Matlab for Simulations

Stanislav Vrána

Systems of differential equations
Transfer function

Conversion of differential equation system

First step is separation of the highest derivations in all equations.

$$\dddot{y} + 3\,\ddot{y} + 2\,\dot{y} + y + \dot{z} = u$$
$$\ddot{z} + 2\,\dot{z} + z + y = v$$

$$\dddot{y} = u - 3\,\ddot{y} - 2\,\dot{y} - y - \dot{z}$$
$$\ddot{z} = v - 2\,\dot{z} - z - y$$

Conversion of differential equation system

Then it is suitable to prepare all variables into scheme.

$$\dddot{y} = u - 3\ddot{y} - 2\dot{y} - y - \dot{z}$$

$$\ddot{z} = v - 2\dot{z} - z - y$$

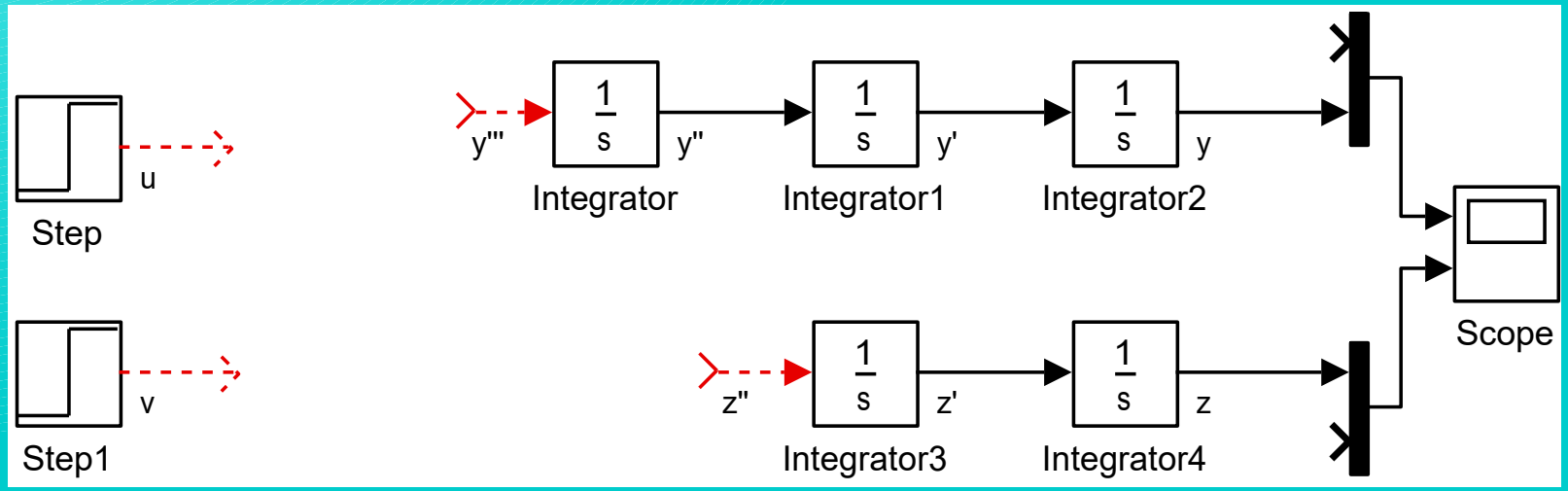
The equation systém is suitable convert as a set of independent equations and connect them in final step. The possible mistakes can be found more easily.

Conversion of differential equation system

Then it is suitable to prepare all variable into scheme.

$$\dddot{y} = u - 3\ddot{y} - 2\dot{y} - y - \dot{z}$$

$$\ddot{z} = v - 2\dot{z} - z - y$$

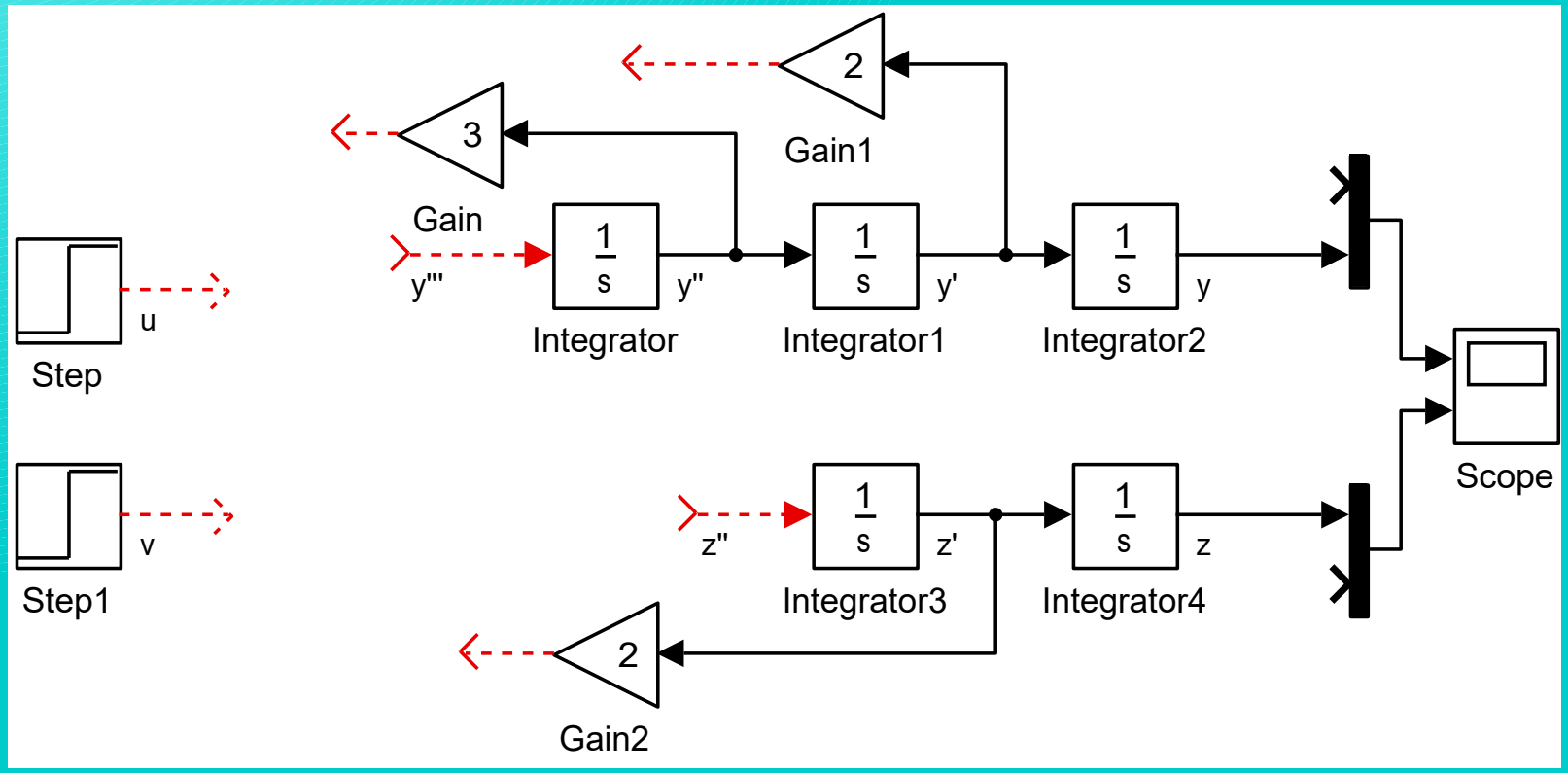


Conversion of differential equation system

Then the variables can be multiplied by respective constants.

$$\dddot{y} = u - 3\ddot{y} - 2\dot{y} - y - \dot{z}$$

$$\ddot{z} = v - 2\dot{z} - z - y$$

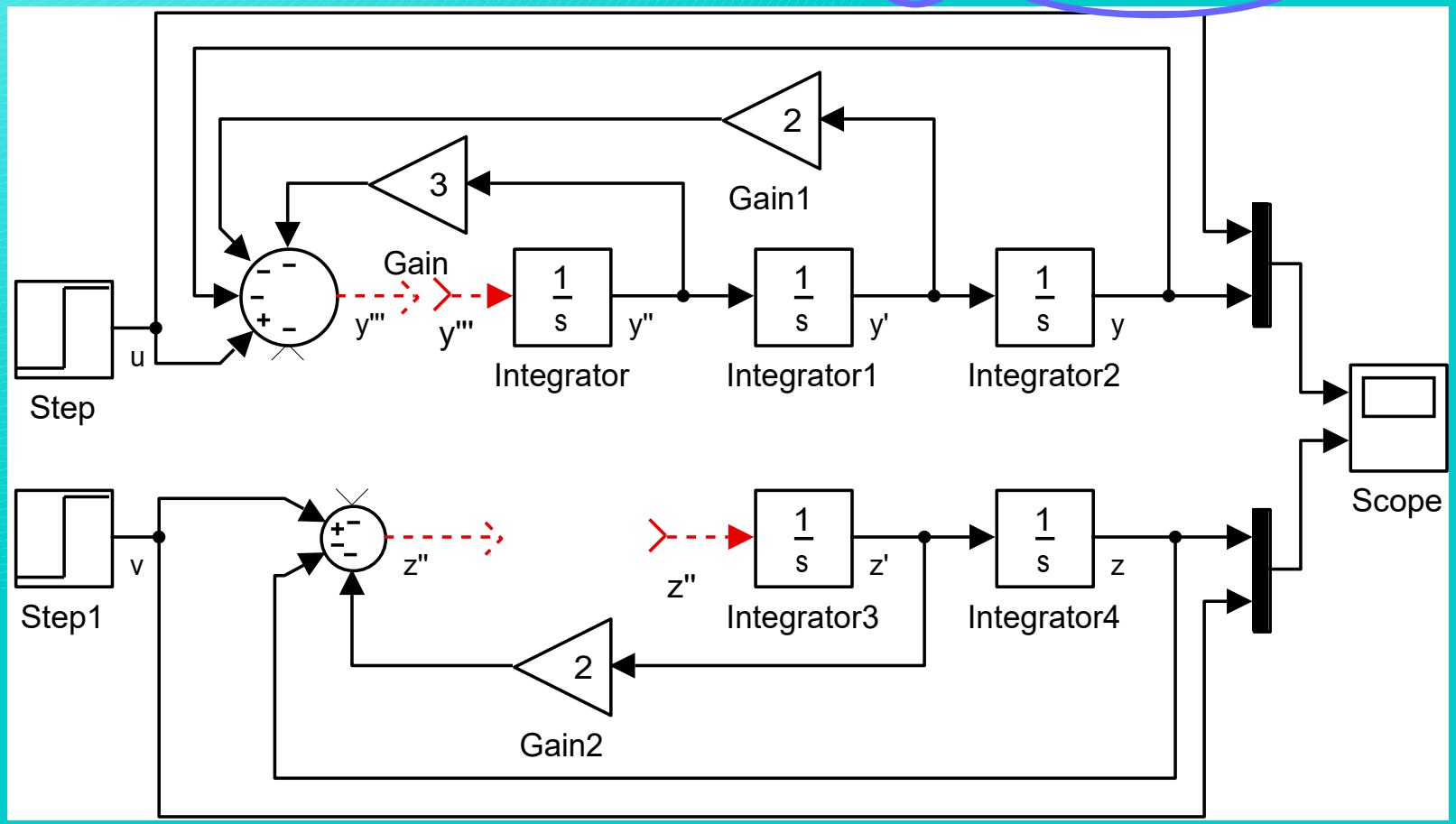


Conversion of differential equation system

Then the right side of equation can be finished.

$$\dddot{y} = u - 3\ddot{y} - 2\dot{y} - y - \dot{z}$$

$$\ddot{z} = v - 2\dot{z} - z - y$$

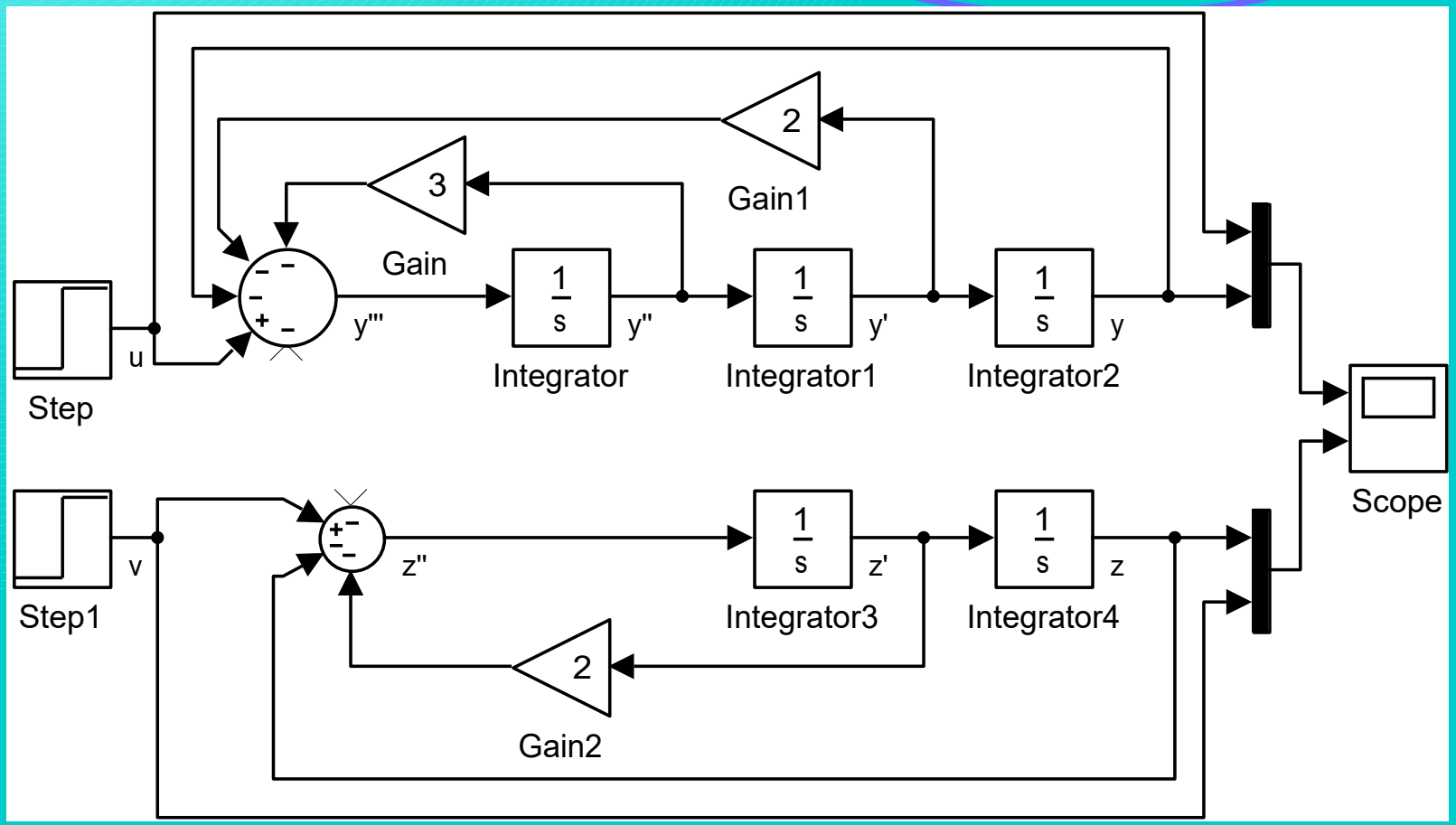


Conversion of differential equation system

… and the left sides can be connected to the right sides.

$$\dddot{y} = u - 3\,\ddot{y} - 2\,\dot{y} - y - \dot{z}$$

$$\ddot{z} = v - 2\,\dot{z} - z - y$$



Conversion of differential equation system

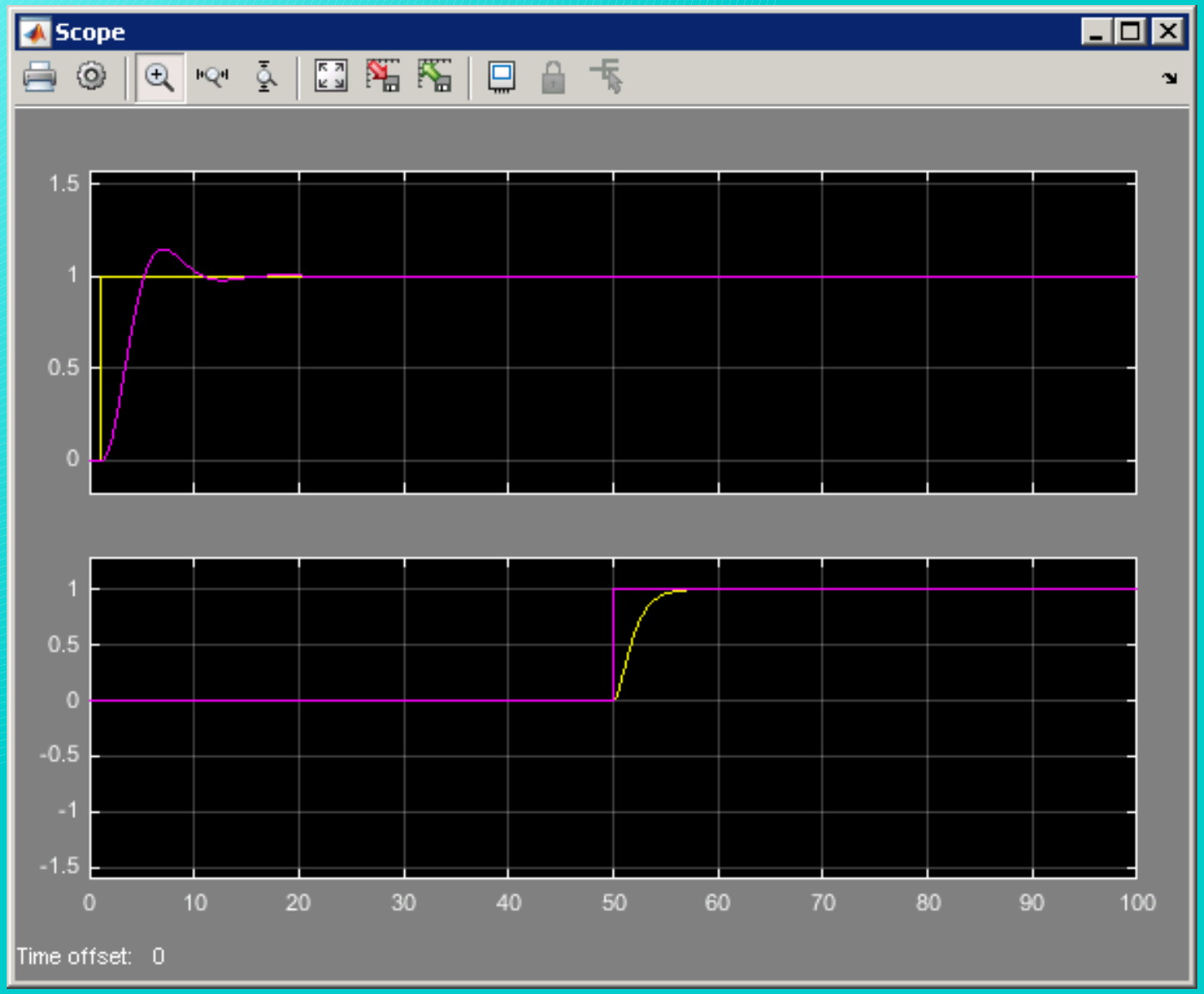
… and the left sides can be connected to the right sides.

$$\dddot{y} = u - 3\,\ddot{y} - 2\,\dot{y} - y + \dot{z}$$

$$\ddot{z} = v - 2\,\dot{z} - z + y$$

Now, the simulink model contains two individual differential equations. It is suitable to verify tha the Simulink model is built correctly.

Conversion of differential equation system

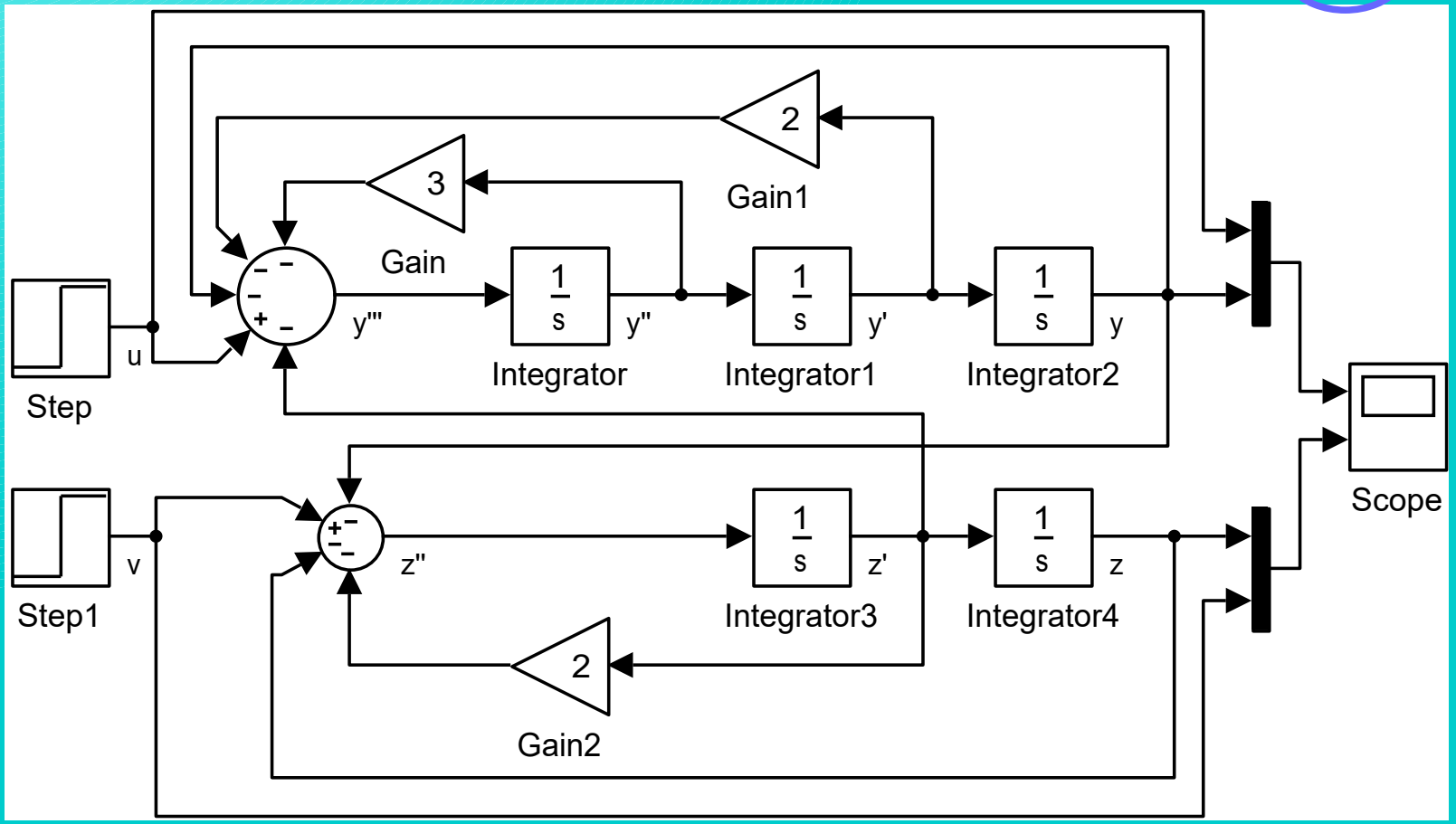


Conversion of differential equation system

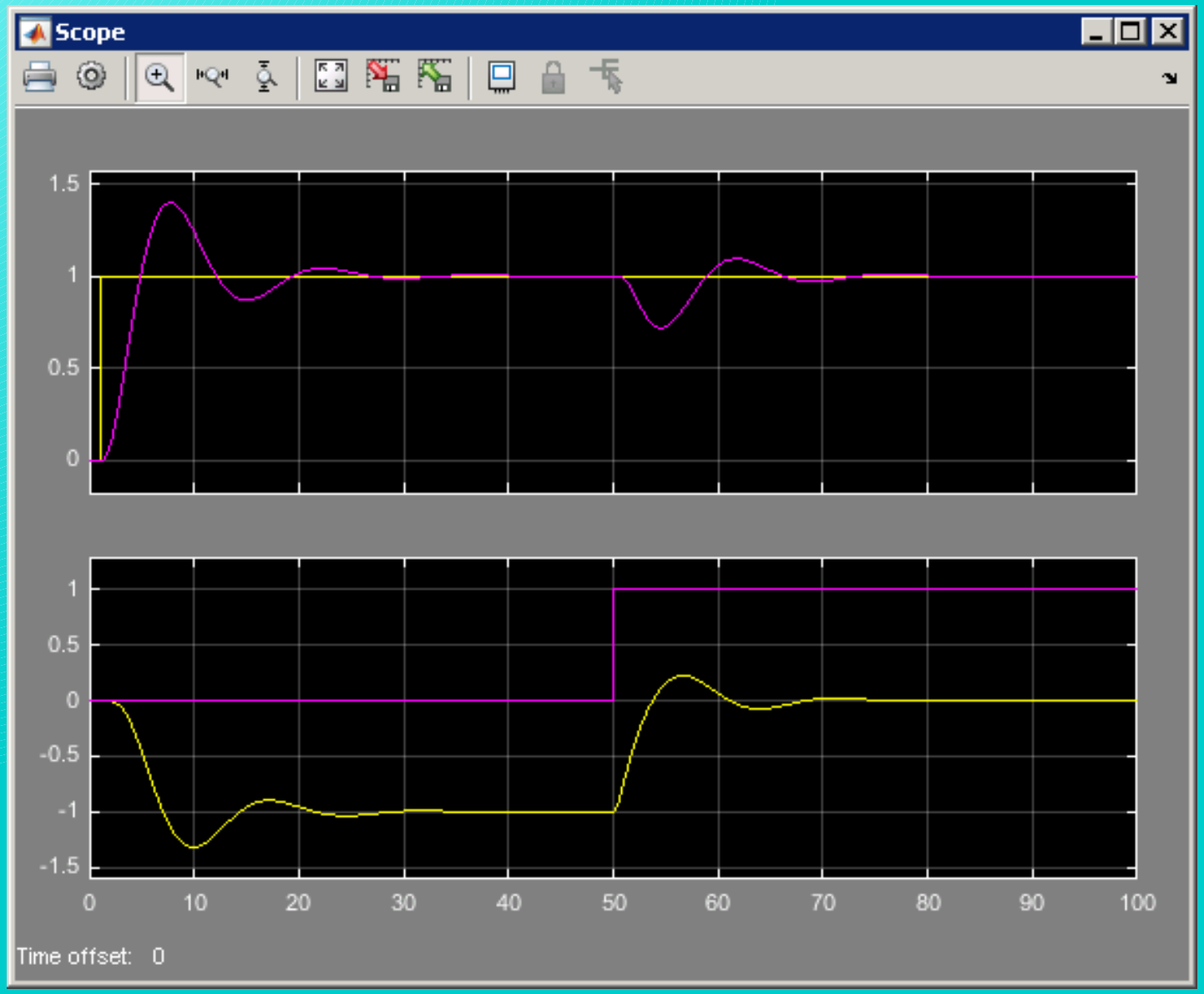
If there are no errors, the equations can be connected.

$$\dddot{y} = u - 3\,\ddot{y} - 2\,\dot{y} - y - \dot{z}$$

$$\ddot{z} = v - 2\,\dot{z} - z - y$$



Převod diferenciální rovnice do Simulinkového modelu



Transfer function

The bockof Transfer function can be used for conversion of differential equatioin into the Simulink model. The Simulink model is then simplier, however more calculations are needed.

The block of Transfer function can be used only for linear equation conversion. If any non-linearity appears, the bloc of Transfer function cannot be used.

Transfer function

First step is separation of all derivatives of the variable with highest derivative.

$$\dddot{y} + 3\ddot{y} + 2\dot{y} + y + \dot{z} = u$$

$$\ddot{z} + 2\dot{z} + z + y = v$$

$$\dddot{y} + 3\ddot{y} + 2\dot{y} + y = u - \dot{z}$$

$$\ddot{z} + 2\dot{z} + z = v - y$$

Transfer function

Then the Laplace transform is applied (typically, the simplest rules are necessary to use)

$$\dot{} \Rightarrow s$$

$$\ddot{} \Rightarrow s^2$$

$$\dddot{} \Rightarrow s^3$$

$$\int \Rightarrow s^{-1}$$

Transfer function

Then the Laplace transform is applied

$$\dddot{y} + 3\ddot{y} + 2\dot{y} + y = u - \dot{z}$$
$$\ddot{z} + 2\dot{z} + z = v - y$$

$$s^3 Y + 3s^2 Y + 2sY + Y = U - sZ$$
$$s^2 Z + 2sZ + Z = V - Y$$

Transfer function

Then the variables at the left side should be separated.

$$s^3 Y + 3 s^2 Y + 2 s Y + Y = U - s Z$$

$$s^2 Z + 2 s Z + Z = V - Y$$

$$Y = \frac{U - s Z}{s^3 + 3 s^2 + 2 s + 1}$$

$$Z = \frac{V - Y}{s^2 + 2 s + 1}$$

Blok přenosu

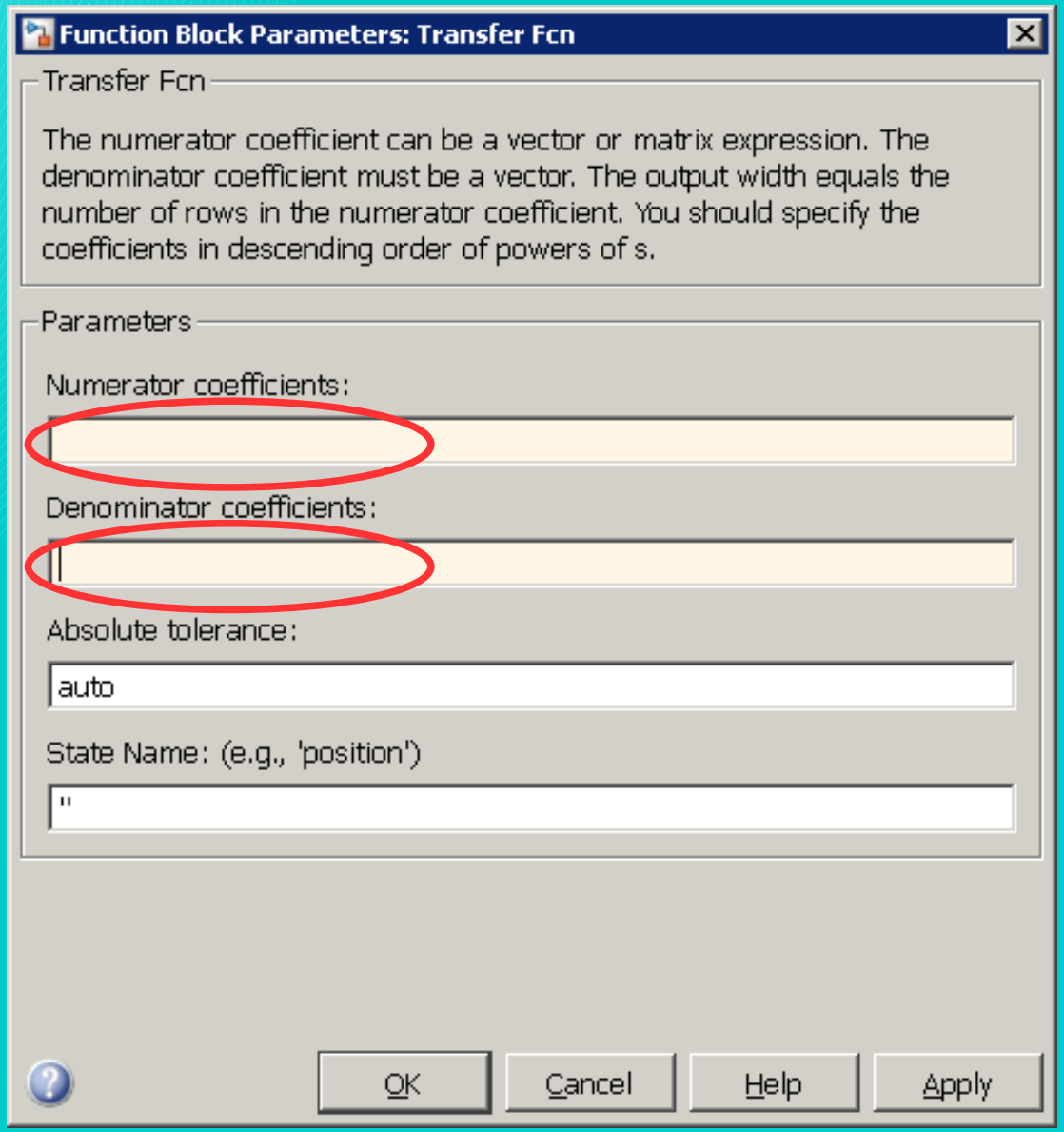
… and modified in a way that every fraction describes the relatioin between only two variables.

$$Y = \frac{U}{s^3 + 3s^2 + 2s + 1} - \frac{sZ}{s^3 + 3s^2 + 2s + 1}$$

$$Z = \frac{V}{s^2 + 2s + 1} - \frac{Y}{s^2 + 2s + 1}$$

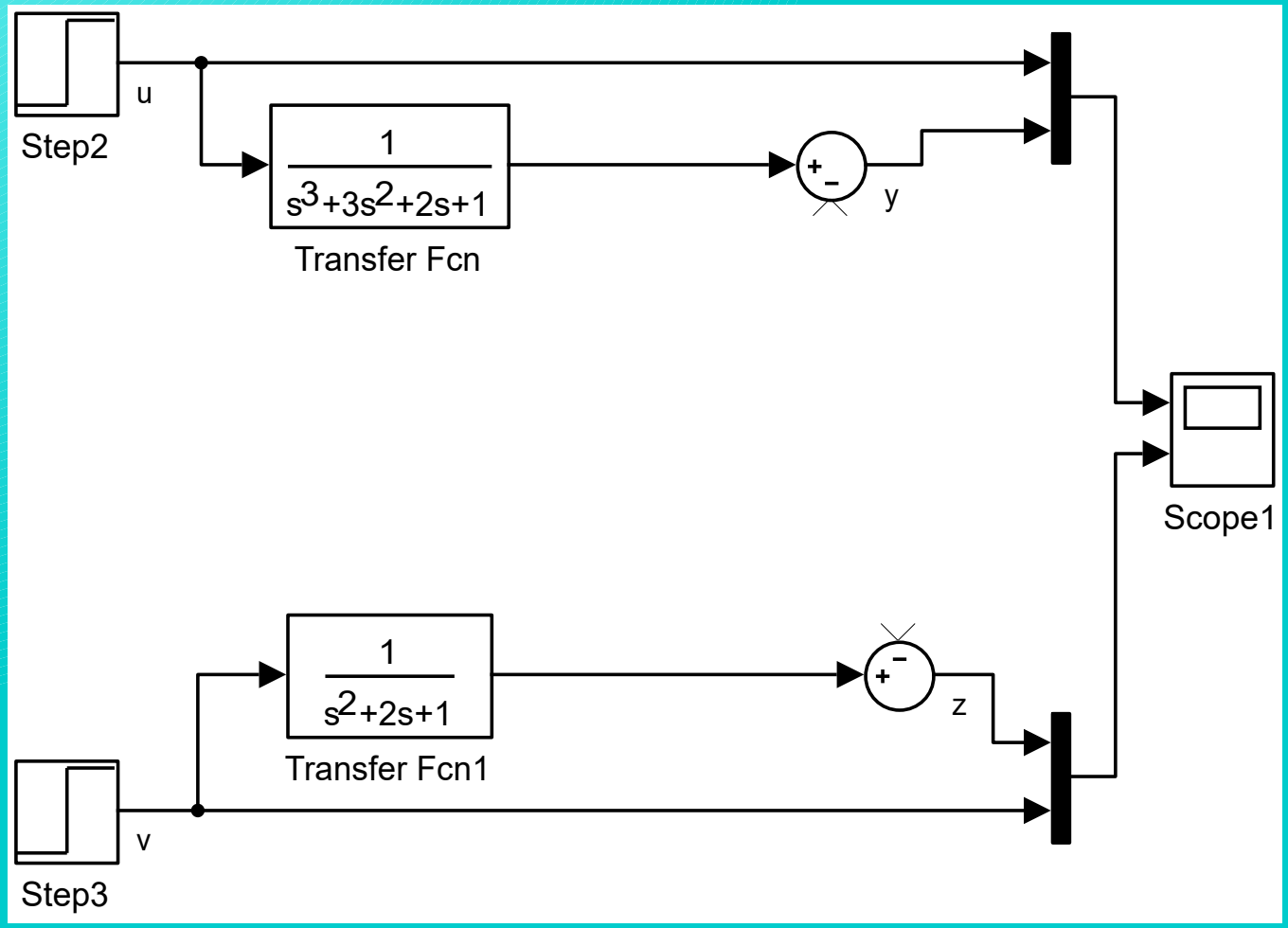
Transfer function
block parameter setting

$$s^3 + 3s^2 + 2s + 1 \Rightarrow [1\ 3\ 2\ 1]$$
$$s^2 + 2s + 1 \Rightarrow [1\ 2\ 1]$$
$$1 \Rightarrow [1]$$
$$s \Rightarrow [1\ 0]$$

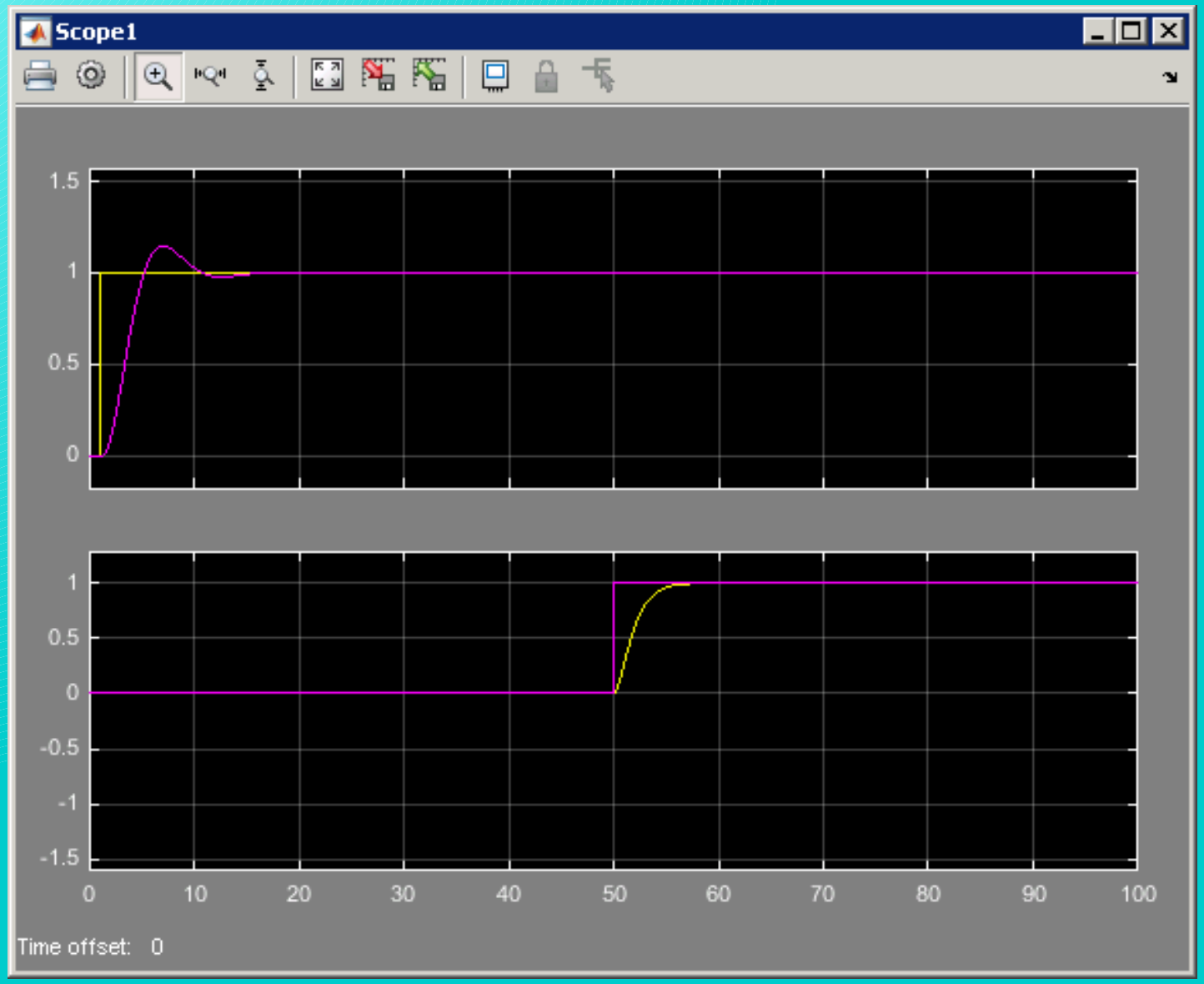


Transfer function

Similarly as in previous case,first, the invidiual equation Simulink modl is build ...

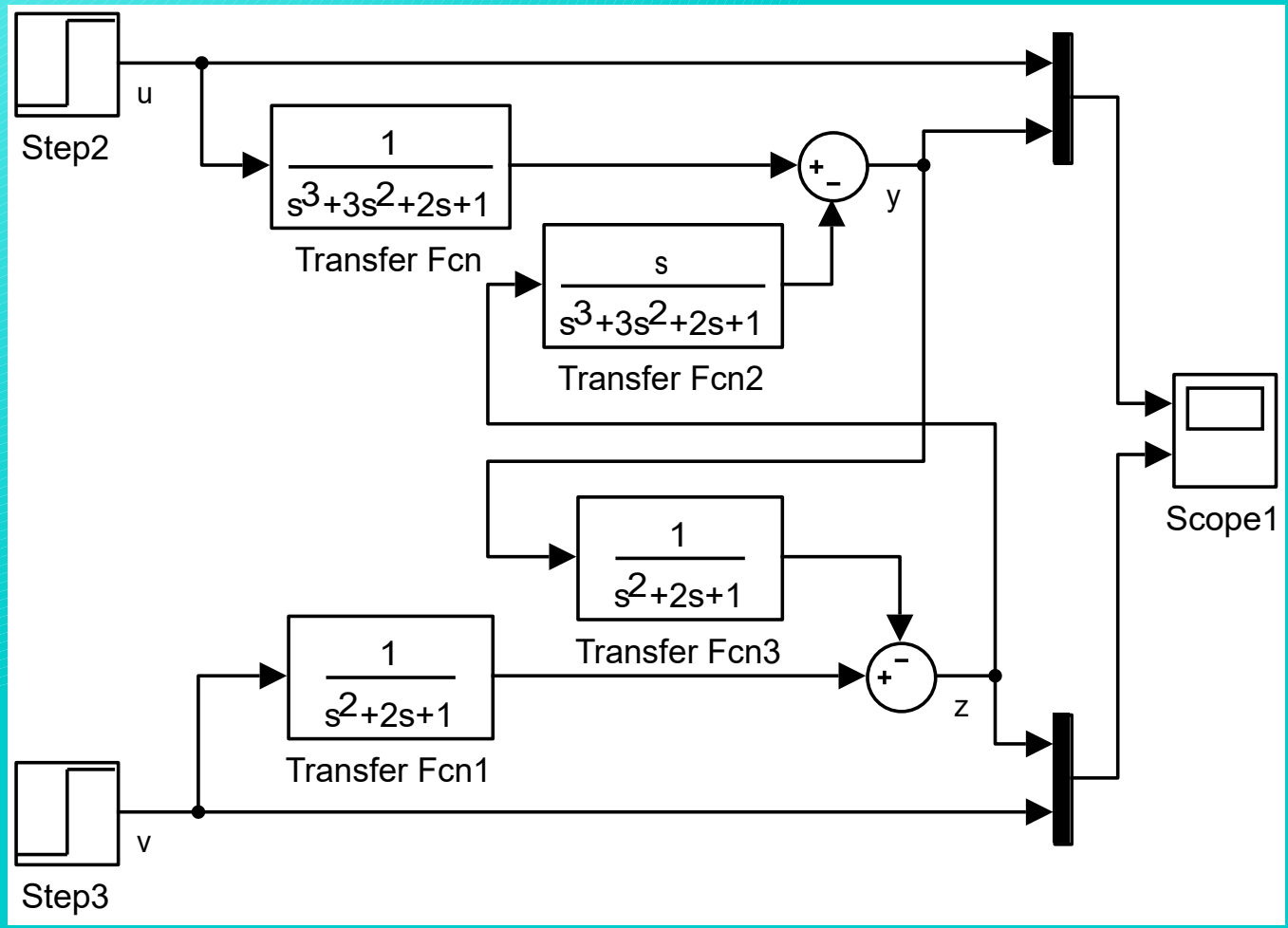


Transfer function

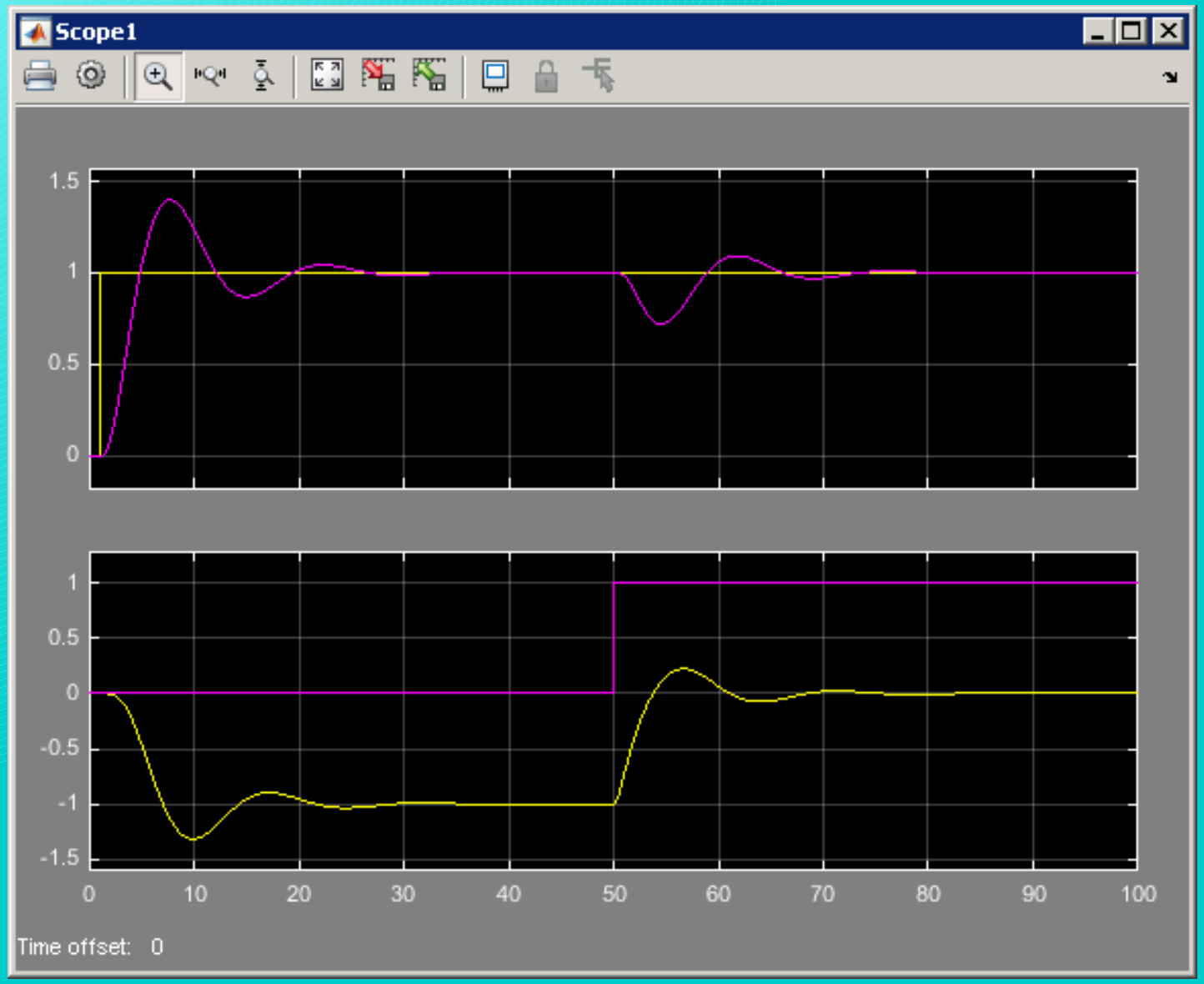


Transfer function

… and after the verification, both equations are connected.



Transfer function



Differences between both models

It is clear, that both models provide equal results. However, if we subtract corresponding variables of both models, their difference is not zero, but a small number.

That is caused by:

- Solution is conducted numerically,
- The results are obtained by a different calculatib process.

The differences between both models