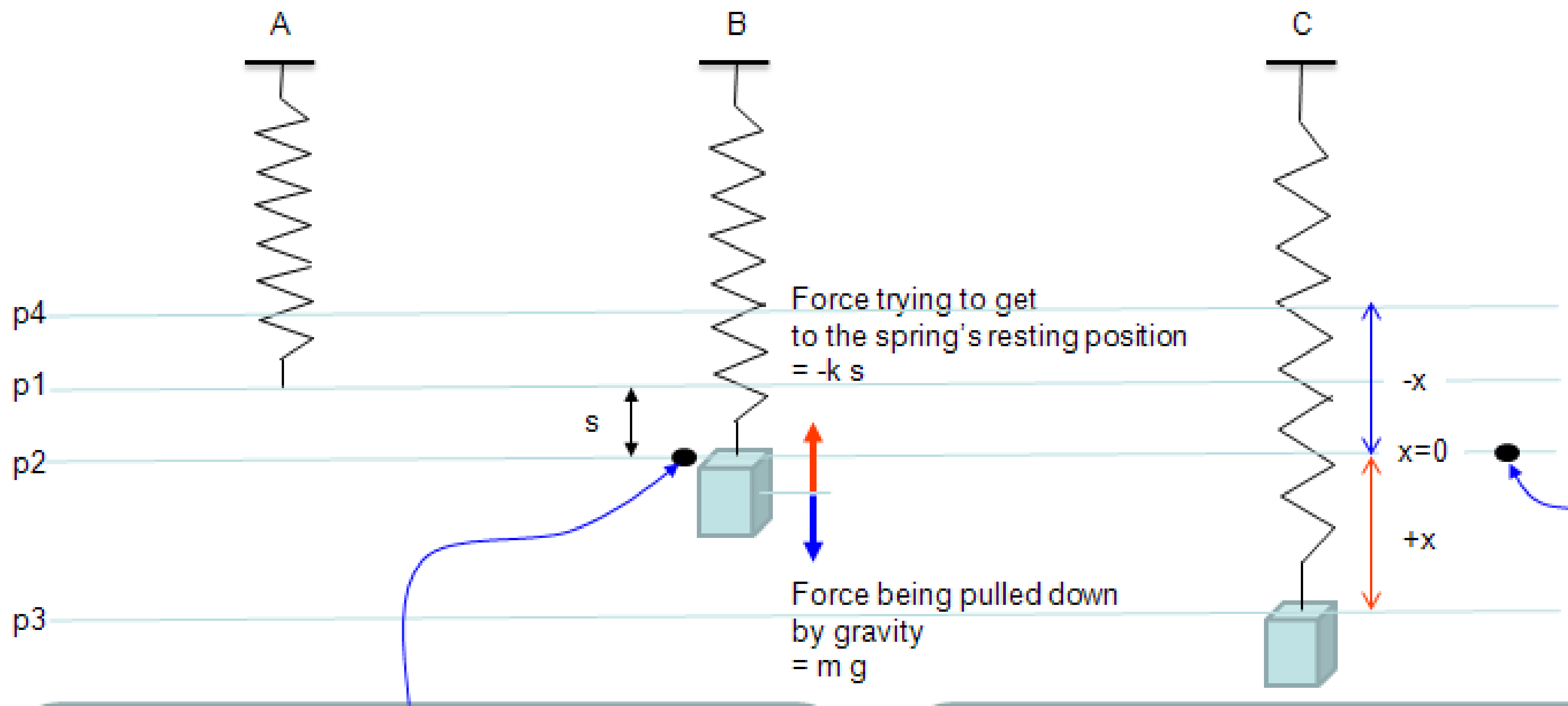
Matlab for Simulations

Stanislav Vrána

Springs

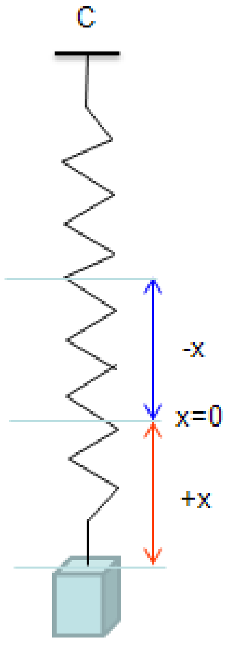
Spring



If you hand a mass to the spring, it would try to fall down and length of the spring would increase, but soon the mass would not fall down anymore because of the restoration force of the spring. This is the point where the springs restoration force and pulling force by gravity become same. We call this point as "Equilibrium Point". At this point, the mass does not move in any direction. So it is the same situation where there is no force being applied to the body (in reality, the two force with the same amount is continuously being applied in opposite direction)

It is very important to know where is the reference point, the point where we define x = 0. It is totally up to you how to define the reference point. You can set any point as a reference point but the final mathematical equation may differ depending on where you take as a reference point. So usually, we set the point where we can get a simplest mathematical model. In vertical spring model, we set the Equilibrium Point as the reference point because we can remove the term –k s and mg since they cancel each other at this point

Spring



Governing Law : Total Force applied to a body = Motion of the body

$$F = ma$$

Q . What kind of Force is there ?

i) Force to makes movement
= Restoration force of the spring trying to get back to the equilibrium position

$$= -kx$$

ii) Force created by Gravity
= Force pulling the object down to the ground

$$= mg$$

iii) Force to oppose the pulling force by gravity
= Restoration force of the spring just to oppose the pulling force by gravity

$$= -ks$$

iv) Force to prevent movement
= damping force

$$= -\beta \frac{dx}{dt}$$

+)

$$-kx + mg - ks - \beta \frac{dx}{dt} \Longrightarrow -kx - \beta \frac{dx}{dt}$$

We can set this part to be '0' by setting 'the equilibrium point' as the reference point of the model.
(Refer to previous figure and comments on it)

Q. Can I convert this into a term related to position of the mass (x = distance from the reference point) ?

A. Yes. Acceleration (a) is the 2nd derivative of distance (x)

$$a = \frac{d^2x}{dt^2}$$

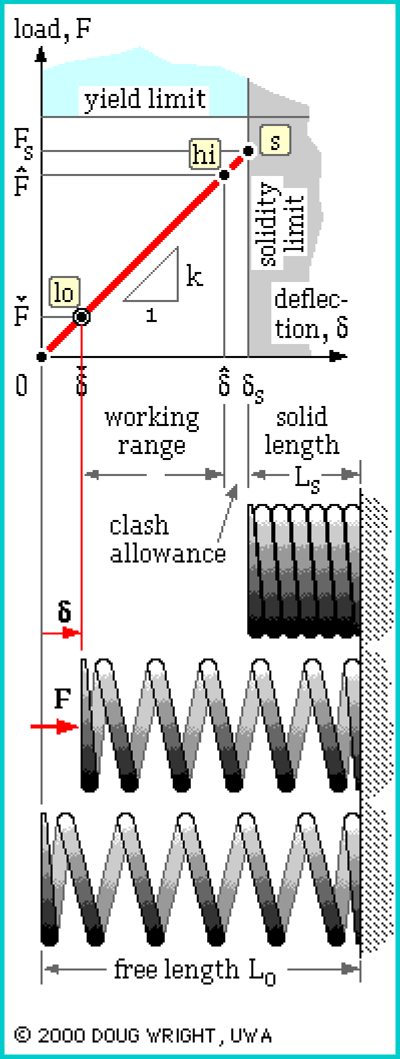
$$ma = m\frac{d^2x}{dt^2}$$

Spring equation

The inner damping force can be very small, so it is possible to consider β = 0. If necessary, the inner damping can be modelled as external damping. Then, if the spring is loaded by external force *F*(*t*)

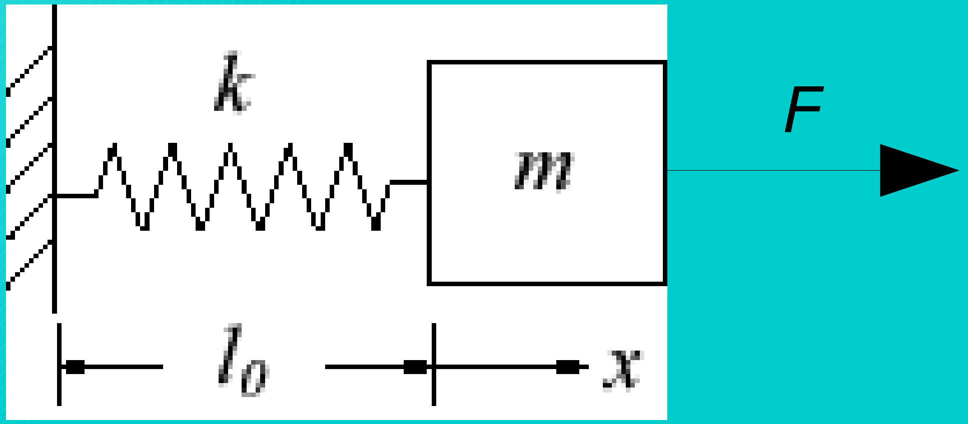
$$\mathrm{k}\,x(t)=F(t)$$

$$k\left(x(t)\right)x(t)=F(t)$$



Spring and mass

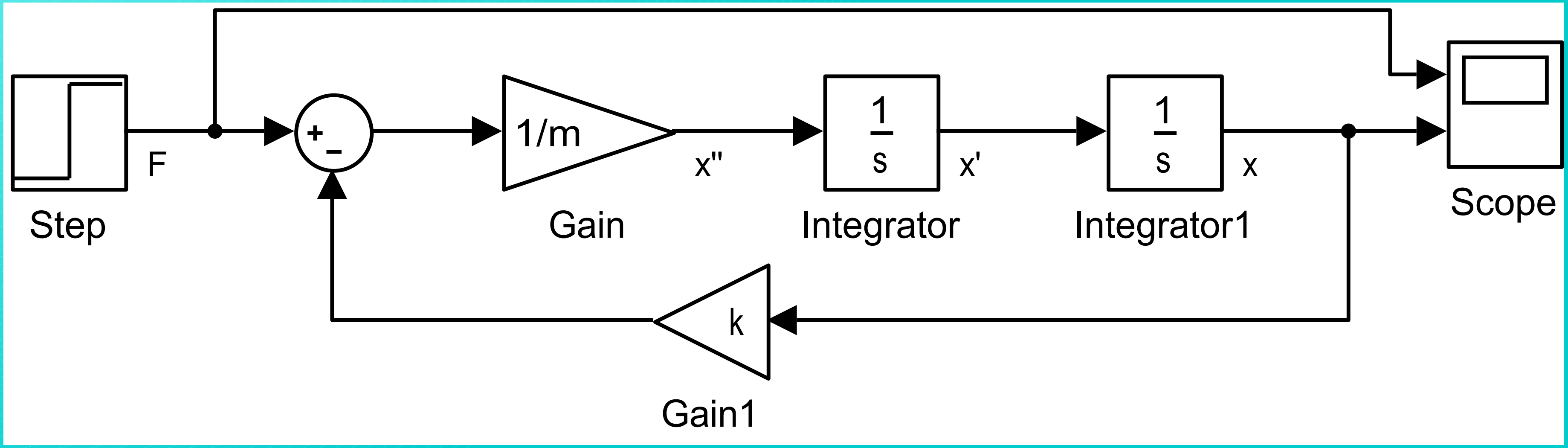
Spring – mass system



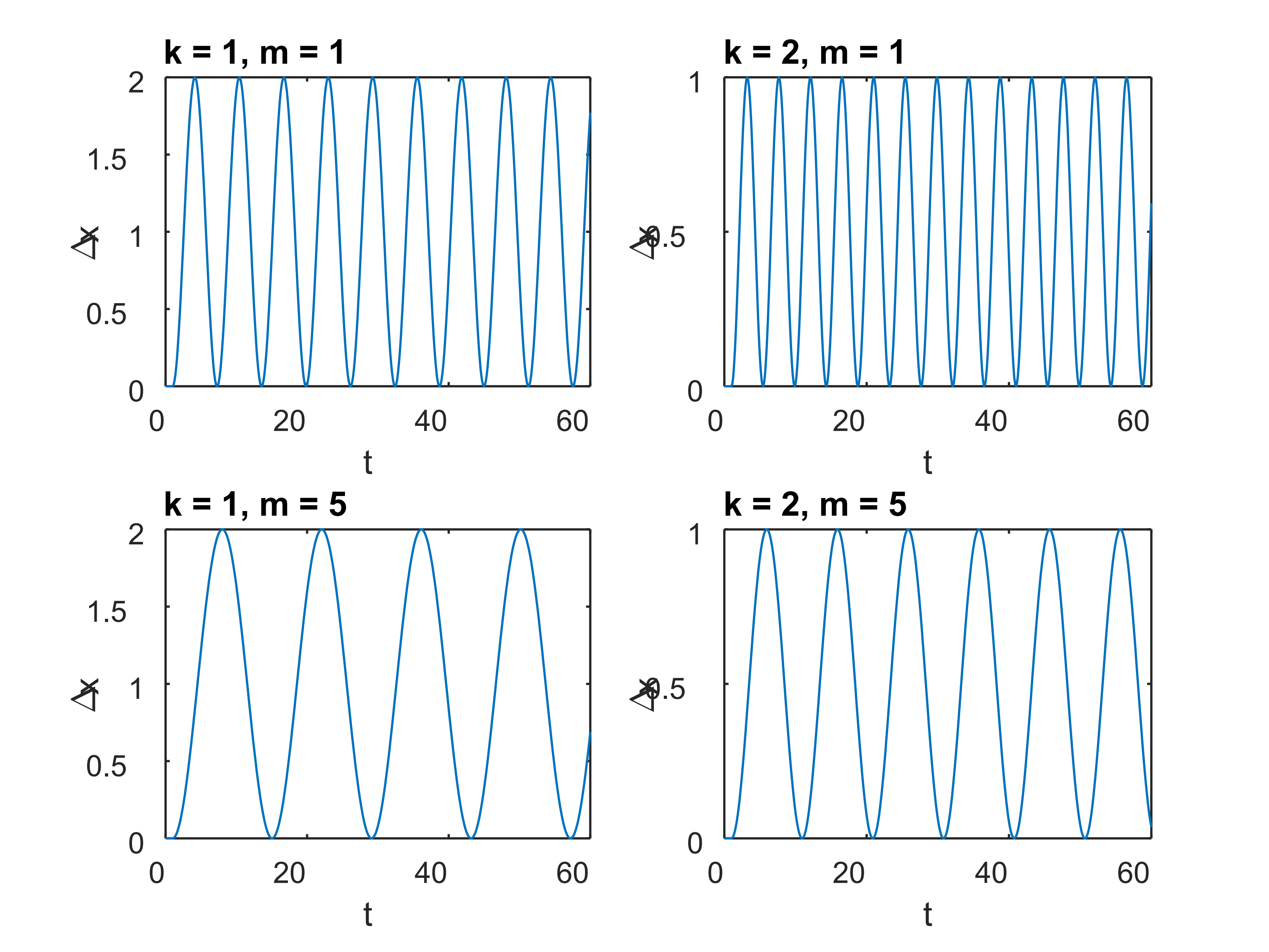
$$m\,\ddot{x}(t)+\mathrm{k}\,x(t)=F(t)$$

$$\ddot{x}(t)=\frac{F(t)-\mathrm{k}\,x(t)}{m}$$

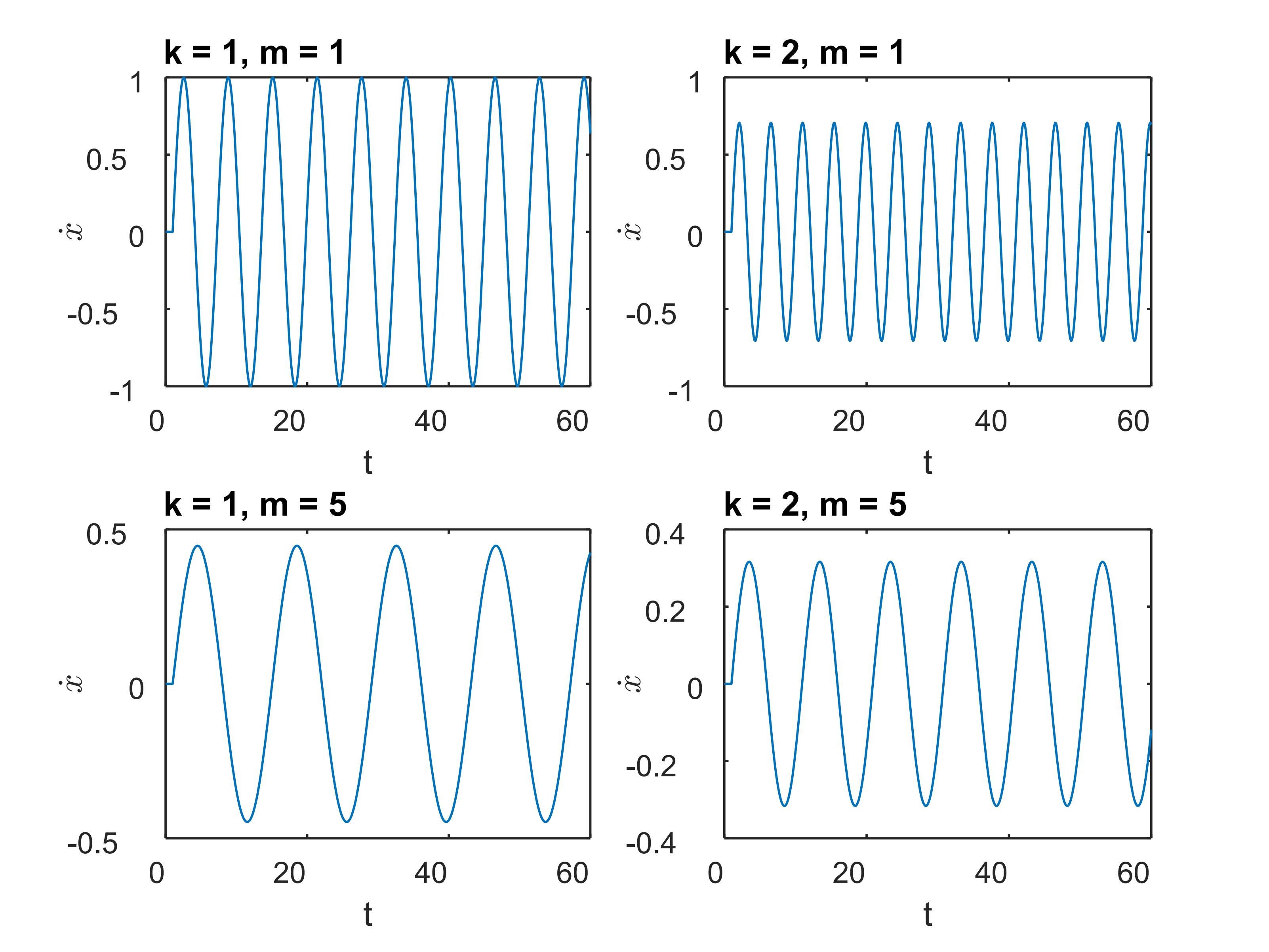
Spring and mass



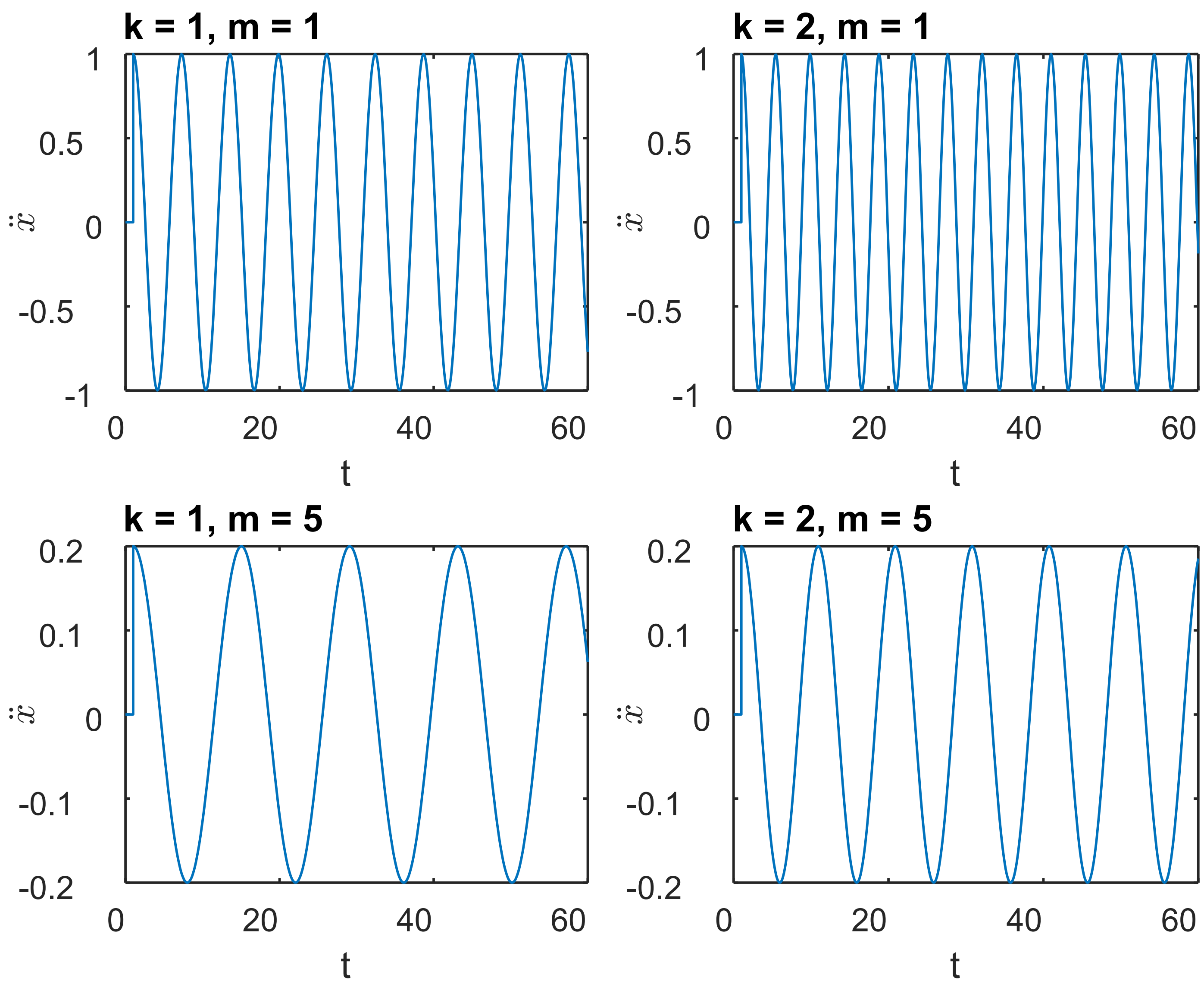
Spring and mass



Spring and mass

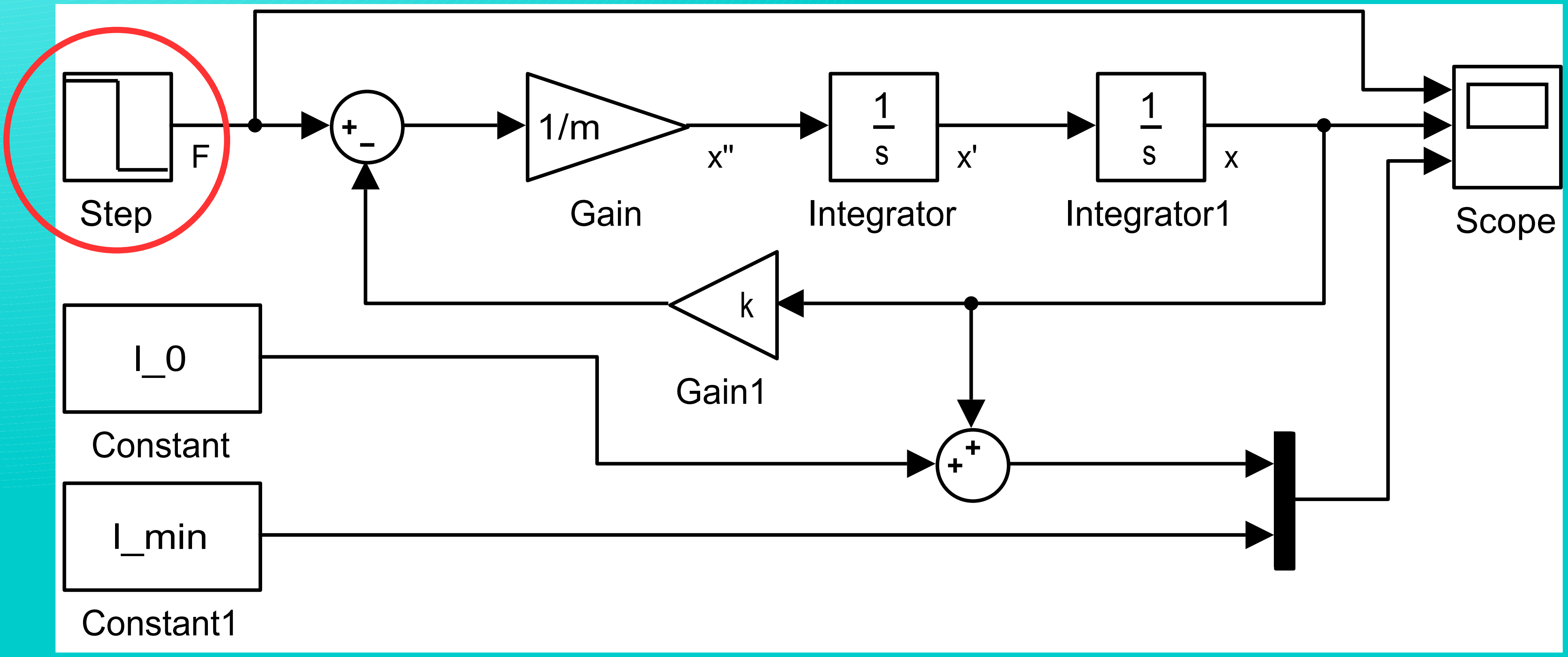


Spring and mass



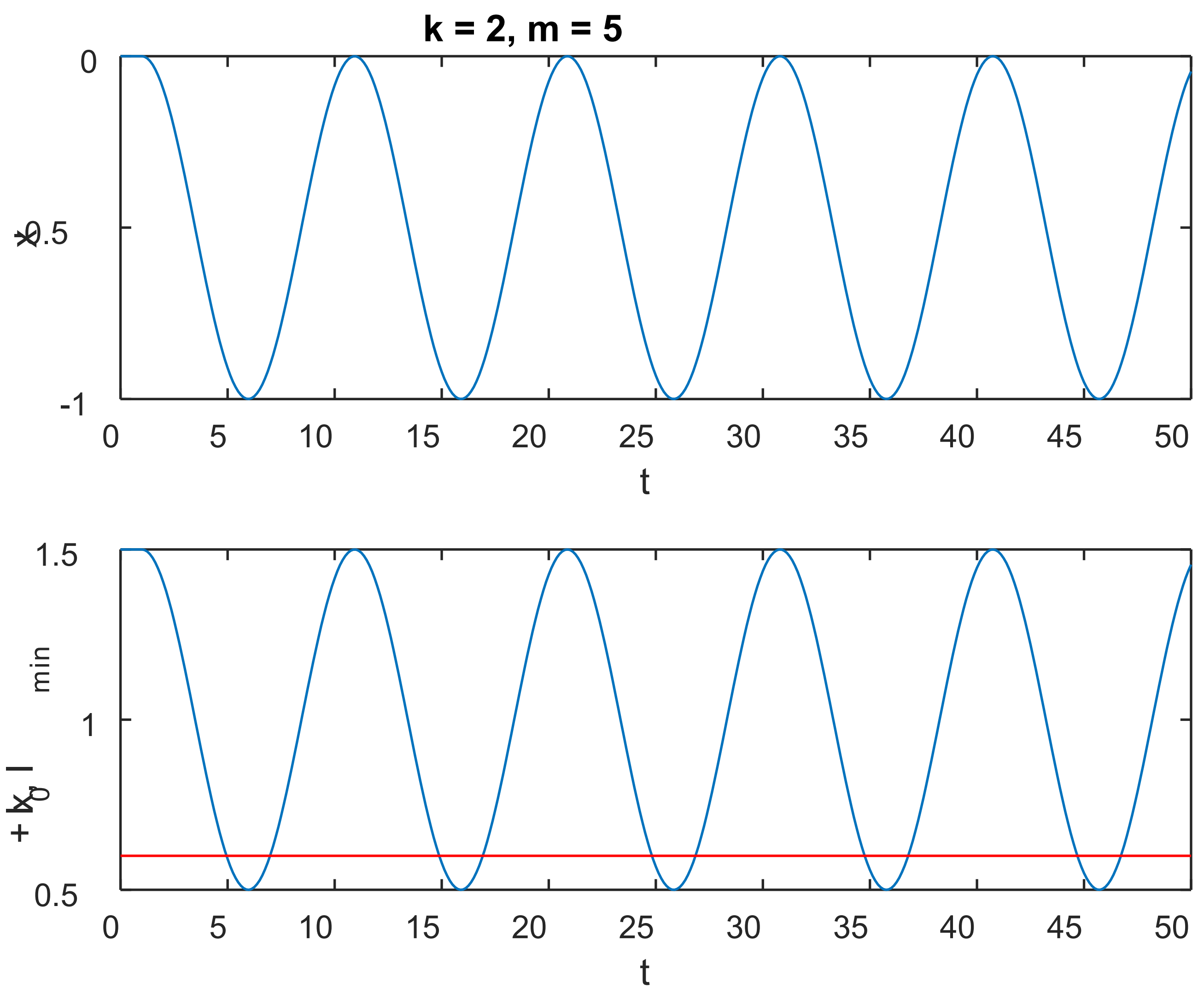
Spring and mass

Till now, only the difference between the length of spring when unloaded and loaded by force is shown. But it is necessary to verify if current spring length does not exceed the limits.



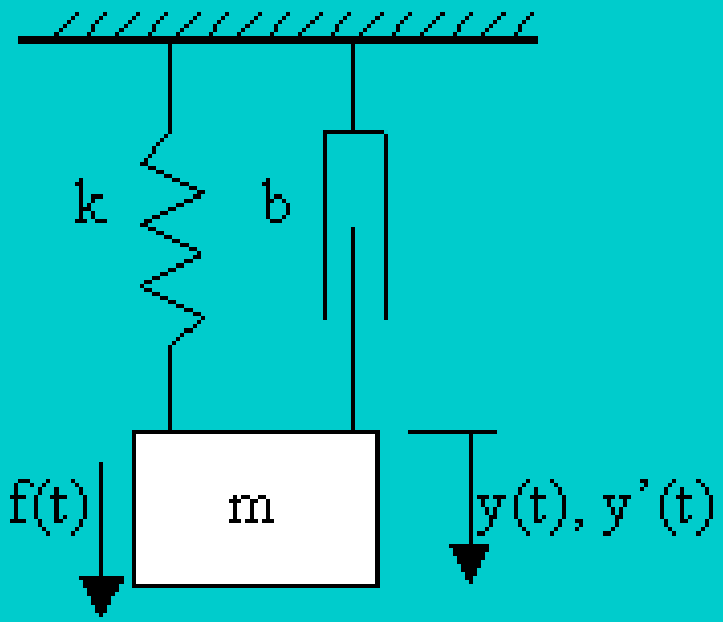
Spring and mass

$l_0 = 1.5$, $l_{min} = 0.6$



Spring and mass with damping

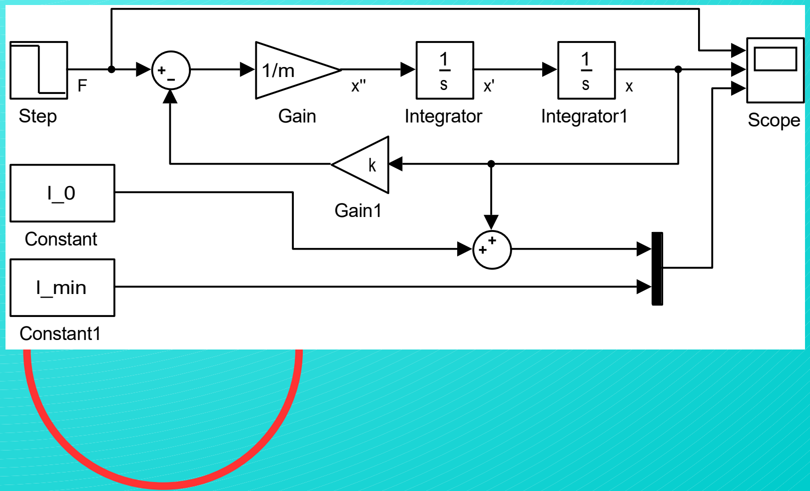
Spring – mass – damping system



$$m\ddot{x}(t)+\mathrm{b}\,\dot{x}(t)+\mathrm{k}\,x(t)=F(t)$$

$$\ddot{x}(t)=\frac{F(t)-\mathrm{b}\,\dot{x}(t)-\mathrm{k}\,x(t)}{m}$$

Spring and mass with damping



Conversion of differential equation system

First step is separation of the highest derivations in all equations.

$$\dddot{y} + 3\ddot{y} + 2\dot{y} + y + \dot{z} = u$$
$$\ddot{z} + 2\dot{z} + z + y = v$$

$$\dddot{y} = u - 3\ddot{y} - 2\dot{y} - y - \dot{z}$$
$$\ddot{z} = v - 2\dot{z} - z - y$$

Conversion of differential equation system

Then it is suitable to prepare all variables into scheme.

$$\dddot{y} = u - 3\ddot{y} - 2\dot{y} - y - \dot{z}$$
$$\ddot{z} = v - 2\dot{z} - z - y$$

The equation systém is suitable convert as a set of independent equations and connect them in final step. The possible mistakes can be found more easily.

Conversion of differential equation system

Then it is suitable to prepare all variable into scheme.

$$\dddot{y} = u - 3\ddot{y} - 2\dot{y} - y - \dot{z}$$
$$\ddot{z} = v - 2\dot{z} - z - y$$