very nonuniform, the shortest residence time of particle flowing at a maximum velocity along the centerline being only one half of the mean residence time. Any obstacle and first of all any flow inverter can only improve the overall residence time distribution (RTD) of pipe. The complete RTD of a tube with flow inverters can be expressed in an analytical form, see Žitný [12], Thýn [10], Střasák [9]. However, from practical point of view it is sufficient to evaluate the effect of inversion using suitable numerical characteristics of RTD, e.g. by using the ratio  $\theta_{min}$  of the first appearance time (the shortest residence time) and the mean residence time. The value  $\theta_{min}=0.5$  corresponds to the worst case of an empty pipe, while the perfect inverter can increase this value up to  $\theta_{min}=0.707$ . It is easy to calculate relationship between the  $\theta_{min}$  and the inversion efficiency  $\varphi$  also for the incomplete inversion:

- convective inverter

- inverter with mixing

$$\theta_{\min} = \frac{1}{\sqrt{2(2-\phi)}},\tag{2}$$

$$\theta_{\min} = \min(\frac{1}{4} + \frac{1}{2\sqrt{2\phi}}, \frac{1}{\sqrt{2(2-\phi)}}),$$
(3)

- inverter with wall layer

$$\theta_{\min} = \frac{1}{4} + \frac{1}{2\sqrt{2(2-\phi)}} \,. \tag{4}$$

It follows from Equations (2) to (4) that the theoretical maximum of the first appearance time  $\theta_{min}=0.707$  can be achieved only by the convective model for complete inversion ( $\varphi=1$ ). Model with mixing predicts the best result  $\theta_{min}=0.645$  for incomplete inversion  $\varphi=0.8$  and the model with wall layer predicts the lowest improvement,  $\theta_{min}=0.603$  for  $\varphi=1$ .

### 3. Heat transfer enhancement

The classical Graetz solution [5] describes temperature profile  $T_k(x,r)$  of a fluid with a fully developed parabolic velocity profile flowing in an empty circular tube with wall held at constant temperature and constant liquid properties (viscosity, thermal diffusivity). The problem is described by the well known energy equation:

$$(1 - r^{*2})\frac{\partial T^{*}}{\partial x^{*}} = \frac{2Gz}{r^{*}}\frac{\partial}{\partial r^{*}}(r^{*}\frac{\partial T^{*}}{\partial r^{*}}), \qquad (5)$$

where  $T^* = (T_w - T)/(T_w - T_0)$  is dimensionless temperature,  $T^* \in <0, 1>, T^* = f(r^*, x_k^*)$ 

$$r^* = r/R$$
 is dimensionless radius,  $r^* \in <0,1>$ 

 $x_k^* = a x_k / (\overline{u} D^2)$  is dimensionless axial coordinate and the distance  $x_k$  is measured from the cross section where a radial temperature profile is specified as a boundary condition.

This paper solves a similar but two-section heat exchanger where a single inverter is inserted in the middle of a long pipe, assuming that the volume of a flow inverter is negligible in comparison with the volume of pipe. The solution to Equation (5) in the form of infinite series can be used in both straight sections of pipe, i.e. in front of (k=1) and behind (k=2) inverter:

$$T_k^* = \frac{T_W - T_k}{T_W - T_o} = \sum_{n=1}^{\infty} C_{kn} Y_n(r^*) \exp(-2\lambda_n^2 x_k^*) \qquad k=1,2 \quad .$$
(6)

Eigenvalues  $\lambda_n$  corresponding to eigenfunctions  $Y_n$  are determined by the boundary condition at wall (constant temperature  $T_k^* = 0$  at  $r^* = 1$ ) and the coefficients  $C_{kn}$  are determined by the inlet temperature profile  $T_{k0}^*(r^*)$  at  $x_k^* = 0$ :

$$C_{kn} = \frac{\int_{0}^{1} T_{k0}^{*}(r^{*})Y_{n}(r^{*})r^{*}(1-r^{*2})dr^{*}}{\int_{0}^{1} Y_{n}^{2}(r^{*})r^{*}(1-r^{*2})dr^{*}}.$$
(7)

The inverter is installed in the middle of pipe at axial position  $x_{1inv} = L/2$  and temperature profile calculated from Equation (6) for  $x_{1inv} = aL/(2\overline{u}D^2)$  represents inlet to the inverter. This inverter rearranges radial temperature profile from  $T_1^*(x_{1inv}^*, r^*)$  before the inverter to the profile  $T_2^*(x_2^* = 0, r^*) = T_{20}^*(r^*)$  immediately behind the inverter. Considering the volume of inverter is small, it is supposed that particles do not change their temperature when passing inverter. Assuming that the velocity profile is immediately stabilised, the temperature profile behind the inverter is described by the Graetz solution (6) again, but with the new boundary condition  $T_{20}^*(r^*)$ . Corresponding coefficients  $C_{2n}$  are obtained from Equation (7) by numerical integration of the radial temperature profile  $T_{20}^*(r^*)$  after the inverter. The whole procedure has been outlined by Nauman [7], however his results were based upon the finite difference method.

Using temperature profiles from Equation (6) it is now possible to calculate the mean-cup mixing temperature at any cross-section of pipe. For a tube of the length L with the single inverter located in the middle of pipe the mean cup mixing temperature at outlet can be expressed as

$$\overline{T}_{2}^{*} = 4 \int_{0}^{1} T_{2}^{*} (1 - r^{*2}) r^{*} dr^{*} = -4 \sum_{n=1}^{\infty} \frac{C_{2n}}{\lambda_{n}^{2}} Y_{n}^{'}(1) \exp\left(-\lambda_{n}^{2} \frac{L/D}{P\acute{e}}\right).$$
(8)

This mean cup mixing temperature is related to the mean value of Nusselt number,

$$Nu_{\rm ln} = -\frac{1}{4}P\acute{e}\frac{D}{L}\ln\overline{T_2}^* \qquad . \tag{9}$$

An improvement in heat transfer of the pipe with single inverter compared with an empty pipe is expressed by the ratio of Nusselt number calculated from Equation (9) to the Nusselt number of an empty pipe (Nu<sub>s</sub>). This relative Nusselt number for CM and MM inverters is shown in Figures 4 and 5 as a function of Graetz number ( $Gz=aL/uD^2$ ) with the inversion efficiency  $\varphi$  as a parameter.



Fig. 4. Relative Nusselt number as a function of 1/Gz and convective inverter efficiency  $\varphi$ 

It is apparent that a single inverter, described either by the convective or the mixing model, can increase the Nusselt number by maximum  $\approx 40\%$  at Gz $\approx 50$ . For a very large Gz $\approx 1000$ , i.e. if the tube is "short" and thermal boundary layer very thin, the effect of inversion is almost independent of inversion efficiency  $\varphi$ , and Nu is increased approximately by constant value 30%. There exists qualitative agreement between the RTD and the heat transfer improvement: Convective model exhibits the best performance for *full inversion* ( $\varphi$ =1) in the both cases, while the mixing model predicts optimum at *incomplete inversion*  $\varphi$ =0.8 for the RTD improvement and even slightly lower  $\varphi$ =0.6÷0.7 for the maximum heat transfer enhancement.



Fig. 5. Relative Nusselt number as a function of 1/Gz and mixing inverter efficiency

The convective and the mixing models can be compared with the model of partial inverter suggested by Nauman [7], see Fig.6. The Nauman's model assumes that the incoming fluid is divided into only two streams, inverted without mixing. The ratio of inner stream flow-rate and the total flow-rate is used as a model parameter  $\varphi$  ( $\varphi$ =0 and  $\varphi$ =1 mean no inversion,  $\varphi$ =0.5 corresponds to the full inversion).



Fig. 6. Relative Nusselt number as a function of 1/Gz and Nauman inverter efficiency

All the previous models are characterised by a relatively high performance at large Gz numbers, which corresponds to the assumption that also the boundary layer is inverted. This need not be fulfilled in practice and then the last model with an intact wall layer, seems to be more suitable for heat transfer modelling, see Fig.7.



Fig. 7. Relative Nusselt number as a function of 1/Gz and wall layer inverter efficiency

The same approach as for the single inverter can be applied to the analysis of heat transfer enhancement by using two or more equidistant inverters and results of the mixing model for  $\varphi=0.6$  are represented in Fig.8.



Fig. 8. Heat transfer enhancement by series of inverters (mixing model  $\varphi$ =0.6) and by distributed static mixers Kenics

We can see that a higher number of inverters not only increases the Nusselt number but also extents the operational range, because optimum is shifted towards lower values  $Gz\approx10$  and a good performance is still preserved at higher values of Gz. The Fig.8 shows also the course Nu(1/Gz) determined experimentally for a tight (distributed) arrangement of static mixers Kenics, Grace [4]:

$$Nu = 3.65 + 3.8 \left( P \acute{e} \frac{D}{L} \right)^{1/3}, \tag{10}$$

nevertheless this course cannot be directly compared with predictions of inverters, because a very high increase of Nu (up to 300%) is probably partly caused also by extended heat transfer surface.

### 4. Invertors based upon local bending of tube

As an example of possible realization of flow inverter we shall analyze very simple arrangement, where the inverter is nothing else than a short, slightly curved pipe, see Fig. 9.



### Fig.9 Geometry of bend

Inversion of streams is accomplished by secondary flow induced by centrifugal forces in bend. The bend angle  $\Phi$  (the angle of straight sections) should be adjusted in such a way, that the secondary flow in curved section causes just one half turn of fluid particles, thus ensuring their transfer between center and wall. Preliminary analysis based upon analytical solution of the secondary flow (Dean [3]) indicates that the optimum angle  $\Phi$  depends primarily upon the Re number (the higher is the Re, the smaller should be the incline  $\Phi$ ) and that the influence of elbow curvature is small, Žitny [12]. Later on we shall see that this assumption is impaired at higher Reynolds numbers.

#### 4.1. First appearance time

Effect of local bend (influence of the angle  $\Phi$ , the ratio  $R/R_c$  and transition between straight and curved parts) upon the residence time distribution and upon the first appearance time was studied numerically using program Fluent with angles of inclines  $\Phi=10^0$ ,  $20^0$ ,  $30^0$ ,  $40^0$ , dimensionless curvature  $R/R_c=0.02$  and Re in the range from 100 to 800. Fluent has been used only for identification of mapping between positions of a particle in inlet and outlet cross-section of pipe  $(r_1,\psi_1) \rightarrow (r_2,\psi_2)$ . Therefore it was sufficient to solve velocity field numerically only in the part of pipe with developing velocity profile, i.e. in a short section before and behind the bend. The RTD and the first appearance time has been calculated from this mapping by special program assuming that the pipe is infinitely long and the volume of inverter (volume of bend) is negligibly small. Resulting characteristics (e.g. the first appearance time) depend generally upon three independent parameters: Re,  $R/R_c$  and the angle  $\Phi$ , however, it was found that results correlate quite well with only one variable, with the product of Re and the angle  $\Phi$ , see Fig.10.



Fig.10 First appearance time as a function of Re. $\phi$  (numerical results by Fluent)

It follows from Fig.10 that optimum, the longest first appearance time  $\theta_{min}=0.6$ , is achieved at Re. $\phi = 110$ , (11)

which corresponds quite well to previously obtained result  $\text{Re.}\phi = 80$ , based upon analytical Dean's solution of fully developed secondary flow in a curved pipe, ignoring transition between the bend and the straight parts of pipe, Žitný [12].

The first appearance time calculated by Fluent can be compared with models of inverters giving values of inverter efficiency  $\varphi$  for each particular case (Re,  $\Phi$ ,  $R/R_c$ ). Fitting numerically obtained data in Fig.10 with prediction of the first appearance time according to Equation (2) it is possible to express the inverter efficiency by empirical correlation

$$\theta_{\min} = 0.5 + (0.012 \,\mathrm{Re}\phi)^3 e^{-0.03 \,\mathrm{Re}\phi} + 0.032(1 - e^{-0.0062 \,\mathrm{Re}\phi}), \qquad (12)$$
  
$$\phi = 2(1 - \theta_{\min}^2)$$

The best performance ( $\theta_{min}=0.6$ ) corresponds to the inverter efficiency  $\phi=0.6$  for this specific realization of flow inverter.

#### 4.2. Heat transfer enhancement

The influence of flow inversion in a bend upon the heat transfer in the following straight pipe has been studied also numerically by Fluent. This time it is necessary to model the whole tube including the long inlet and outlet sections. Analyzed geometry are summarized in Tab.1



Sufficiently long straight sections ensures small relative volume of bend  $(0.6 \div 5\%)$  and negligible pressure drop increase (<1%, within the limits of numerical errors). With the aim to eliminate the heat transfer enhancement in the inverter itself, the boundary condition has been defined as adiabatic in the bend, and  $T_w$ =const has been applied only in the straight heat transfer sections (each 600 mm long). Fully developed velocity profile has been ensured by inserting a short (50 mm) adiabatic section before the entry to the heat transfer section and specifying parabolic velocity profile at inlet. Symmetry has been utilized when possible, thus reducing number of hexagonal cells (typically 200 000).

Results for an elbow ( $\Phi=\pi/2$ ,  $R_c/R=2$  and 16), calculated for different Gz are shown in Fig.11 (points), together with the model prediction (the curves are taken from Fig.7).



Fig.11 Relative Nusselt number for  $(\phi = \pi/2)$ ,  $R_c/R = 2$  (squares) and 16 (circles). Re=70. Comparison with model of flow inverter with wall layer for  $\phi = 0.9$ , 0.96, 0.97.

When calculating reference  $Nu_s$  number in a straight empty pipe almost the same 3D numerical model, with the same sections, including isolated straight part, which corresponds

to the isolated bend, has been used. However, the reference  $Nu_s$  can be also calculated quite accurately from Hausen's correlation (error <2% for data in Fig.11)

$$Nu_s = 3.66 + \frac{0.0668Gz}{1 + 0.04Gz^{2/3}}.$$
(13)

What follows from Fig.11: Even if the model of the flow inverter with wall layer cannot describe the heat transfer enhancement within the broad range of Gz accurately, it is by far better than other models considered previously (compare with Figs.4,5,6). Unfortunately, it also means that it is not possible to use some general-purpose one parametric model with a specific value of parameter  $\varphi$  for optimal description of residence times and heat transfer simultaneously. On the other hand the good message is, that the optimum heat transfer enhancement is near the value Gz  $\approx$  50, which corresponds to the model prediction. And just for this value Gz=50 the increase of mean logarithmic Nu, as a function of Re,  $\Phi$ ,  $R_c/R$ , are presented in Fig.12 (Re has been changing from 7 up to 520).



Fig.12 Relative Nusselt number at Gz=50 as a function of Re. $\phi$ . Data denoted by S were calculated by Střasák [9] with shorter straight section (L/D only 80).

Several conclusions can be drawn from these numerical experiments:

- Heat transfer enhancement can be rather high, approaching theoretical limit 40% predicted by the flow inverters models. It means, that the effect of bend cannot be explained or modeled by only a simple ideal mixing unit (see Fig.5 for φ=0) and that the flow inversion must be considered. Numerical results confirmed that the heat transfer enhancement is not accompanied by a significant pressure drop increase (≈1%).
- For a high curvature of bend ( $R_c/R < 4$ ) the heat transfer enhancement is a monotonically increasing function of  $\Phi$ Re, which can be approximated by an empirical formula

$$\frac{Nu}{Nu_s} = 1 + 0.37(1 - e^{-0.01\phi \operatorname{Re}}).$$
(14)

• At higher values of  $R_c/R>4$  the effect of curvature starts to be important and there exists apparent decrease of heat transfer enhancement for  $\Phi Re>110$ . This phenomenon can be

qualitatively explained by Dean's theory, because a higher value of angle  $\Phi$  ( $\Phi$ >110/Re) corresponds to more than one half-turn of secondary vortices, thus annihilating the effect of inversion. This is in agreement with the residence time distribution analysis, see Fig.10. Nevertheless, there is still an open question why this effect is suppressed when the curvature of bend is very high; possible explanation is that the curved section is very short in this case, the flow field is not fully developed and temperature field is very asymmetric.

• Numerical experiments with double-bend (two 90° elbows mutually rotated by 90°, see Tab.1), exhibits almost the same performance as the simple U tube ( $\Phi=\pi$ ). It was surprising, because this arrangement proves good for residence times improvement.

The correlation (14) together with Hausen's correlation (13) enables to estimate Nusselt number for Gz within the range approximately 30 to 100. The correlation (14) can be used also for identification of inversion efficiency  $\varphi$  and thus to extent the range of Gz, using e.g. Fig.7 and a suitable relationship  $\varphi(\Phi Re)$  for the inverter model with wall layer. This relationship can be approximated by empirical but rather precise formula

$$\varphi = 1 - 0.638 e^{-0.216\sqrt{\phi \, \text{Re}}} \,. \tag{15}$$

There are other influences that can be even stronger than the inversion effect when real fluids are considered: natural convection and temperature dependent viscosity. As an example we studied heat transfer in the previous geometry of rectangular bend (D=5 mm, L=1200 mm,  $R_c=20 \text{ mm}$ , full 3D model without symmetry, 400 thousands cells) using sunflower oil as a model fluid, see Tab.2

| 140.2 Thermophysical parameters of sanflower on used in numerical simulation by Praem |                     |                          |                             |          |                    |          |  |  |  |  |
|---|---------------------|--------------------------|-----------------------------|----------|--------------------|----------|--|--|--|--|
| ρ <sub>o</sub> [kg.m <sup>-3</sup> ]  | β[K <sup>-1</sup> ] | $c_p [J.kg^{-1}.K^{-1}]$ | $\lambda [W.m^{-1}.K^{-1}]$ | μ [Pa.s] | T <sub>0</sub> [K] | $T_w[K]$ |  |  |  |  |
| 900   | 0.00075             | 2000                     | 0.16                        |          | 300                | 373      |  |  |  |  |

Tab.2 Thermophysical parameters of sunflower oil used in numerical simulation by Fluent

For the mean velocity  $\overline{u} = 0.233$  m/s (Re $\approx$ 70) the inversion effect increases Nu 1.3 times, superposed inversion and variable viscosity 1.6 times, and combined effect of inversion, variable viscosity and natural convection in horizontally oriented pipe increases heat transfer1.89 times. It means that in this case the three influences are of the same importance, each of them increases the heat transfer by about 30%.

# 5. Conclusion

It is a common practice to take into account corrections for variable viscosity and natural convection when calculating heat transfer in laminar flows in pipes, the effect of singularities (e.g. elbows) being neglected. This article is an attempt to submit simple methods how to take into account also the influence of flow inversion in a bend upon the residence time distribution and the heat transfer coefficient in a straight section of pipe. It has been found, that the most important parameter is  $\Phi$ Re, and this parameter enables to estimate the first appearance time using Equation (12), or heat transfer using Equation (14). The parameter  $\Phi$ Re serves also for identification of parameter  $\phi$  characterizing flow inverters models, which can be used for more detailed analysis (e.g. as a part of programs for calculation tubular heat exchangers).

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