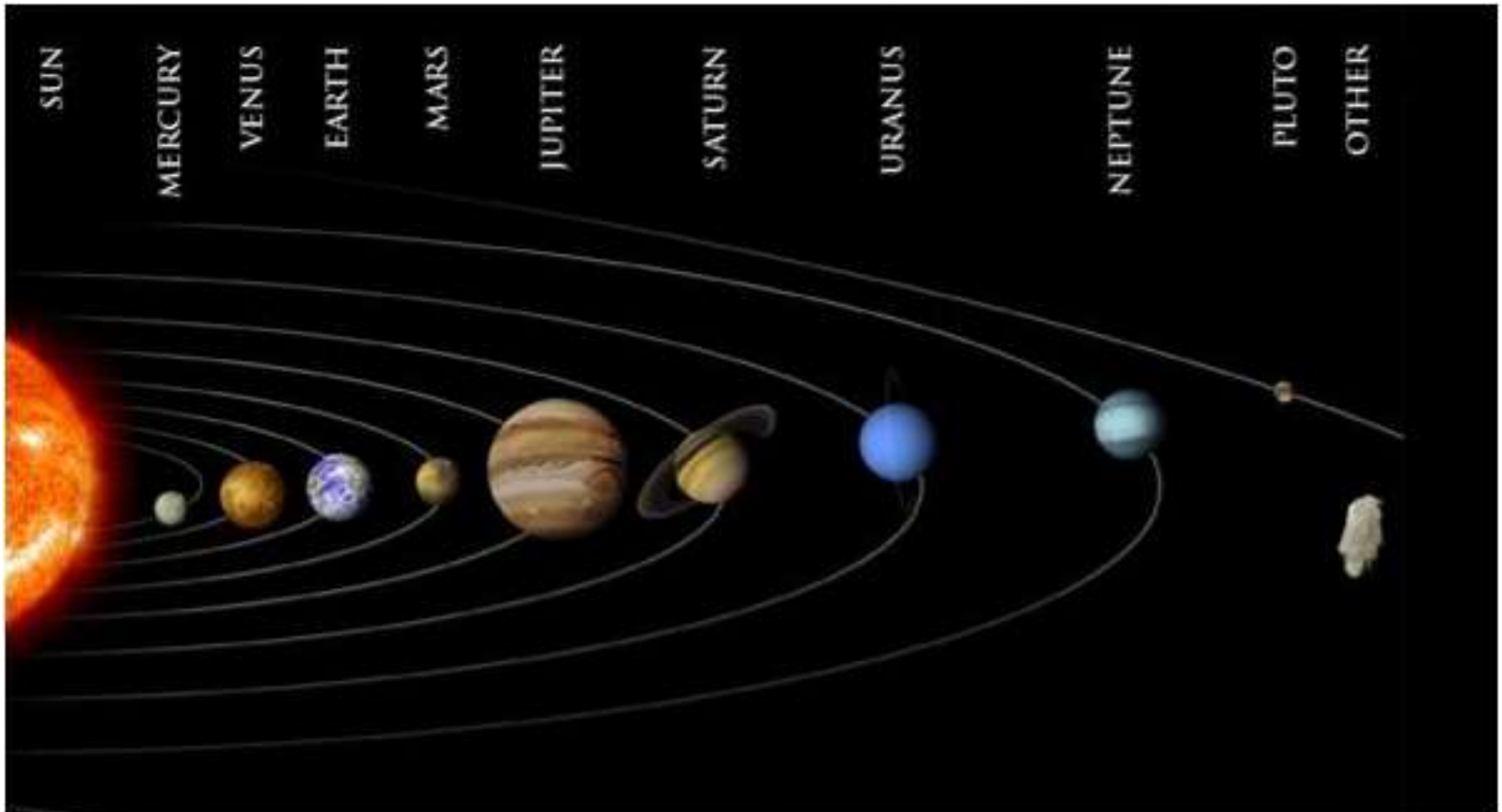
Solar energy

- solar radiation
- definitions
- incident solar energy

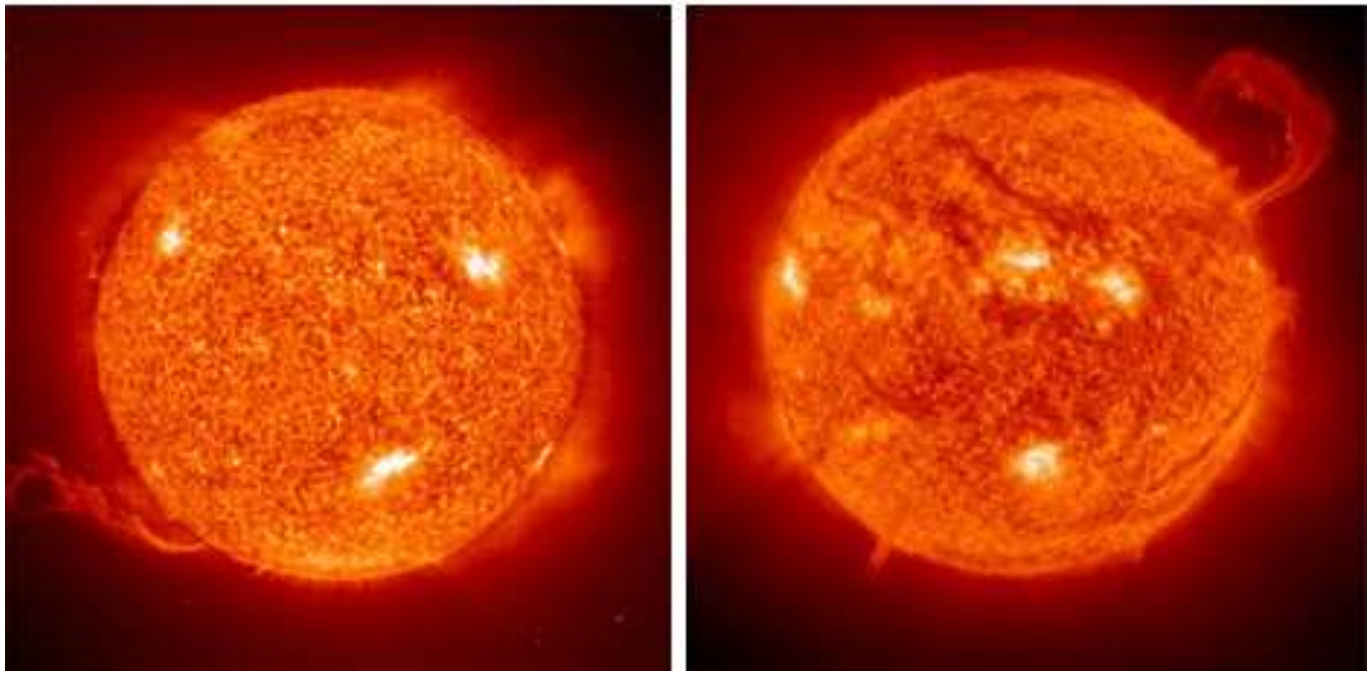
Sun

- closest star
- centre of our planetary system – solar system



Sun

- diameter 1 392 000 km 109 x larger than Earth
- weight 2 x 10^{30} kg 330 000 x greater than Earth
  99,86 % weight of the solar system
- consists of: 70 % hydrogen **H**, 28 % helium **He**, 2 % other elements



Sun

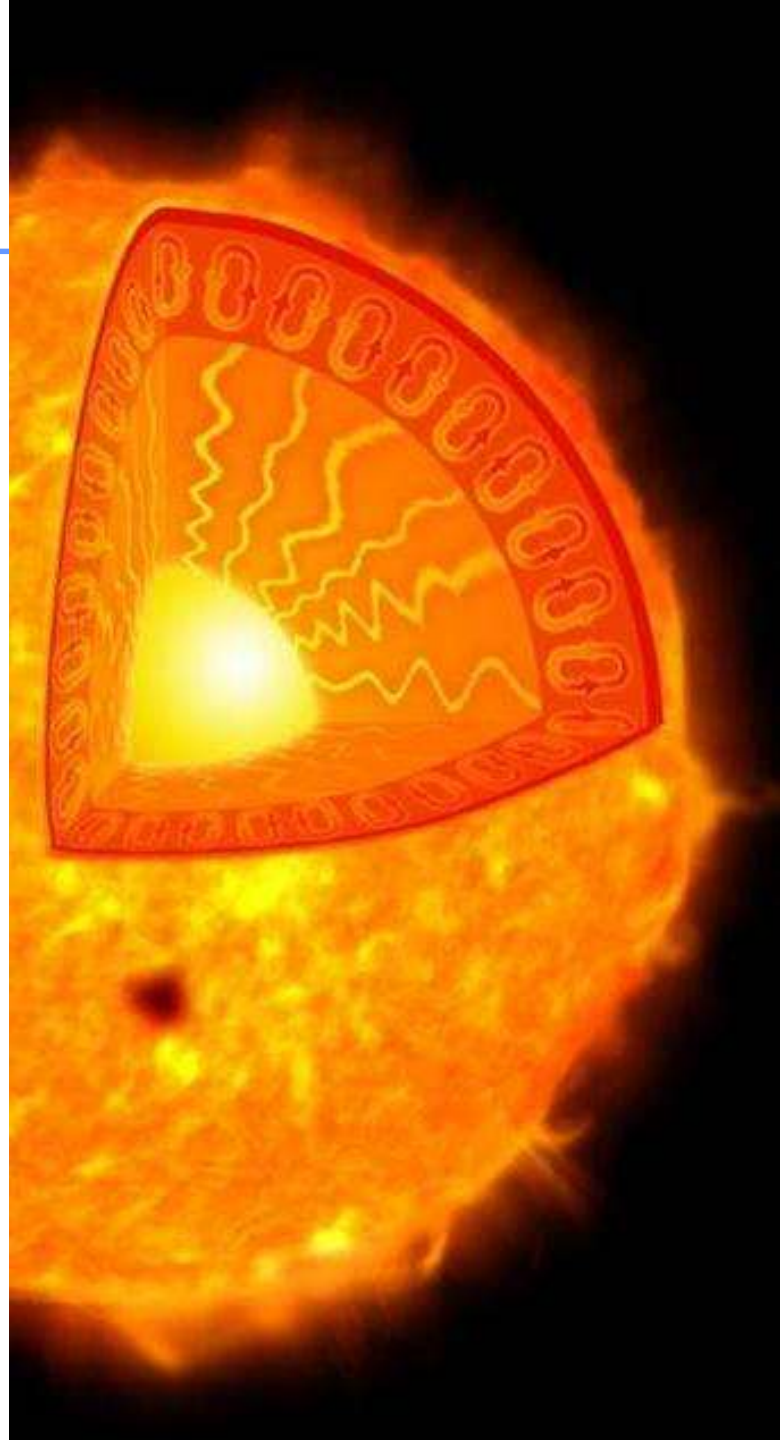
- origin of solar energy in **nuclear** reactions
- nuclear fusion takes place inside the Sun
  at high temperatures **10^6 K** and pressures **10^{10} Mpa,** atoms are ionised
  synthesis of hydrogen nuclei (H) $\rightarrow$ helium nuclei (He)
- **564 x 10^9 kg/s Hydrogen** transforms to **560 x 10^9 kg/s Helium**
- mass difference **4 x 10^9 kg/s** is radiated in the form of energy

$$E = m.c^2$$

- radiative power: **3,6 x 10^{26} W**
- specific radiated power (density): **6 x 10^7 W/m^2**

Sun

- **core** (to 23 % radius)
  temperature 10^6 K, X-ray radiation
  90 % of Sun's energy generated
- **radiative zone** (from 23 to 70 % radius)
  temperature falls down to 130 000 K
  radiative energy transfer (fotons)
- **convective zone** (from 70 to 100 % radius)
  lower density, convective energy transfer
- **photosphere** (visible surface of Sun)
  temperature 5800 K, solar radiation



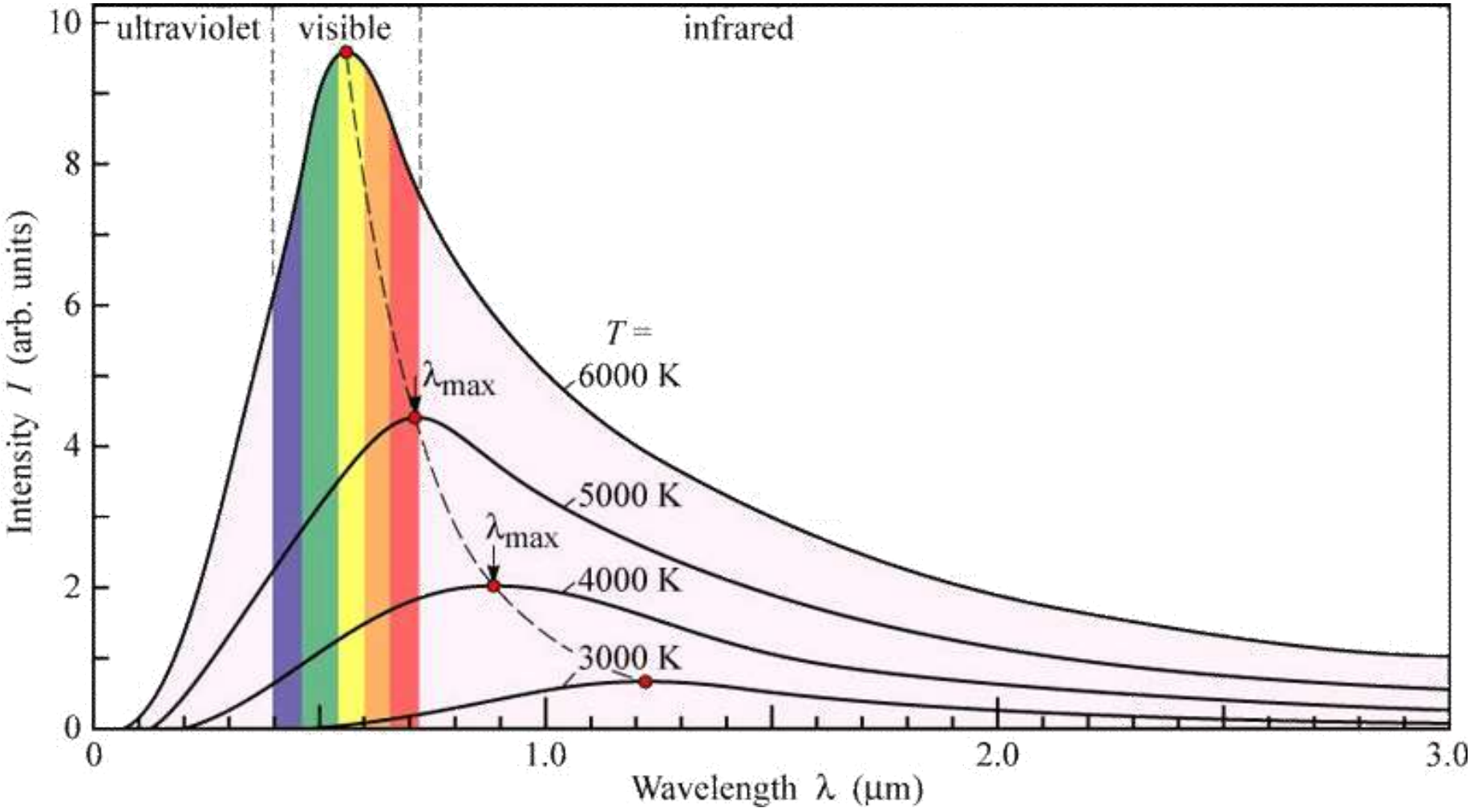
Spectral density of radiative flux

- Sun radiates as **perfect black body** with surface temperature **5800 K**
- spectral density of radiative solar flux (Planck's law)

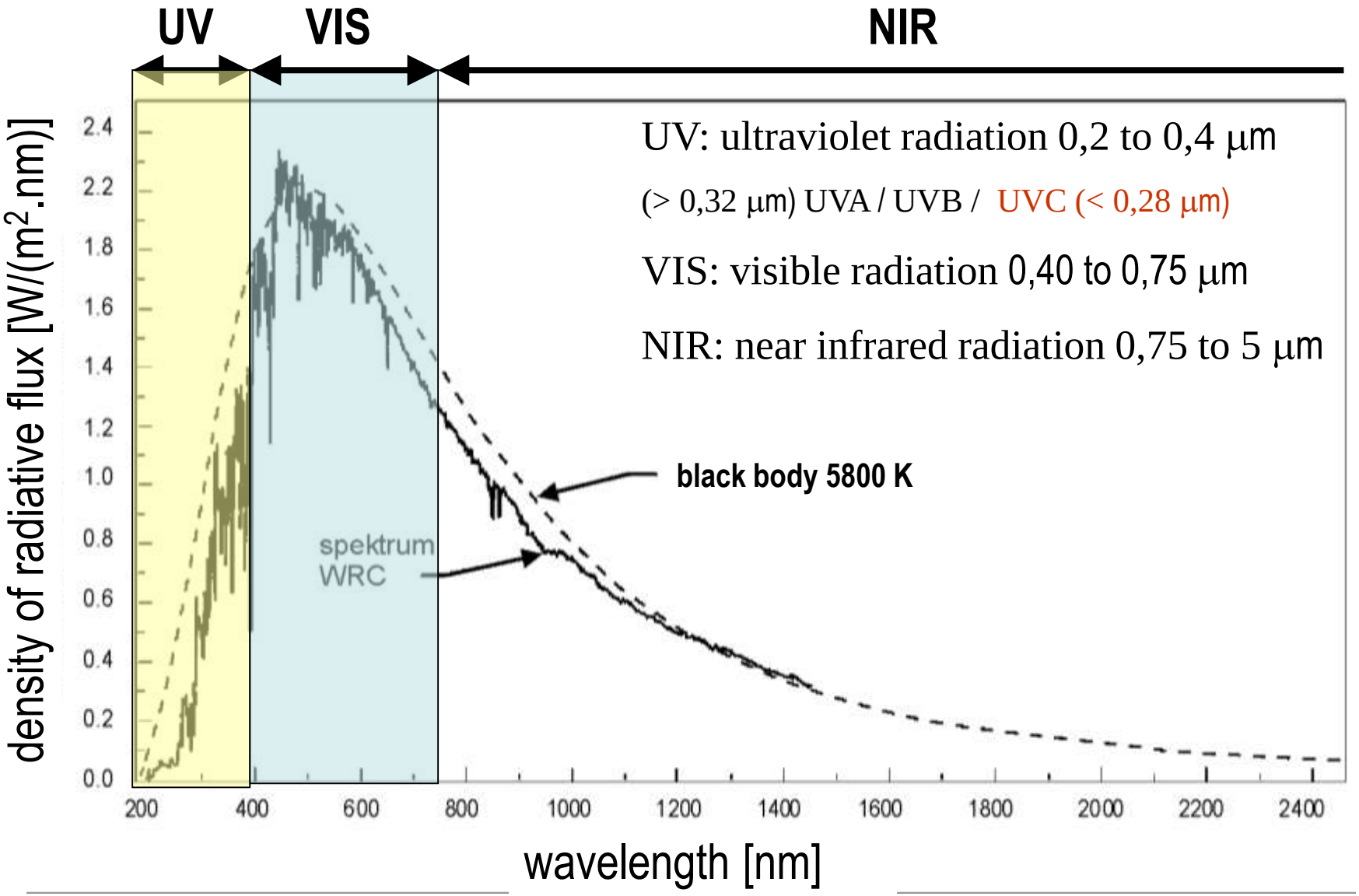
$$E_{\check{c}}(\lambda,T) = \frac{2\cdot\pi\cdot h\cdot c^2}{\lambda^5}\left[e^{\frac{h\cdot c}{k\cdot\lambda\cdot T}} - 1\right]^{-1} \qquad [W/m^2.\mu m]$$

h = 6,6256 x 10^{-34} J.s	Planck's constant
k = 1,3805 x 10^{-23} J/K	Boltzmann's constant
c = 2,9979 x 10^{8} m/s	light velocity in vacuum
T	thermodynamic surface temperature [K]

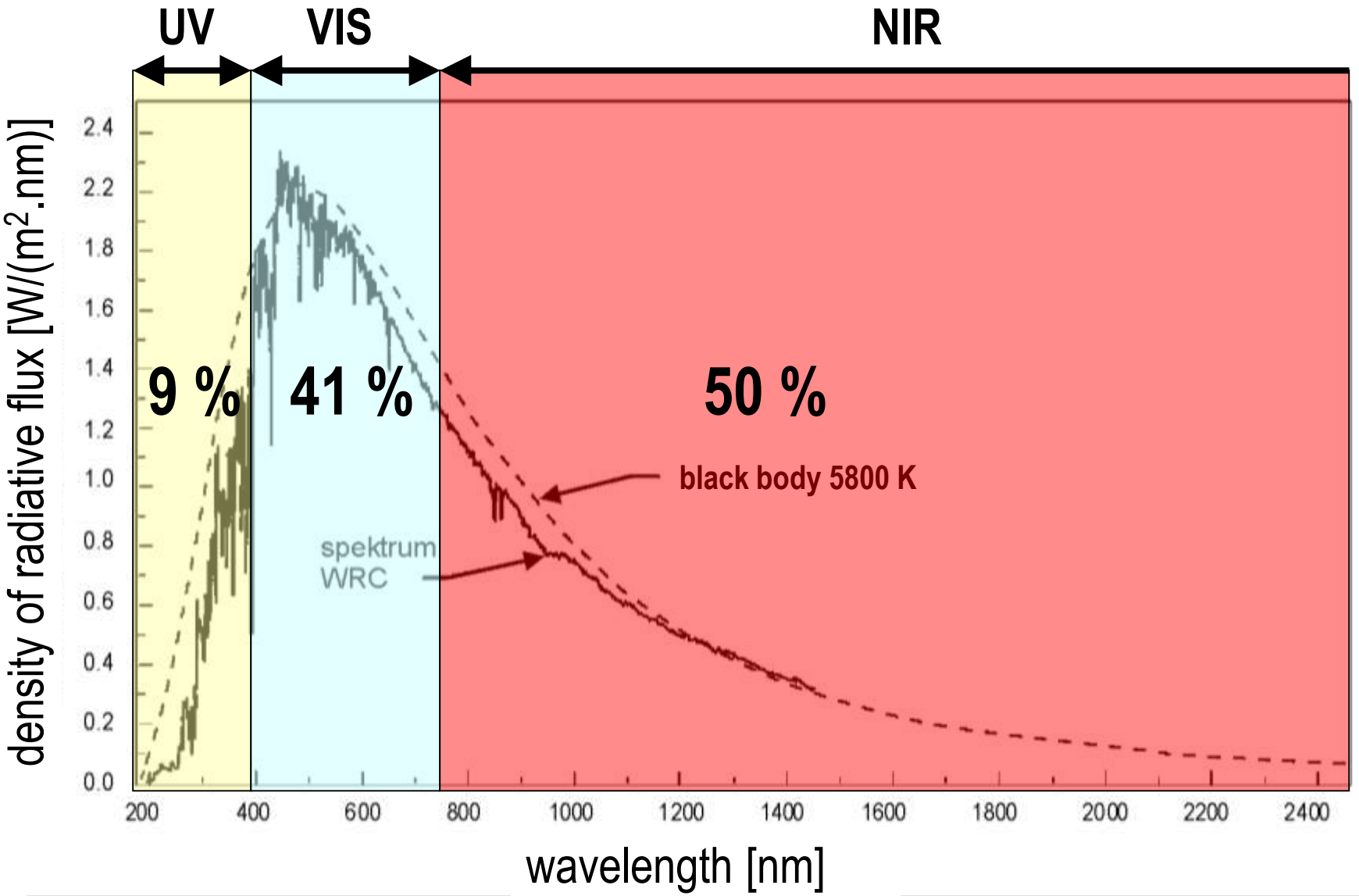
Planck's law


ultraviolet
visible
infrared
Intensity I (arb. units)
Wavelength λ (μm)
T =
6000 K
5000 K
4000 K
3000 K
λ_{max}
λ_{max}
0
1.0
2.0
3.0
0
2
4
6
8
10

Spectral density of radiative flux



Solar energy irradiated

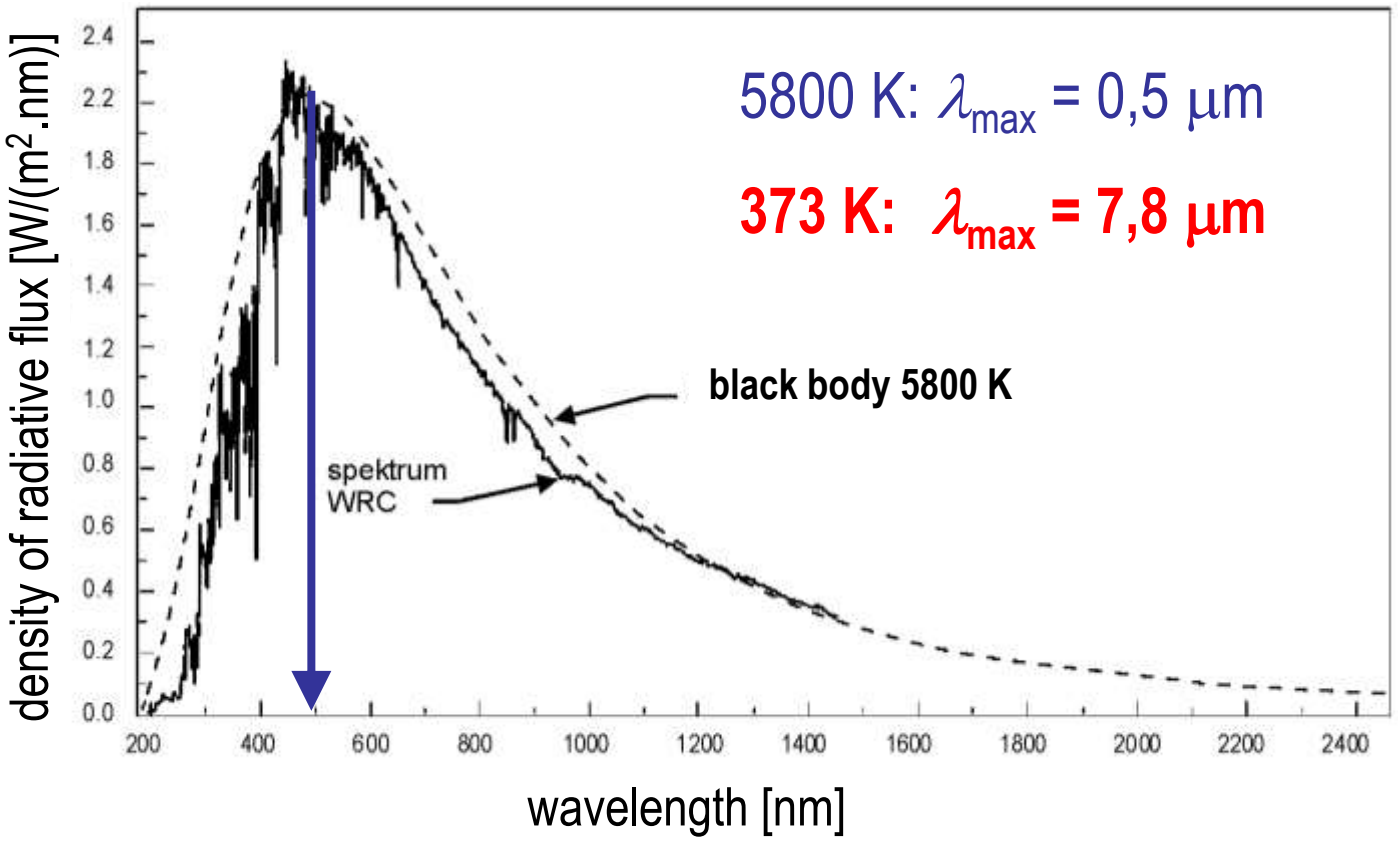


Wien's law

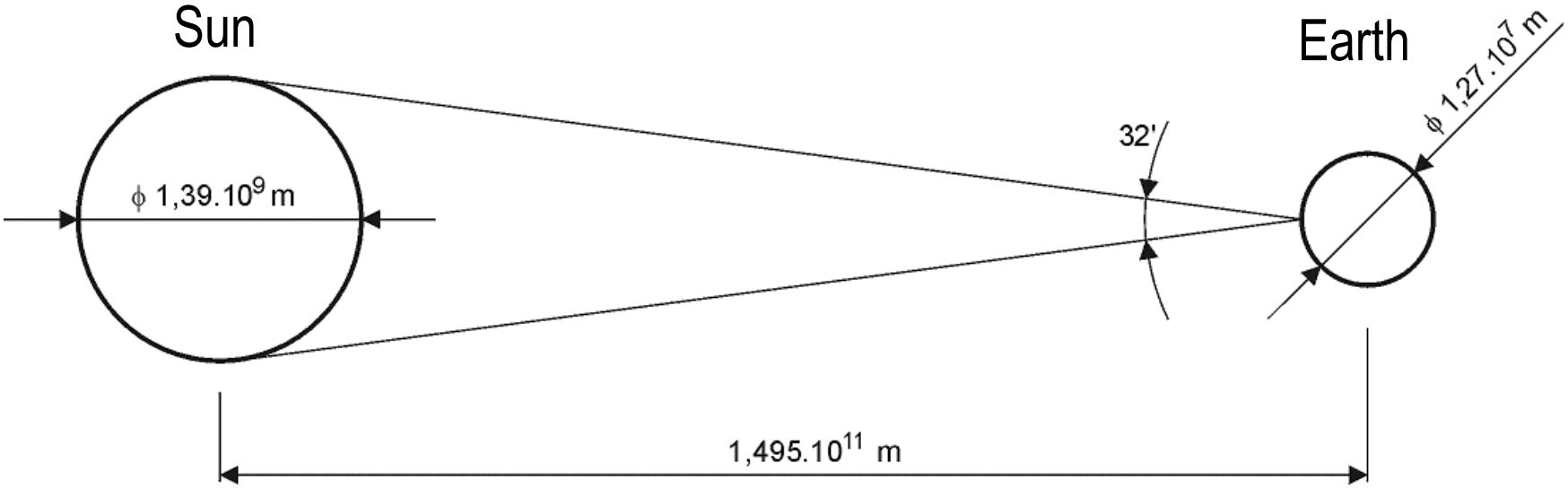
- maximum of radiative flux – seeking the extreme of Planck's function

$$\frac{\partial E_{č}(\lambda,T)}{\partial \lambda} = 0 \quad \rightarrow \quad \lambda_{max} \cdot T = 2898 \quad [\mu m.K]$$

Wien's displacement law



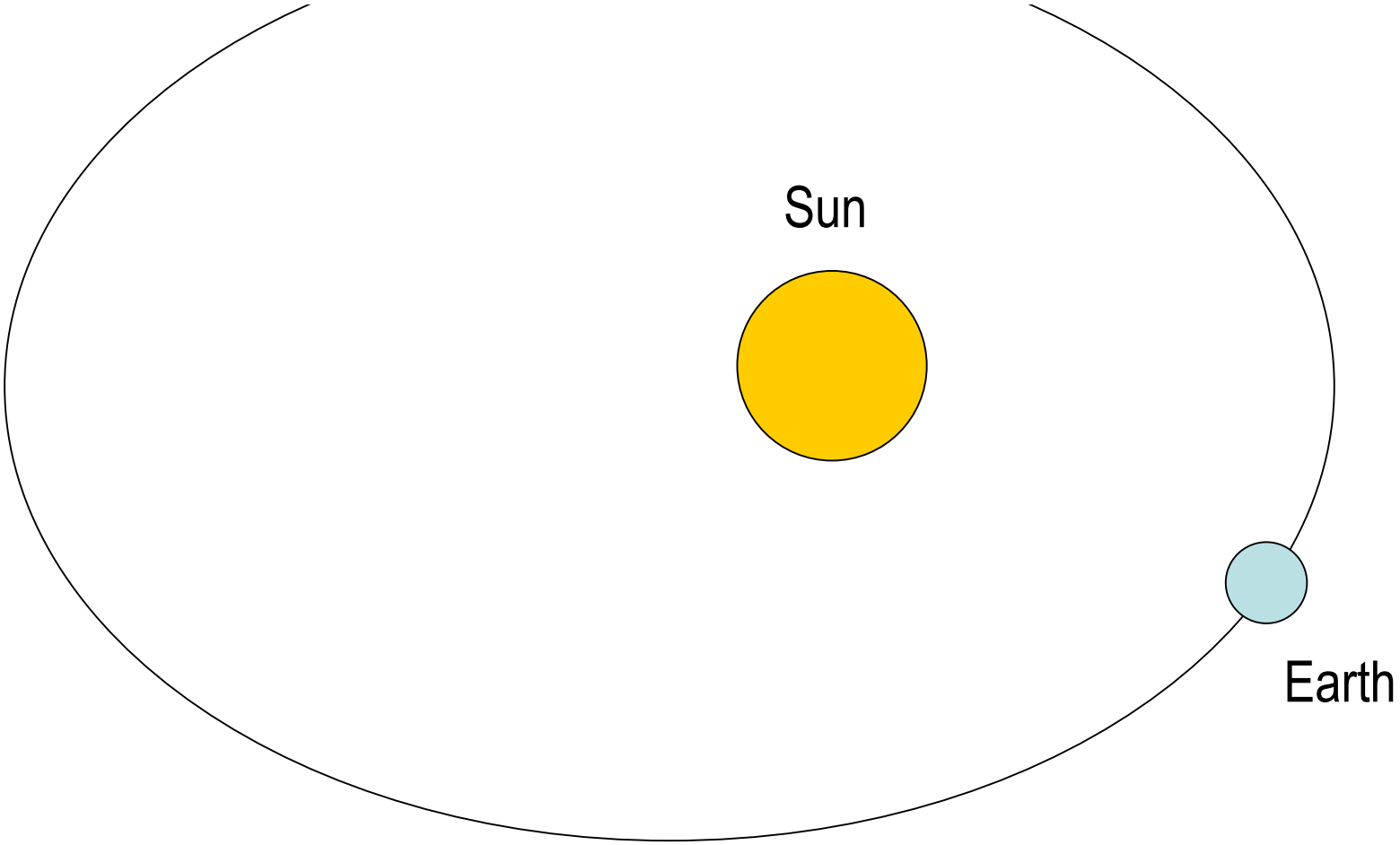
Propagation of solar energy



- power spreads to larger area with increasing distance from Sun
- 0,5 x 10^{-9} of the irradiated Sun's power is incident on Earth
  radiative flux **1,7 x 10^{17} W**
- solar „beams" considered as parallel (32')

Earth rotates around Sun ...

eliptic orbit (almost circular), Sun in one of focuses



Radiative flux density out of atmosphere

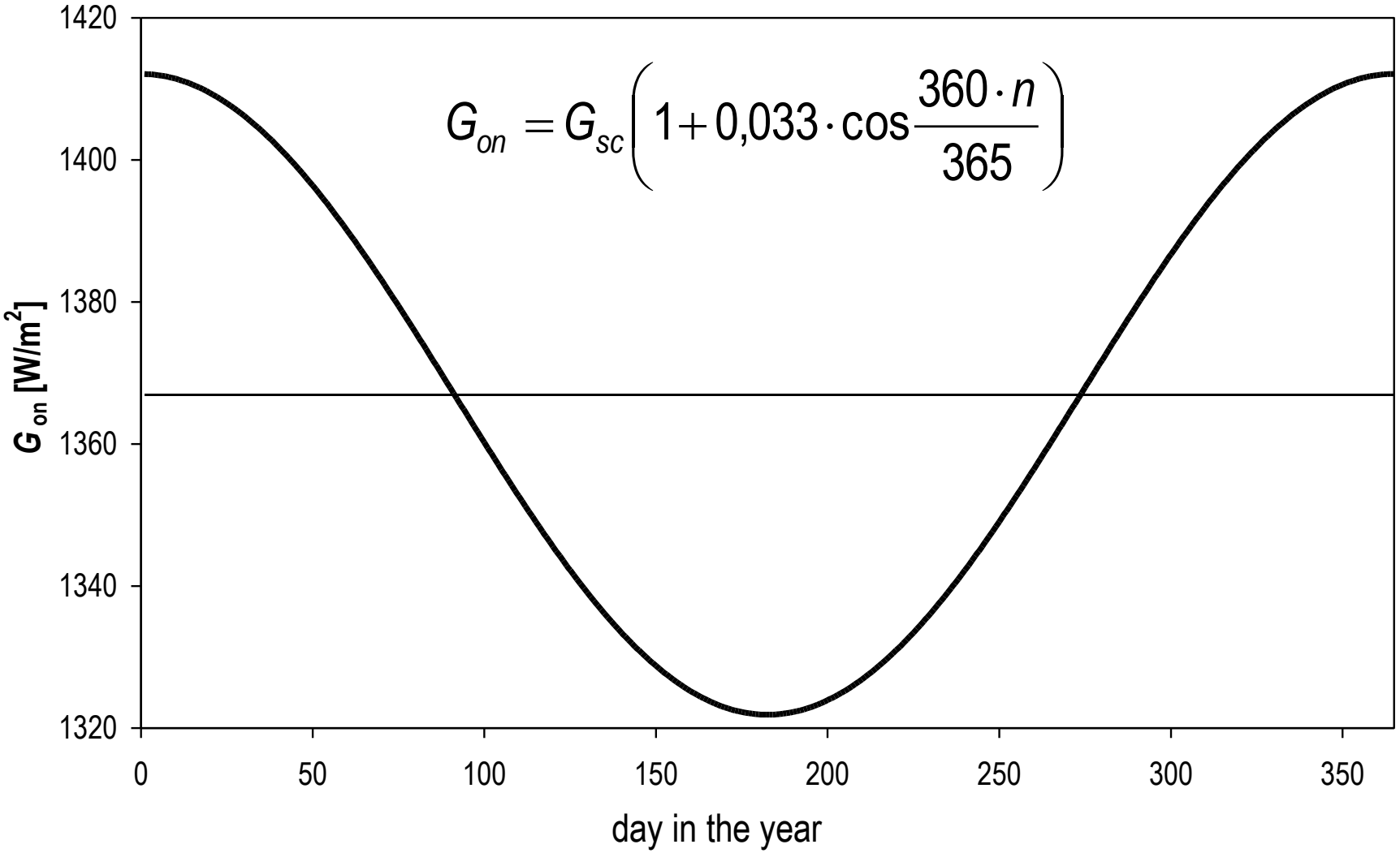
- solar radiative flux incident on area unit perpendicular to direction of propagation
- changes during the year, variable distance Sun-Earth (eliptic orbit)

  change of distance ± 1,7 %, change of flux ± 3,3 %
- value for mean distance Sun-Earth

  solar constant G_{sc} = 1367 W/m^2 (value from WRC, 1 %)

  original measurements Ch. Abbot in mountains 1322 W/m^2, today satellites
- Merkur: 9040 W/m^2 ... Neptun: 1,5 W/m^2

Radiative flux density out of atmosphere



Solar radiation passing the atmosphere

… solar radiation enters the atmosphere
(no definite boundary, exosphere continuosly fading to interplanetary space)

- **ionosphere** (60 km)

  atmospheric gases **O_2, N_2** absorb ***x-ray*** and **ultraviolet** radiation, becoming ionised

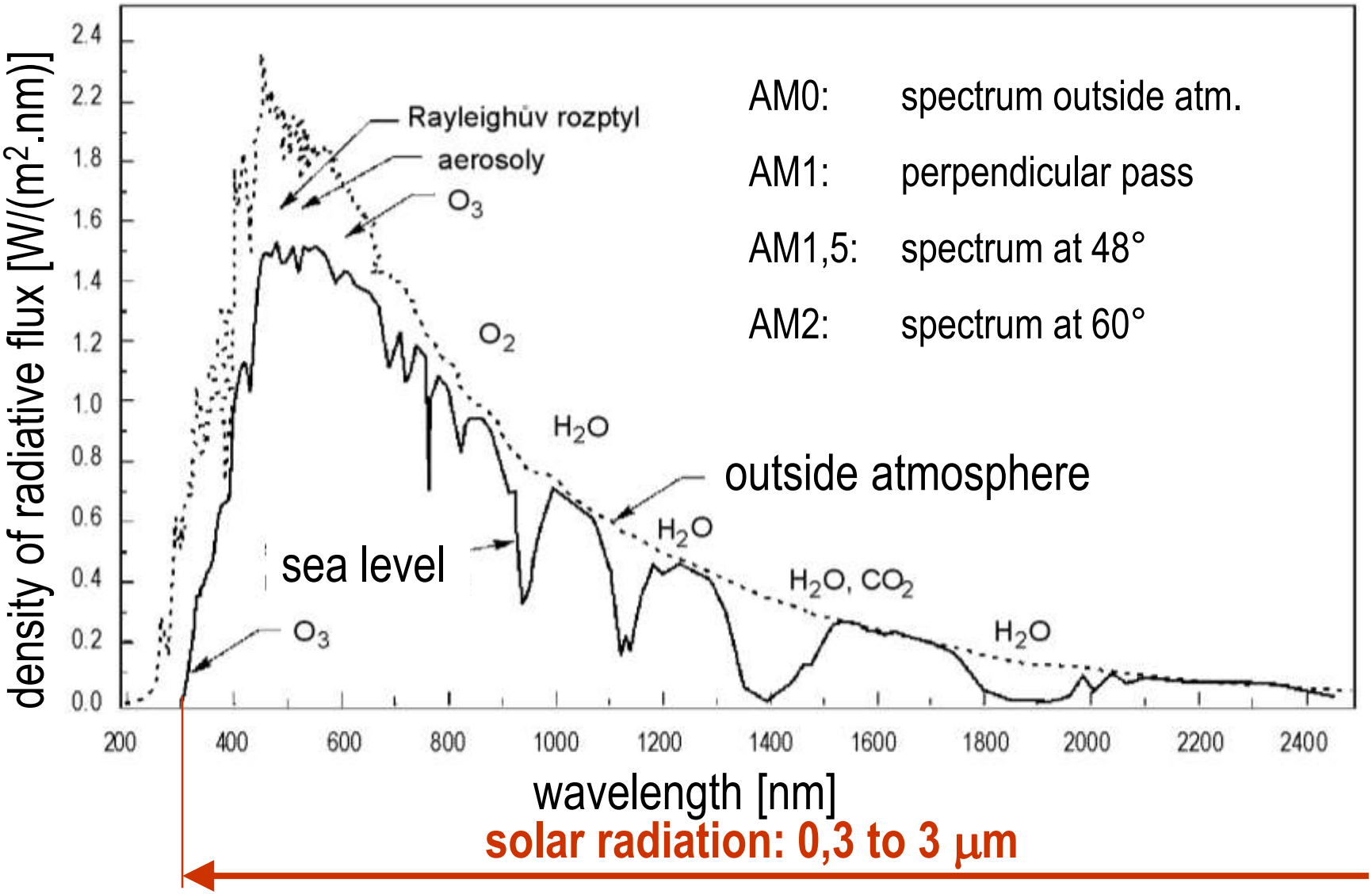
- **ozonosphere** (20 to 30 km)

  ozon **O_3** absorbs rest of harmfull **ultraviolet** radiation (UVC)

- **troposphere** (lowest layer, clouds)

  water vapor, **CO_2**, dust, water droplets absorb **infrared** radiation

Solar radiation passing the atmosphere



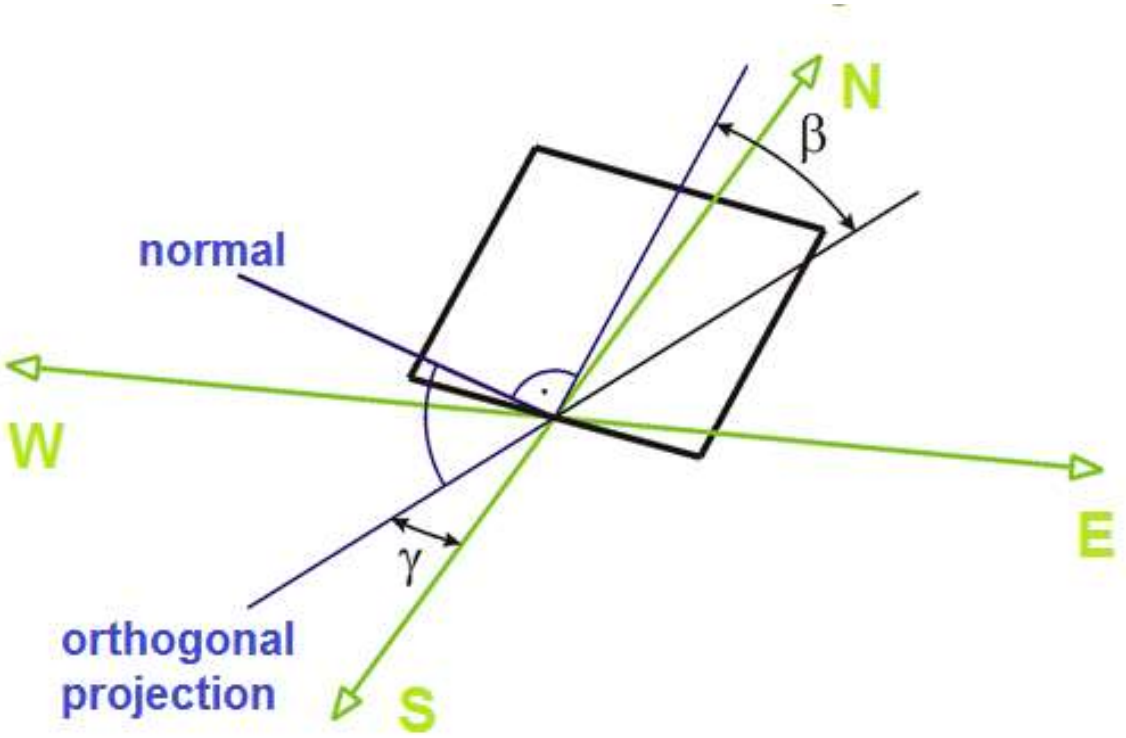
Annual balance of energy flows

- reflection from atmosphere 34 %
- absorption in atmosphere 19 %
- **incident and absorbed by Earth surface 47 %**

absorbed at surface

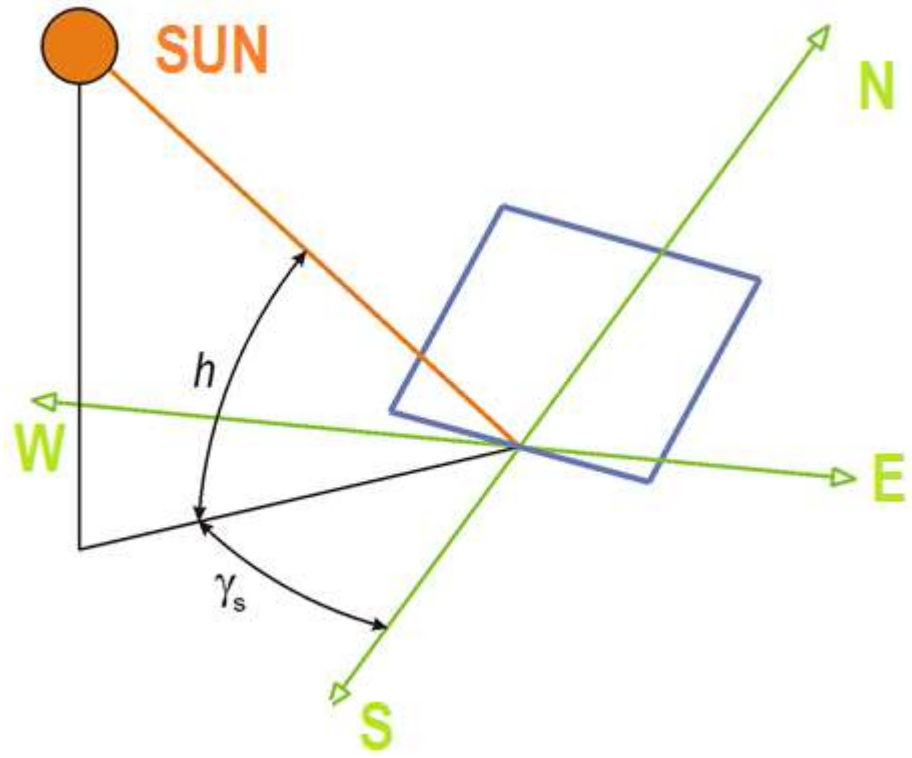
emitted back	14 %	energy of environment
evaporation (oceans)	23 %	water energy
convection, winds	10 %	wind energy
biologic reactions, photosynthesis	**0,1 %**	**energy of biomass**

Solar geometry - angles



- surface slope β
- surface azimuth γ
- latitude ϕ

Solar geometry - angles



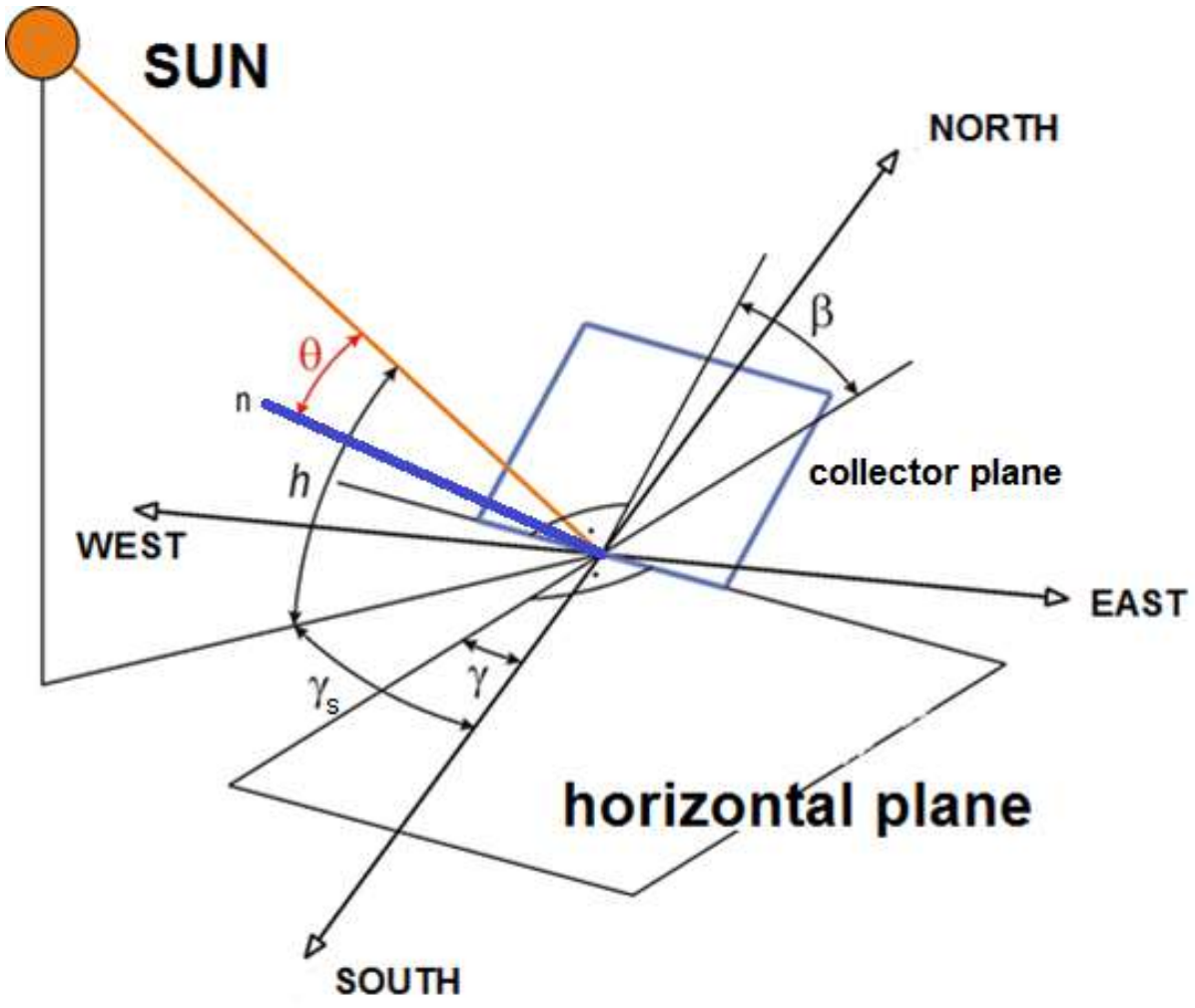
(time, date)

(time angle τ)

(declination δ)

- Sun altitude *h*
- Sun azimuth γ_s

Solar geometry - angles

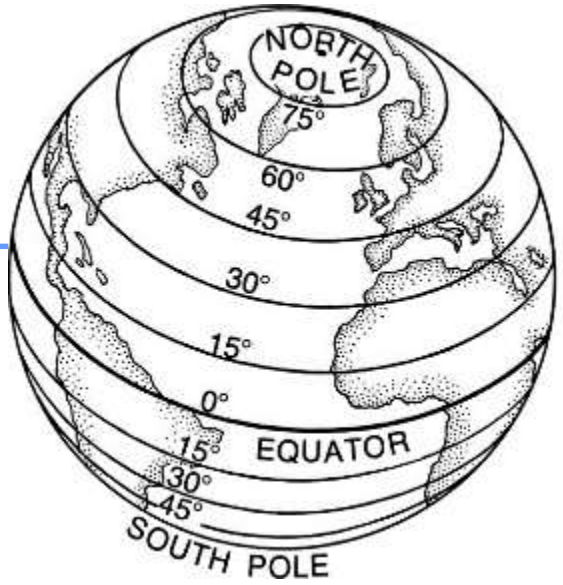


- surface slope β
- surface azimuth γ
- latitude ϕ

- time, date
- time angle τ
- declination δ
- Sun altitude h
- Sun azimuth γ_s

- incidence angle θ

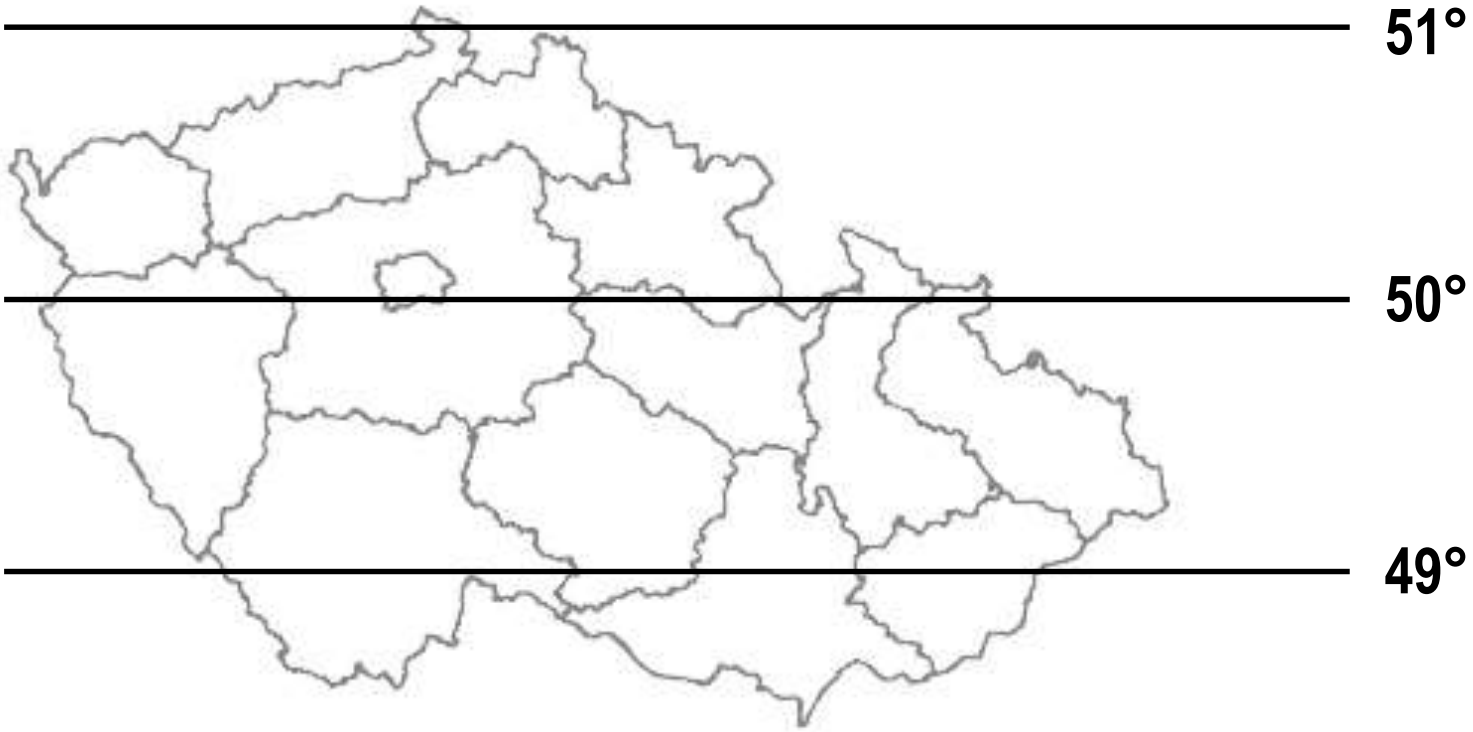
Surface position



- **latitude ϕ** <u>convention:</u> north (+), south (-)

  angle between plane of equator

  and a line connecting Earth centre and given place on Earth surface



Surface position

high difference between north and south in one country

Norway: 57 to 71°

Sweden: 55 to 68°

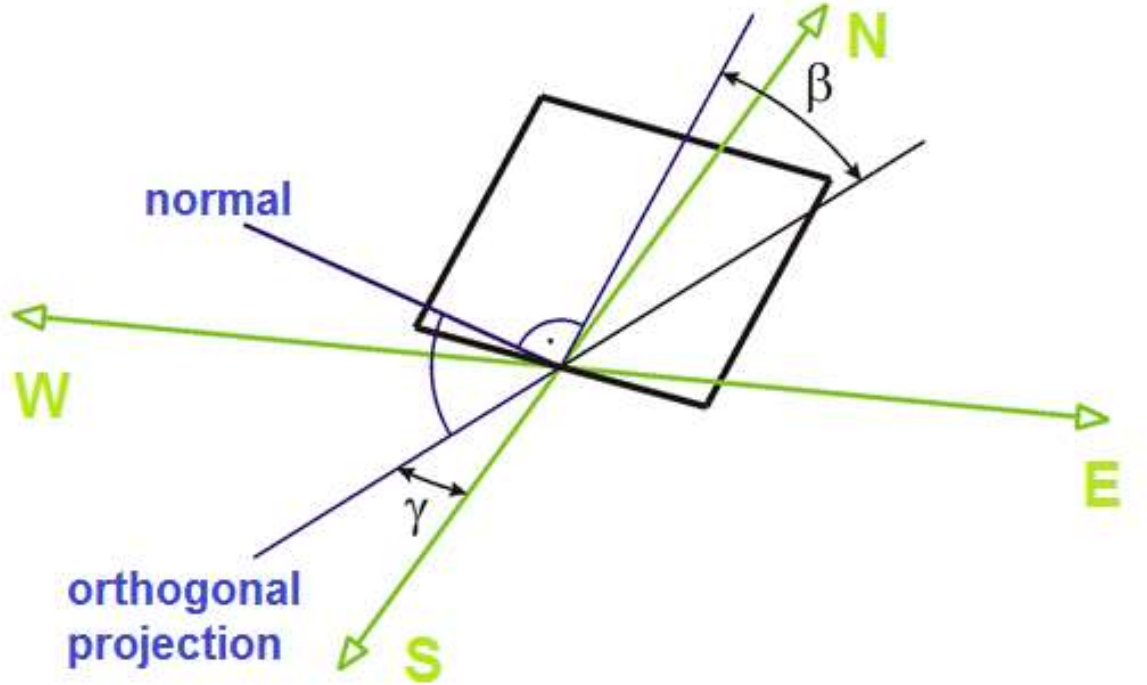
Surface orientation

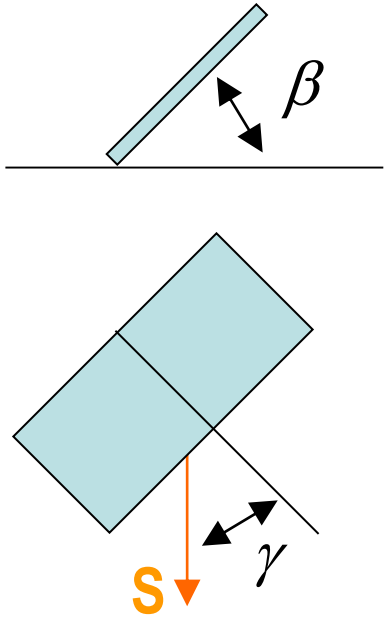
- **slope angle β** convention: horizontal 0°, vertical 90°

  angle between horizontal plane and surface plane

- **surface azimuth γ** convention: east (-), west (+), south (0°)

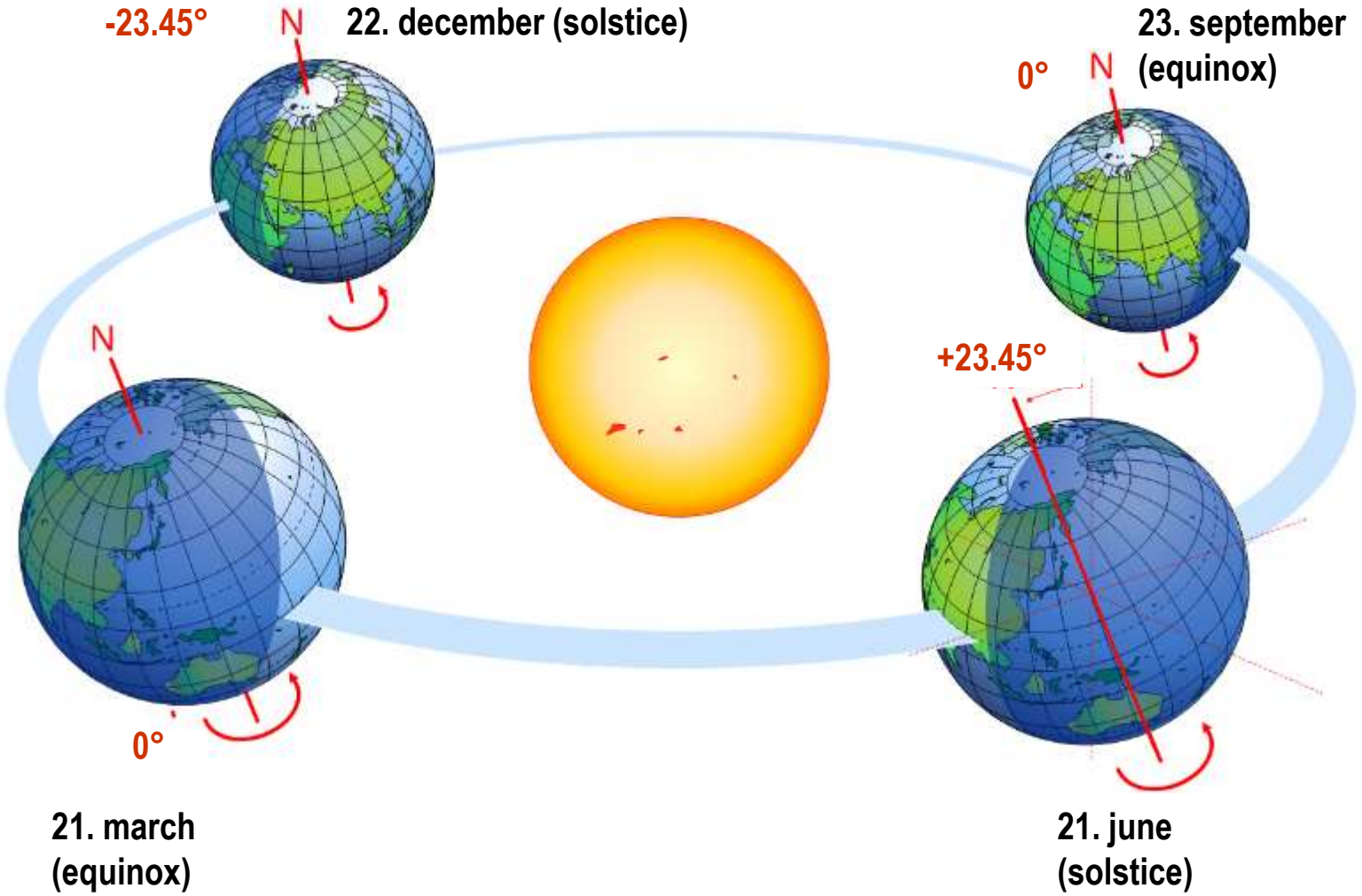
  angle between projection of surface normal and local meridian (south)



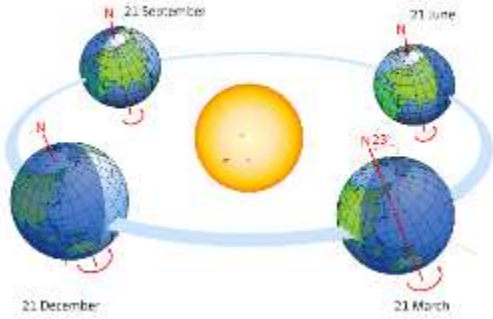


Declination (tilt of Earth axis)

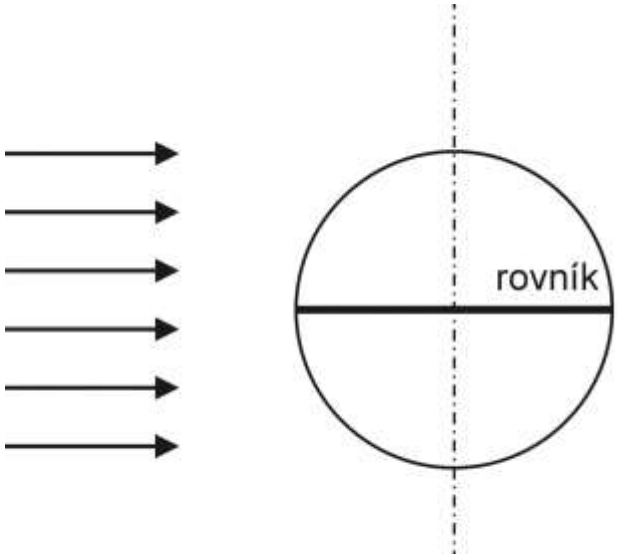
- declination δ

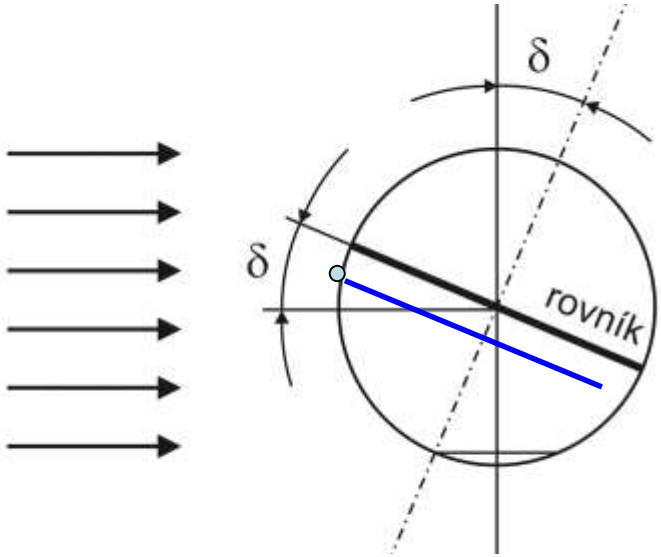


Declination δ

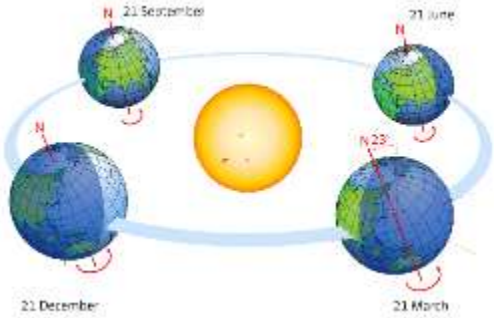


- tilt angle of Earth's axis due to precession movement during rotation
- angle between the line (connecting centres of Earth and Sun) and equator plane
- latitude of place on Earth where in given day in the noon the Sun is in zenith





Calculation of declination δ



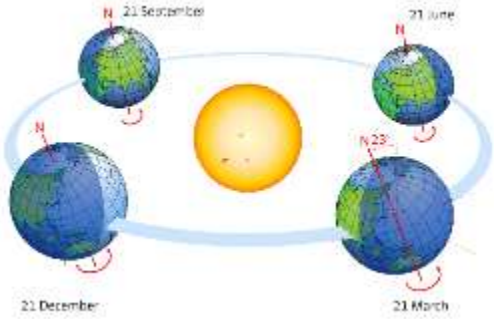
- calender date ***DD.MM.***

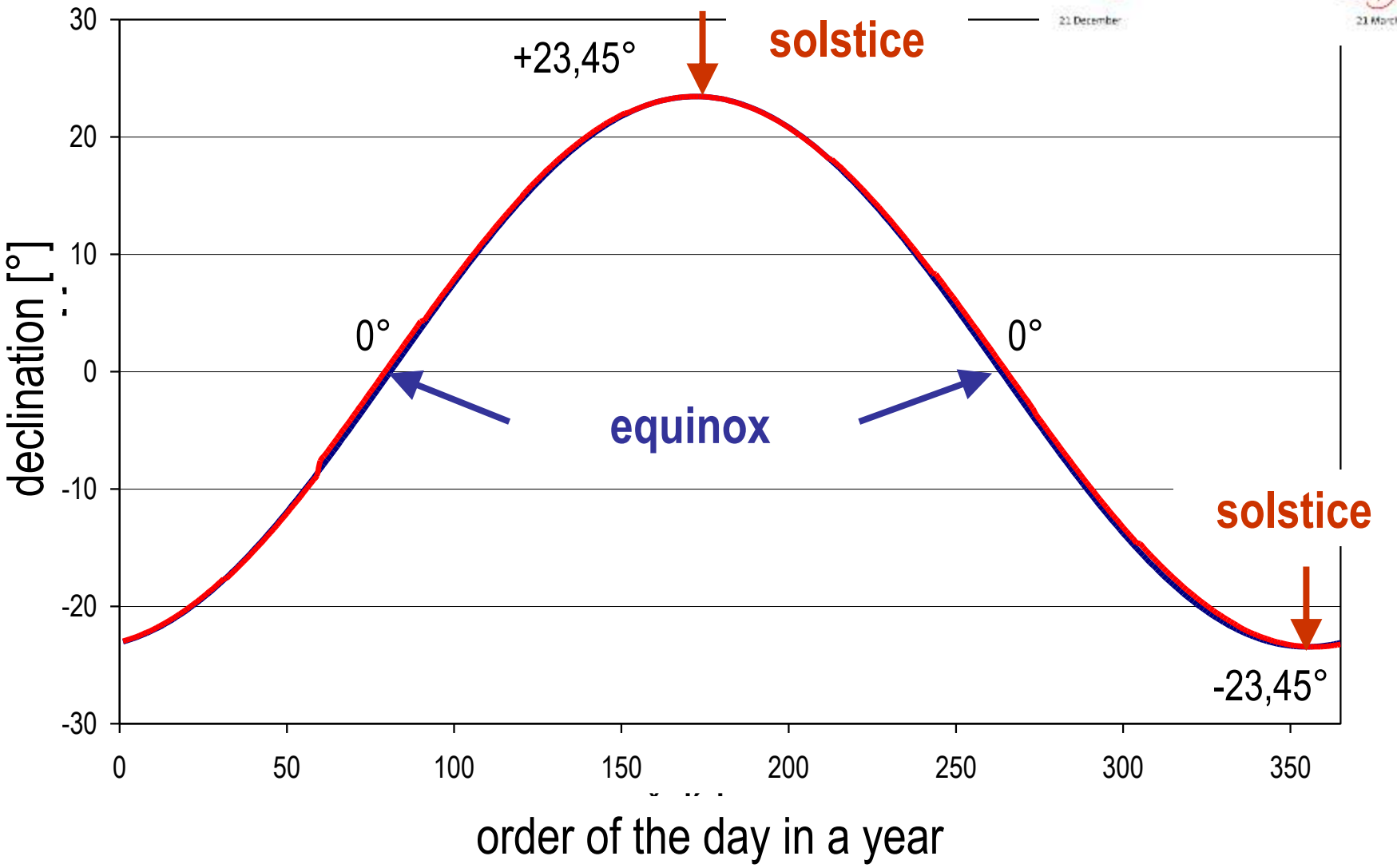
$$\delta = 23{,}45° \sin\left(0{,}98 \cdot DD + 29{,}7 \cdot MM - 109°\right)$$

- number of the day in the year ***n***

$$\delta = 23{,}45° \sin\left(360 \frac{284 + n}{365}\right)$$

Calculation of declination δ





time (hour) angle τ

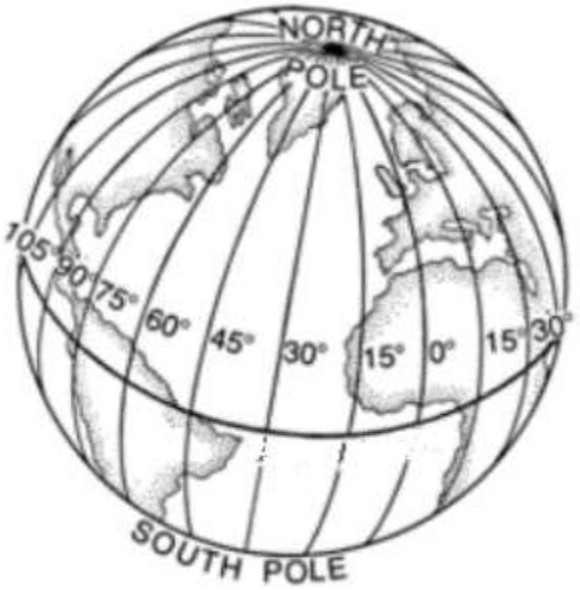
- angle of **virtual** translation of Sun above local meridians due to Earth rotation, related to solar noon
- Earth is rotating around its axis (360°) once for 24 hours

  → translation motion of Sun by 15° over 1 hour

- time angle is calculated from solar time *ST*

$$\tau = 15° \cdot (ST - 12)$$

convention: before noon (-), after noon (+)

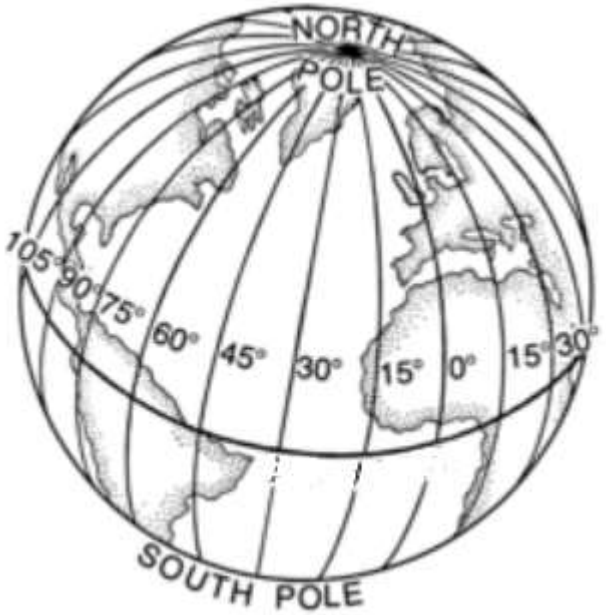


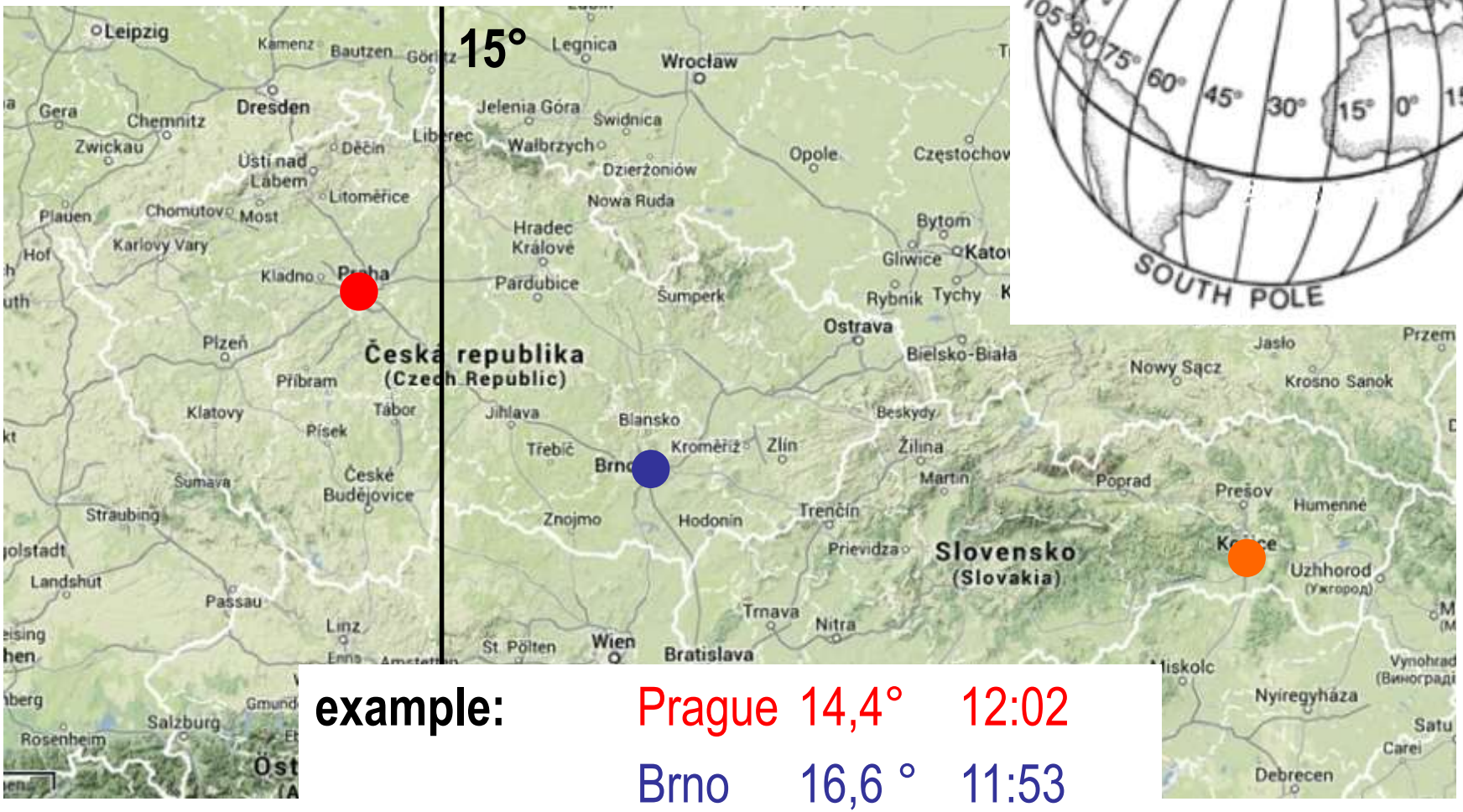
Solar time *ST*

- each timezone has a time related to local (standard) meridian
  timezones of 1 h ~ meridians of 15°
- CET: local standard time at meridian 15° east longitude

- **solar time:** daily time defined from virtual translation of Sun
- observer at reference meridian: local time = solar time
- observer out of reference meridian: local time ≠ solar time
  shift up to 30 minutes

Solar time *ST*





example:

Prague	14,4°	12:02
Brno	16,6 °	11:53
Košice	21,2°	11:35
	longitude	**solar noon**

Sun altitude *h*

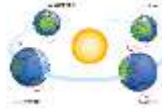
- angle between line connecting surface--Sun and horizontal

$$\sin h = \sin\delta \cdot \sin\phi + \cos\delta \cdot \cos\phi \cdot \cos\tau$$

- latitude ϕ

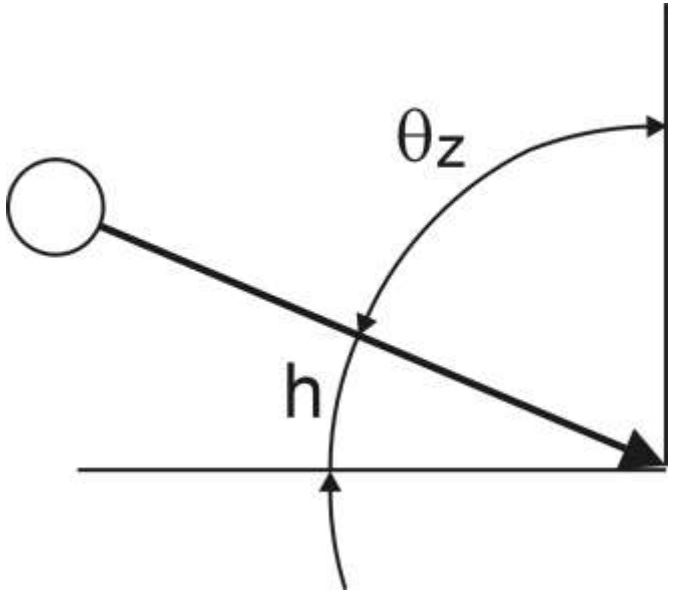
- time angle τ

- declination δ

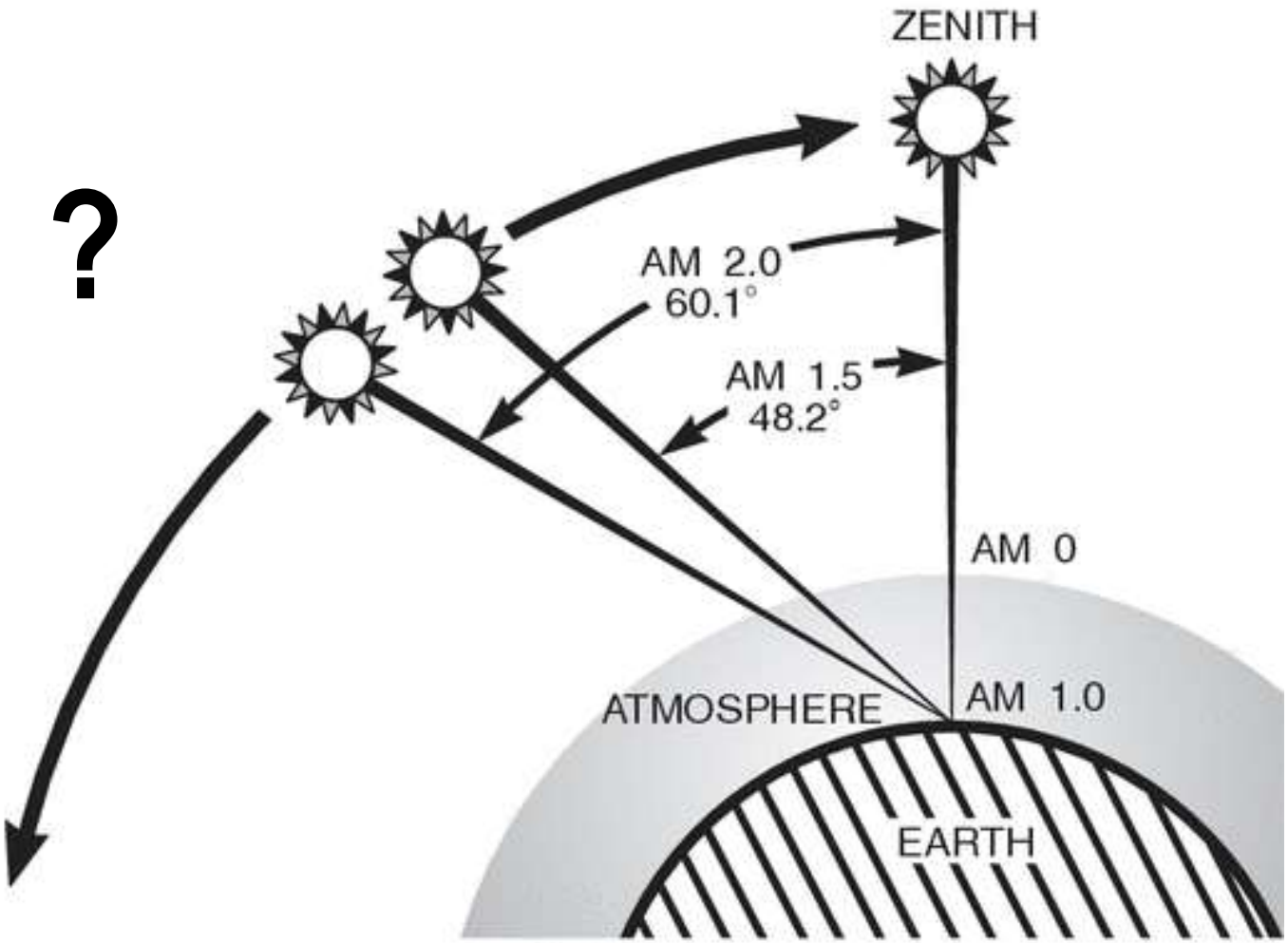


- complement angle to 90°: zenith angle θ_z

$$\theta_z = 90° - h$$



Air mass

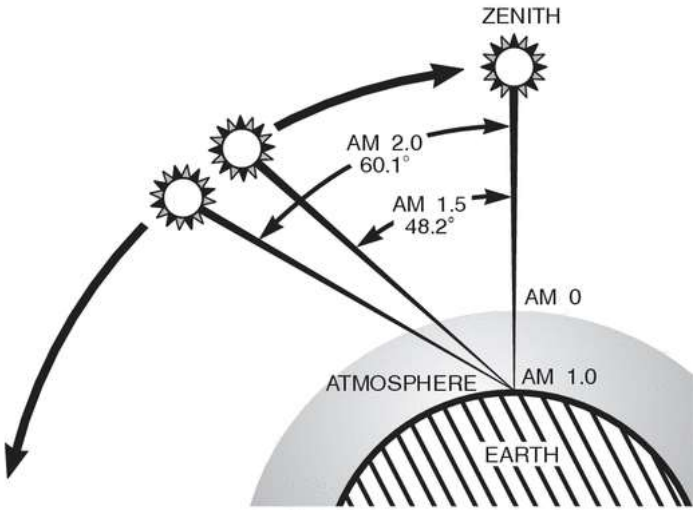


Air mass

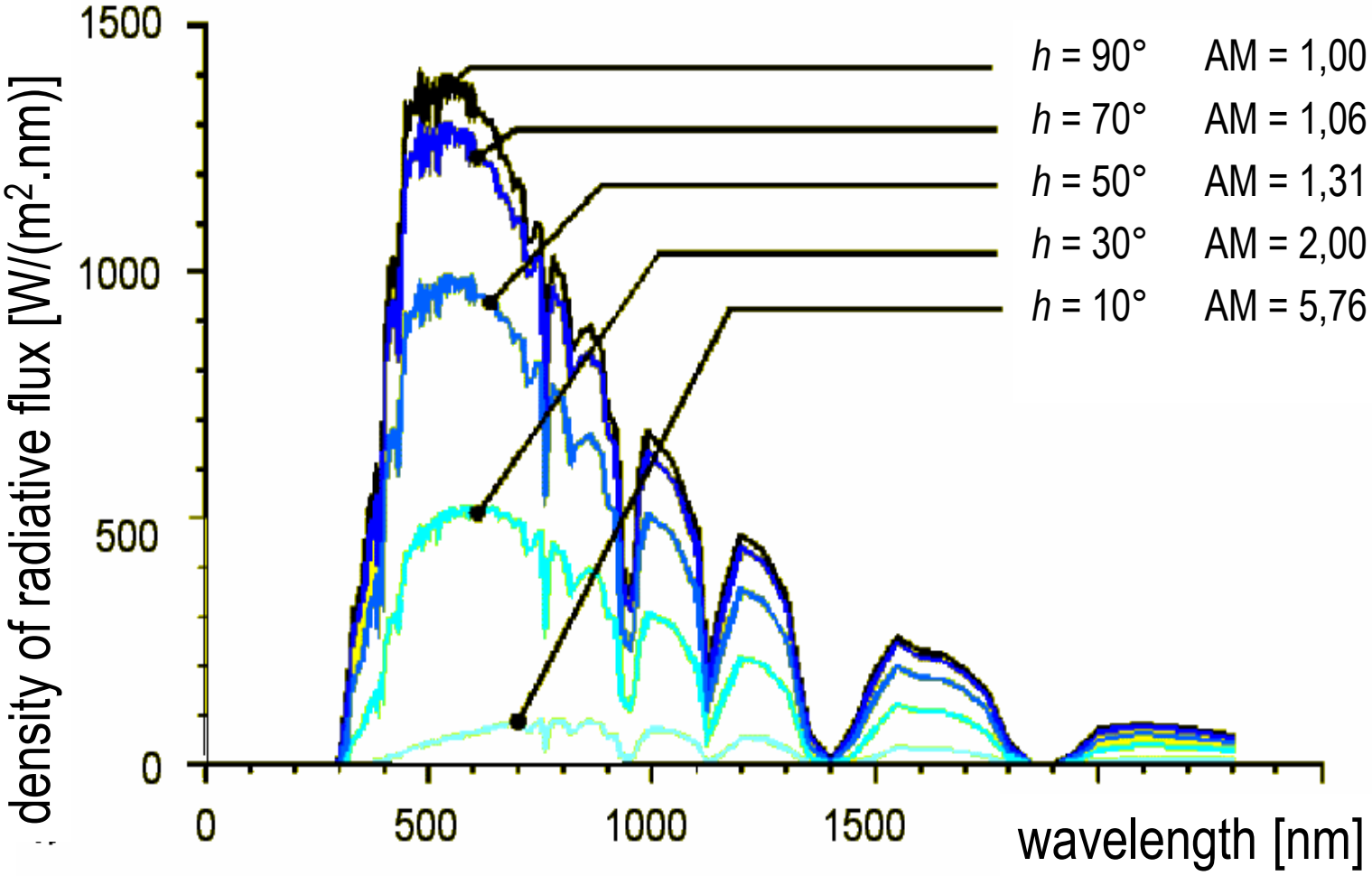
- ratio between mass of atmosphere passed by solar radiation to mass, which would be passed if Sun is in zenith

$$AM = \frac{1}{\cos\theta_z} = \frac{1}{\sin h}$$

- $AM = 0$ outside atmosphere
- $AM = 1$ zenith $h = 90°$
- $AM = 1{,}5$ $\theta_z = 48°$ $h = 42°$
- $AM = 2$ $\theta_z = 60°$ $h = 30°$



Change of spectrum with air mass



$h = 3°$ … AM=21

Time of sunrise and sunset

- sunrise / sunset: altitude angle = 0°

$$\sin h = \sin\delta \cdot \sin\phi + \cos\delta \cdot \cos\phi \cdot \cos\tau = 0$$

- time angle of sunrise / sunset

$$\tau_{1,2} = \arccos\left(-tg\phi \cdot tg\delta\right)$$

- theoretical period of sunshine = time between sunrise and sunset

$$\tau_t = \frac{2 \cdot \tau_{1,2}}{15°}$$

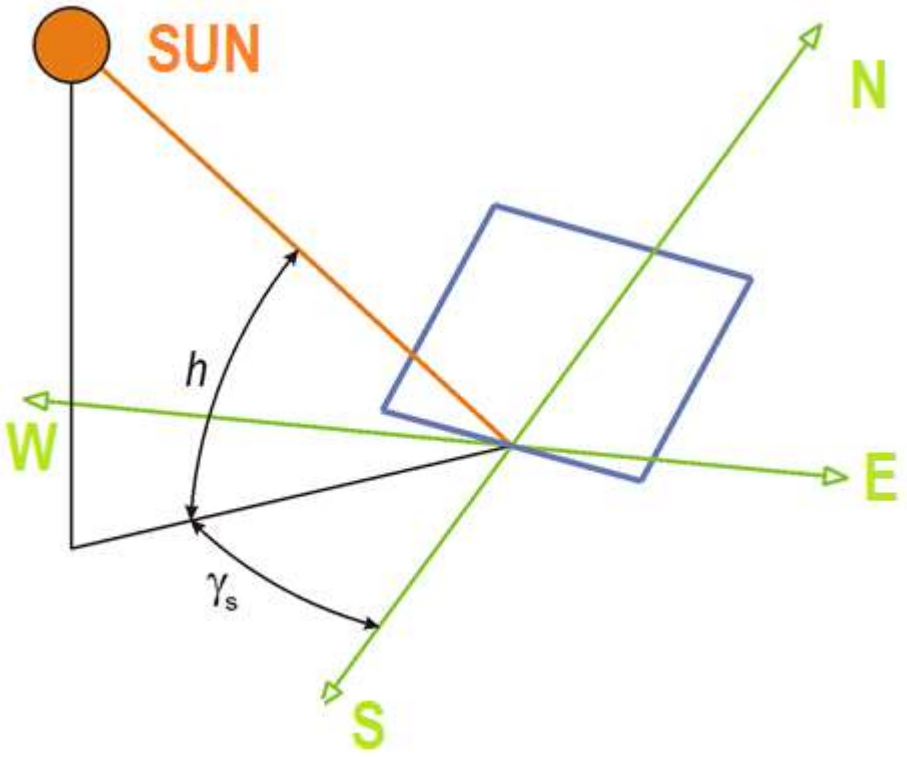
Azimuth of Sun γ_s

- angle between projection of line connecting surface-Sun and local meridian (south)

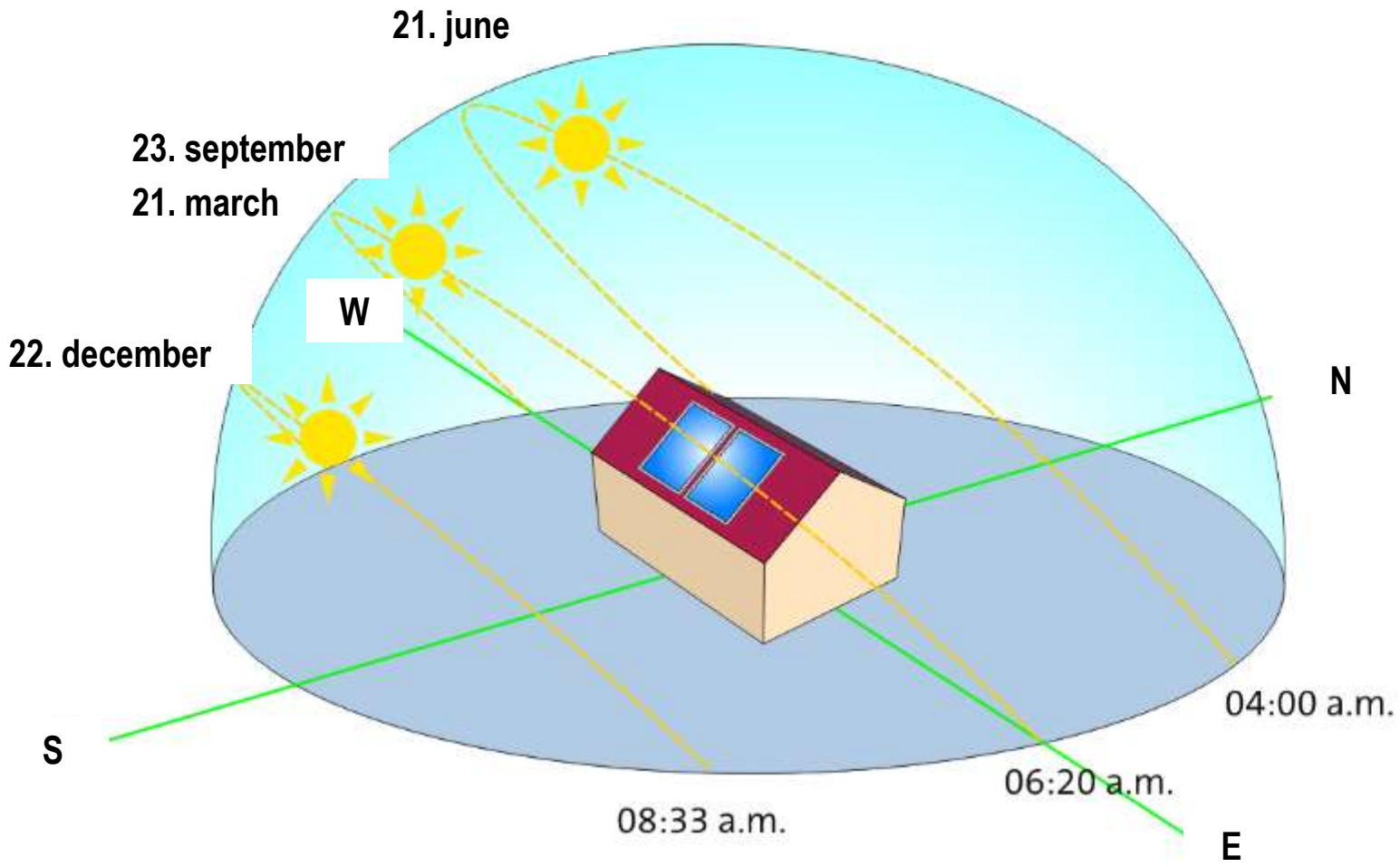
convention: measured from south

east (-), west (+)

$$\sin\gamma_s = \frac{\cos\delta}{\cos h}\sin\tau$$

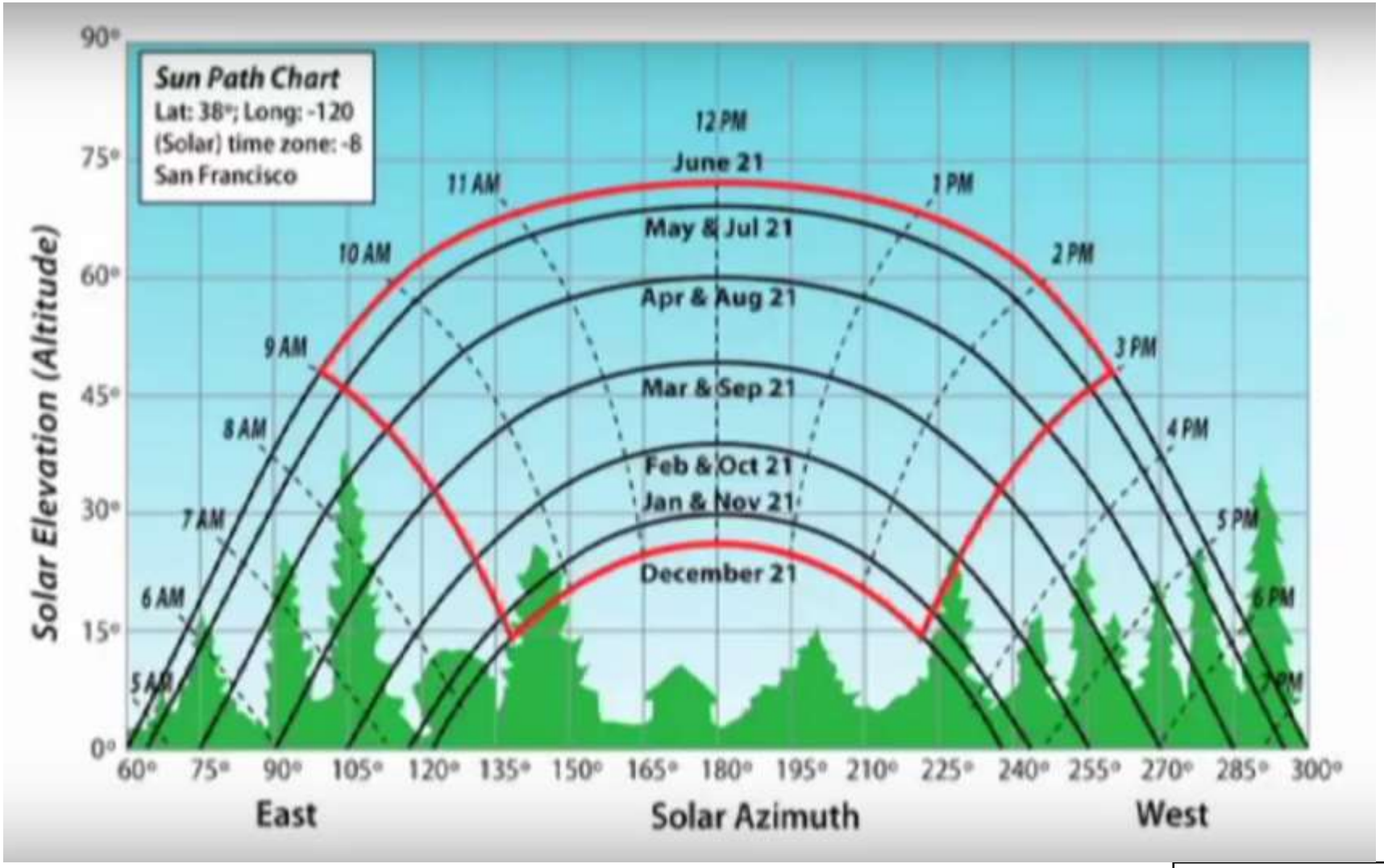


Altitude and azimuth of Sun



source: solarpraxis

Altitude and azimuth of Sun



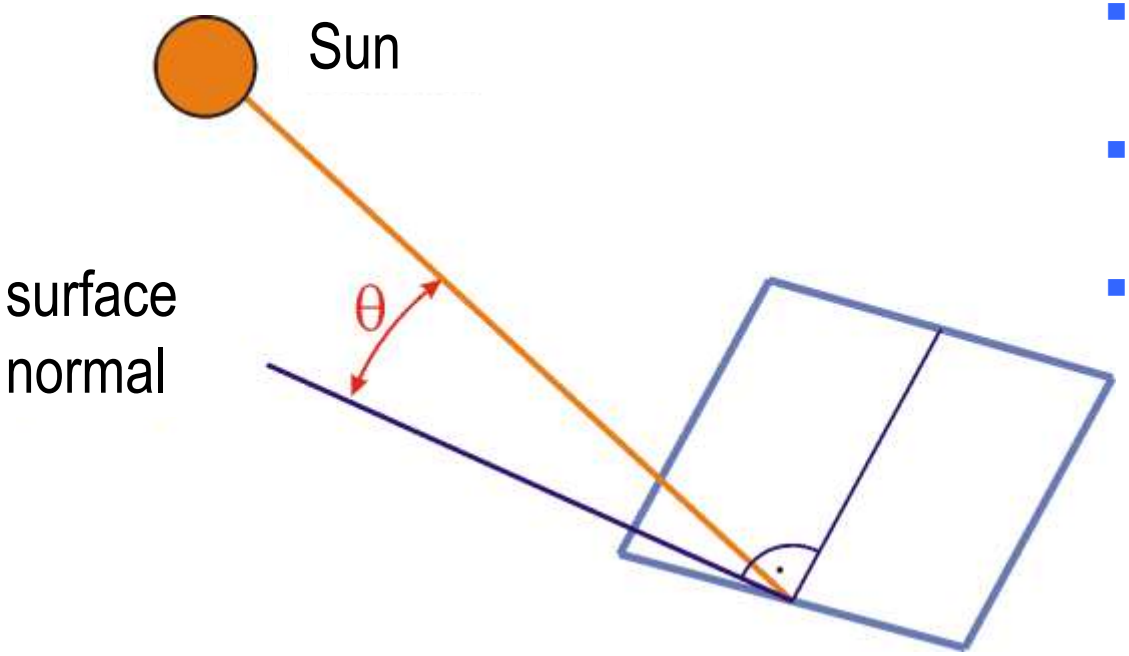
source: solarpraxis

Incidence angle

- angle between line connecting surface-Sun and surface normal

$$\cos\theta = \sin h \cdot \cos\beta + \cos h \cdot \sin\beta \cdot \cos(\gamma_s - \gamma)$$

- surface slope β
- surface azimuth γ
- Sun altitude h
- ¨Sun azimuth γ_s



Example for February 20th 10:00

- **collector plane:**

  latitude $\phi = 50°$

  slope $\beta = 45°$

  azimuth $\gamma = +15°$

declination $\delta = 23{,}45° \sin(0{,}98 \cdot DD + 29{,}7 \cdot MM - 109°)$

$$\delta = 23{,}45° \sin(0{,}98 \cdot 20 + 29{,}7 \cdot 2 - 109°) = -11.7°$$

Example for February 20th 10:00

time angle

$$\tau = 15° \cdot (10 - 12) = -30°$$

Sun altitude $\sin h = \sin\delta \cdot \sin\phi + \cos\delta \cdot \cos\phi \cdot \cos\tau$

$$h = \arcsin(\sin(-11.7°) \cdot \sin 50 + \cos(-11.7°) \cdot \cos 50 \cdot \cos(-30))$$
$$= 23°$$

Sun azimuth $\sin\gamma_s = \frac{\cos\delta}{\cos h}\sin\tau$

$$\gamma_s = \arcsin\left(\frac{\cos(-11.7°)}{\cos 23°}\sin(-30°)\right) = -32.1°$$

Example for February 20th 10:00

incidence angle $\cos\theta = \sin h \cdot \cos\beta + \cos h \cdot \sin\beta \cdot \cos(\gamma_s - \gamma)$

$$\theta = \arccos\left(\sin 23° \cdot \cos 45° + \cos 23° \cdot \sin 45° \cdot \cos\left(-32.1° - 15°\right)\right)$$
$$= 44°$$

time angle of sunrise / sunset $\tau_{1,2} = \arccos(-tg\phi \cdot tg\delta)$

$$\tau_{1,2} = \arccos\left(-tg50° \cdot tg(-11.7°)\right) = 75.8°$$

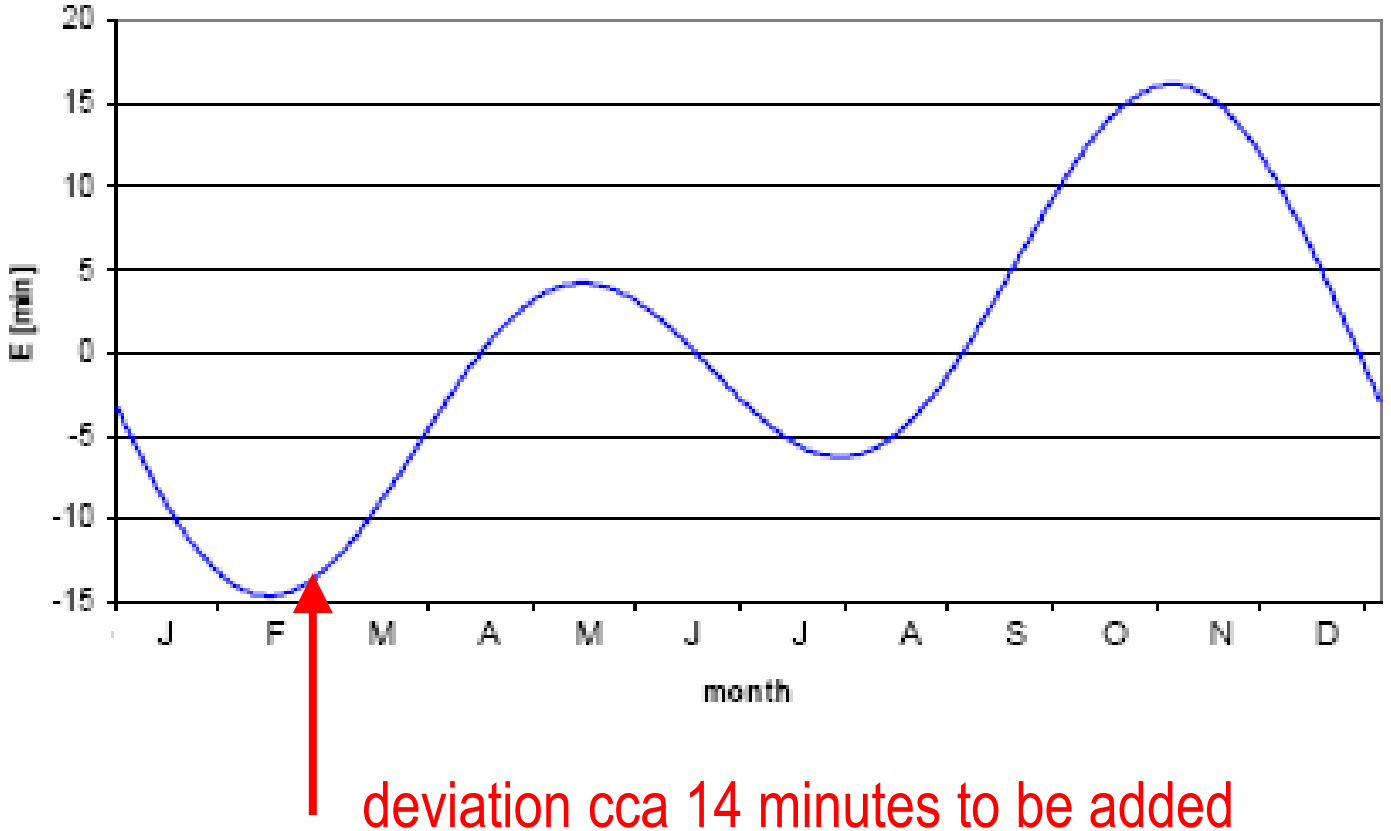
time of sunrise

$$t_1 = 12 - \frac{\tau_{1,2}}{15°} = 6.9 = \quad 6{:}54$$

BUT ...

Deviation of solar time

due to nonuniform rotation of Earth ...



Solar radiation - definitions

- **solar irradiance *G* [W/m^2]** - radiative ***power*** incident at area unit, density of solar radiative flux

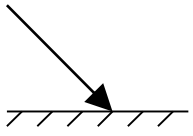
- **solar irradiation *H* [kWh/m^2, J/m^2]** – density of radiative ***energy***, integral of flux density per time period, e.g. hour, day, ...

$$H = \int_{\tau_1}^{\tau_2} G.d\tau$$

Solar radiation - definitions

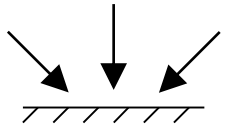
- **direct** solar radiation (index „b“, beam) – without scattering in atmosphere

  angle dependent, significant intensity in one direction



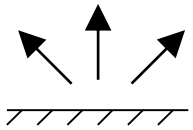
- **diffuse** solar radiation (index „d“, diffuse) – scattered in atmosphere

  all-directions, isotropic: identical intesity in all direction

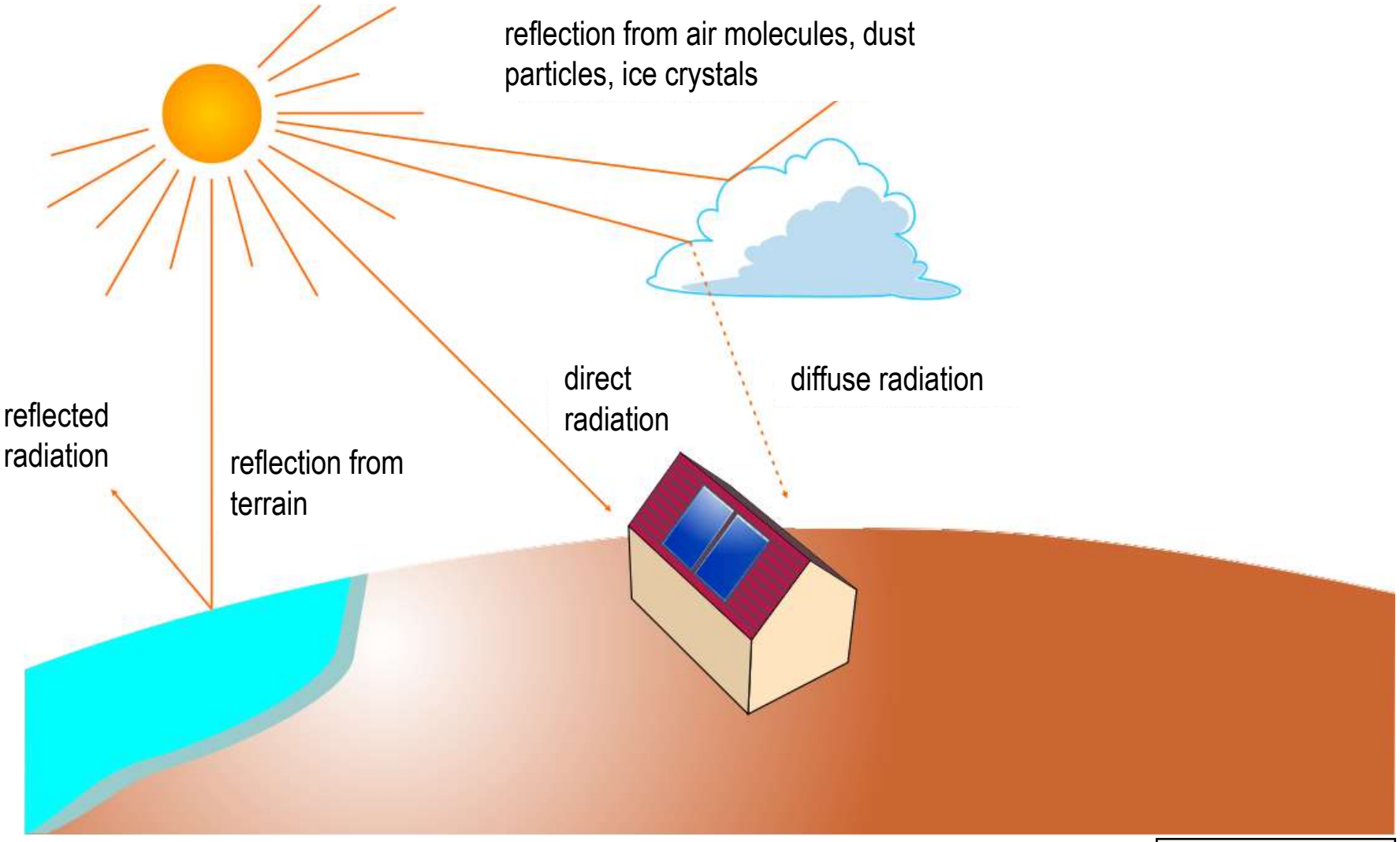


- **reflected** solar radiation (index „r“, reflected) – reflection from terrain, buildings

  usual surfaces reflect diffusively – considered together with diffuse radiation



Solar radiation - definitions



source: solarpraxis

Solar radiation passing the atmosphere

- **direct normal** solar irradiance (on surface perpendicular to direction of propagation) after passing atmosphere … G_{bn}

$$G_{bn} = G_{on} \cdot \exp\left(-\frac{Z}{\varepsilon}\right) \qquad [W/m^2]$$

G_{on} normal solar irradiance above atmosphere

Z attenuation factor

ε factor depends on the height of the Sun above the horizon and the elevation above m. sea l.

$$\varepsilon = \frac{9{,}38076 \cdot \left[\sin h + (0{,}003 + \sin^2 h)^{0{,}5}\right]}{2{,}0015 \cdot (1 - L_v \cdot 10^{-4})} + 0{,}91018$$

h Sun altitude [°]

L_v elevation above sea-level [m]

Attenuation factor *Z*

- how many times the clear atmosphere should be „optically thicker“, to have the same transmissivity as the real polluted atmosphere
- polluted means also **water vapor** not only **dust**, **emissions**, etc.
- gives attenuation of solar flux when passing the real atmosphere

$$Z = \frac{\ln G_{0n} - \ln G_{bn}}{\ln G_{0n} - \ln G_{b0}}$$

G_{0n} normal solar irradiance above atmosphere

G_{bn} **direct** normal solar irradiance after passing atmosphere

G_{b0} direct irradiance after passing **completely clear atmosphere** (with $Z = 1$)

Attenuation factor

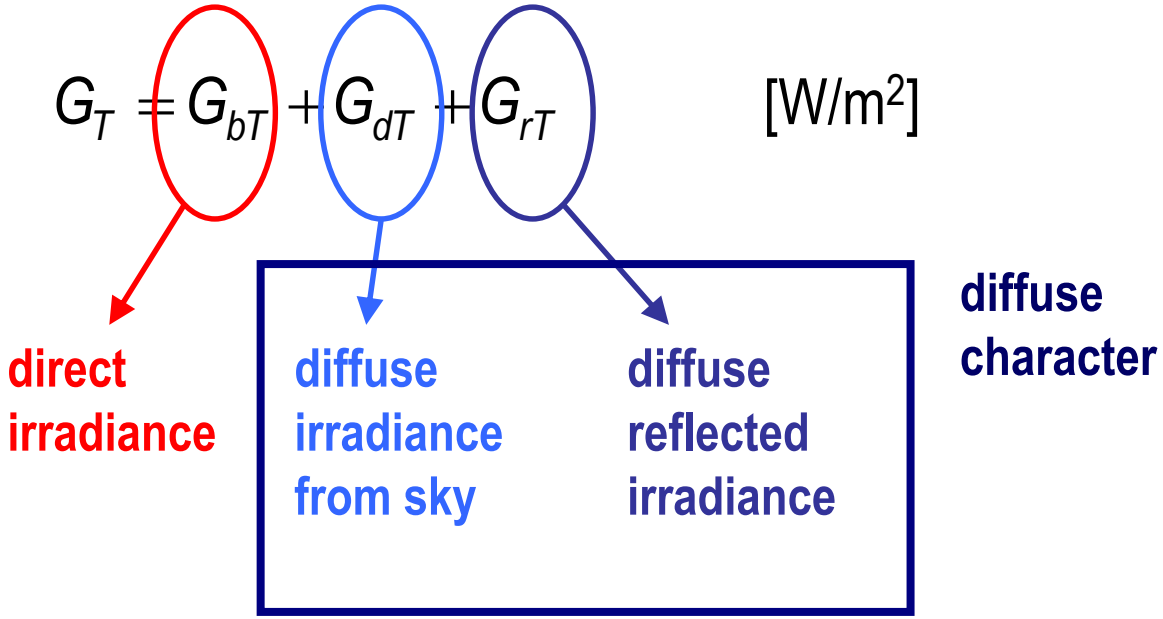
Month	Average monthly values for *Z* for locations with different environment			
	mountains	country side	cities	industrial
I.	1,5	2,1	3,1	4,1
II.	1,6	2,2	3,2	4,3
III.	1,8	2,5	3,5	4,7
IV.	1,9	2,9	4,0	5,3
V.	2,0	3,2	4,2	5,5
VI.	2,3	3,4	4,3	5,7
VII.	2,3	3,5	4,4	5,8
VIII.	2,3	3,3	4,3	5,7
IX.	2,1	2,9	4,0	5,3
X.	1,8	2,6	3,6	4,9
XI.	1,6	2,3	3,3	4,5
XII.	1,5	2,2	3,1	4,2
annual average	**1,9**	**2,75**	**3,75**	**5,0**

simplified:

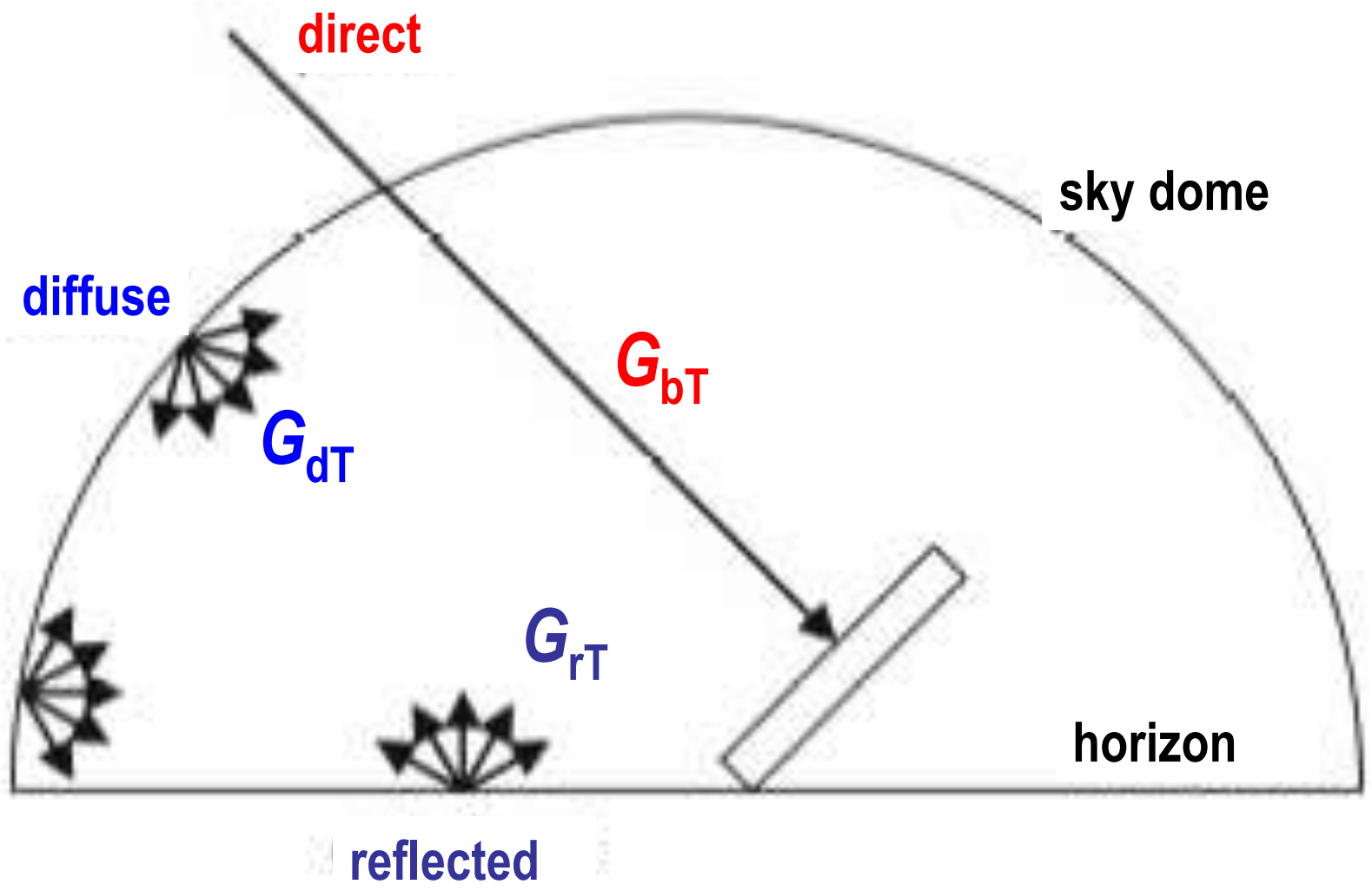
mountains	$Z = 2$
countryside	$Z = 3$
cities	$Z = 4$
industrial	$Z > 5$

Solar irradiance on general surface

- total solar irradiance on generally sloped and oriented surface



Solar irradiance on general surface



Solar irradiance on general surface

- direct solar irradiance on given surface

$$G_{bT} = G_{bn}\cos\theta = G_b\frac{\cos\theta}{\sin h} = G_b\frac{\cos\theta}{\cos\theta_z} \quad [W/m^2]$$

direct normal; Incidence / sun altitude; Incidence / zenith angle

- sky diffuse solar irradiance on given surface

$$G_{dT} = \left(\frac{1+\cos\beta}{2}\right)\cdot G_d \quad [W/m^2]$$

- reflected diffuse solar irradiance on given surface

$$G_{rT} = \rho_g\left(\frac{1-\cos\beta}{2}\right)\cdot(G_b + G_d) \quad [W/m^2]$$

Terrain reflectance (albedo)

- ratio between reflected and incident solar irradiance
- for calculations considered ρ_g **= 0,20**

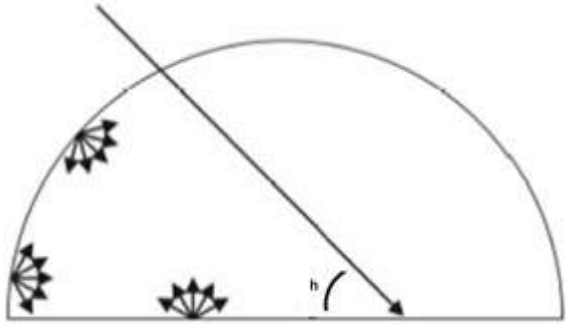
usual surfaces	0,10 to 0,15
snow	0,70 to 0,90
Earth albedo (planet)	0,30 (average)

Solar irradiance on horizontal plane

- direct solar irradiance on horizontal plane

$$G_b = G_{bn} \sin h \qquad [W/m^2]$$

direct normal … sun altitude



- diffuse solar irradiance on horizontal plane

$$G_d = 0{,}33 \cdot (G_{on} - G_{bn}) \cdot \sin h \qquad [W/m^2]$$

simplified model: 1/3 of solar radiatin „lost“ in atmosphere comes to horizontal plane (sin *h*) as a diffuse isotropic radiation

Example for February 20th 10:00

- angles:

  Sun altitude h = 23.0°

  Sun azimuth γ_s = -32.1°

  incident angle θ = 44.0°

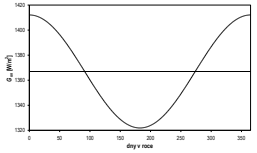
from the date and time

- solar constant **1367 W/m²**
- attenuation factor ***Z* = 4**
- altitude 200 m

local parameters

Example for February 20th 10:00

- **extraterrestrial normal solar irradiance**



$$G_{on} = 1367\left(1+0{,}033\cdot\cos\frac{360\cdot 51}{365}\right) = 1396\ \mathrm{W/m^2}$$

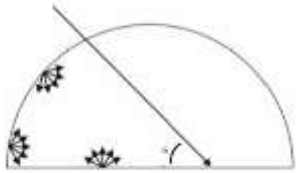
ε factor depends on the height of the Sun above the horizon and the elevation AMSL

$$\varepsilon = \frac{9{,}38076\cdot\left[\sin 23° + (0{,}003+\sin^2 23°)^{0{,}5}\right]}{2{,}0015\cdot(1-200\cdot 10^{-4})} + 0{,}91018 = 4.67$$

- **direct normal irradiance (after passing atmosphere)**

$$G_{bn} = G_{on}\cdot\exp\left(-\frac{Z}{\varepsilon}\right) = 1396\cdot\exp\left(-\frac{4}{4.67}\right) = 593\ \mathrm{W/m^2}$$

Example for February 20th 10:00



- **direct solar irradiance on horizontal plane**

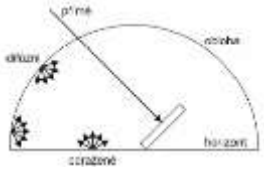
$$G_b = G_{bn} \sin h = 593 \cdot \sin 23° = 231 \text{ W/m}^2$$

- **diffuse solar irradiance on horizontal plane**

$$G_d = 0{,}33 \cdot (G_{on} - G_{bn}) \cdot \sin 23°$$

$$= 0{,}33 \cdot (1396 - 593) \cdot \sin 23° = 103 \text{ W/m}^2$$

Example for February 20th 10:00



- **direct solar irradiance on given surface**

$$G_{bT} = G_{bn}\cos\theta = 593\cos 44° = 426\ \text{W/m}^2$$

- **sky diffuse solar irradiance on given surface**

$$G_{dT} = \left(\frac{1+\cos\beta}{2}\right)\cdot G_d = \left(\frac{1+\cos 45°}{2}\right)\cdot 103 = 88\ \text{W/m}^2$$

- **reflected diffuse solar irradiance on given surface**

$$G_{rT} = \rho_g \cdot \left(\frac{1-\cos\beta}{2}\right)\cdot (G_b + G_d)$$

$$= 0.2\cdot\left(\frac{1-\cos 45°}{2}\right)\cdot (231+103) = 10\ \text{W/m}^2$$

Example for February 20th 10:00

$$G_T = G_{bT} + G_{dT} + G_{rT}$$

$$= 426 + 88 + 10$$

$$= 524 \text{ W/m}^2$$

... for given time and date

... for clear sky (no clouds)

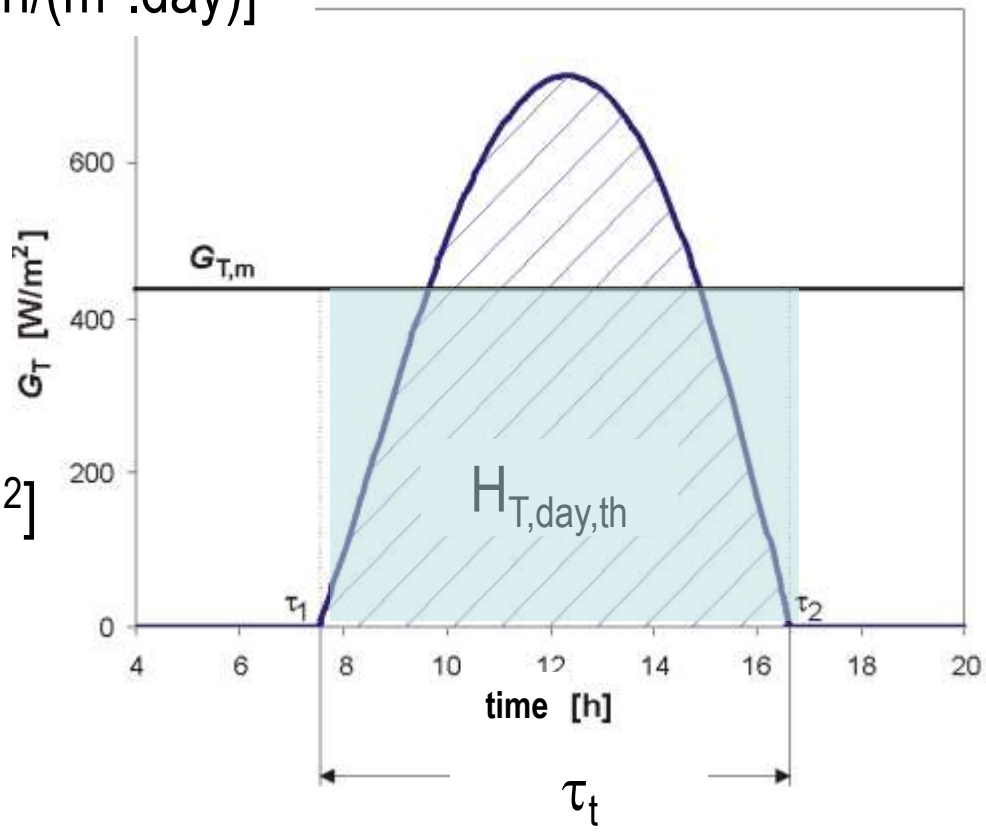
Solar irradiation on general surface

- theoretical daily solar irradiation, integration of irradiance on a plane from sunrise τ_1 to sunset τ_2

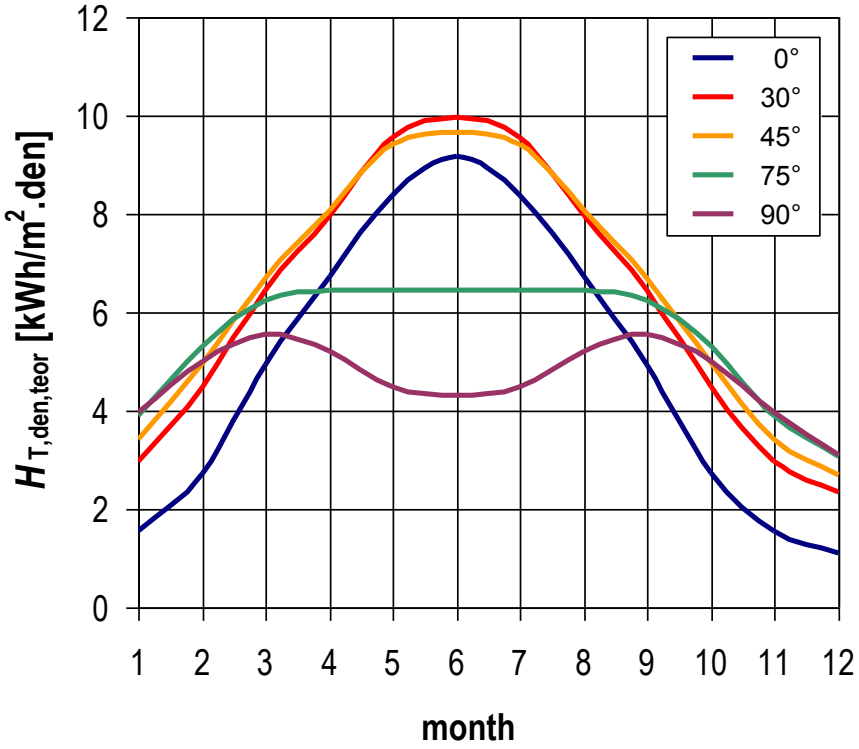
$$H_{T,day,th} = \int_{\tau_1}^{\tau_2} G_T d\tau \qquad [\mathrm{kWh/(m^2.day)}]$$

- mean daily solar irradiance

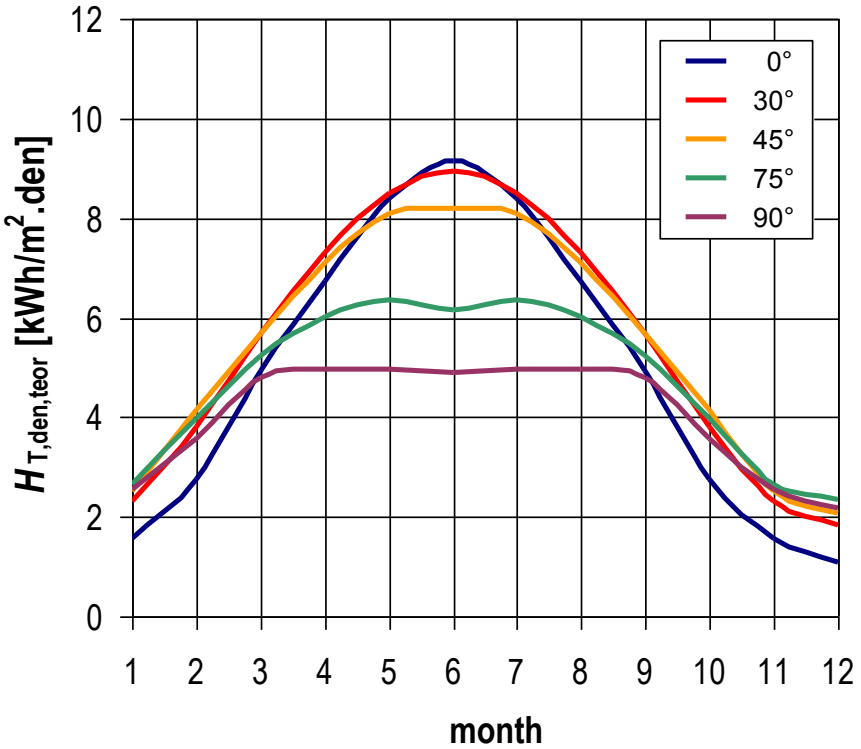
$$G_{T,m} = \frac{H_{T,day,th}}{\tau_t} \qquad [\mathrm{W/m^2}]$$



Influence of slope



azimuth 0° (south)



azimuth 45° (SW, SE)

optimum slope: summer 20-30° winter 75-90° annual 35-45°

Solar irradiation on general surface

- diffuse daily solar irradiation, integration of diffuse solar irradiance on a plane from sunrise τ_1 to sunset τ_2

$$H_{T,day,dif} = \int_{\tau_1}^{\tau_2} G_{dT} d\tau \qquad [\mathrm{kWh/(m^2.day)}]$$

$H_{T,day,th}$

$H_{T,day.dif}$

$G_{T,m}$

} tabelled in literature for given:

slopes, azimuths, locations (attenuation factors)

Real duration of sunshine

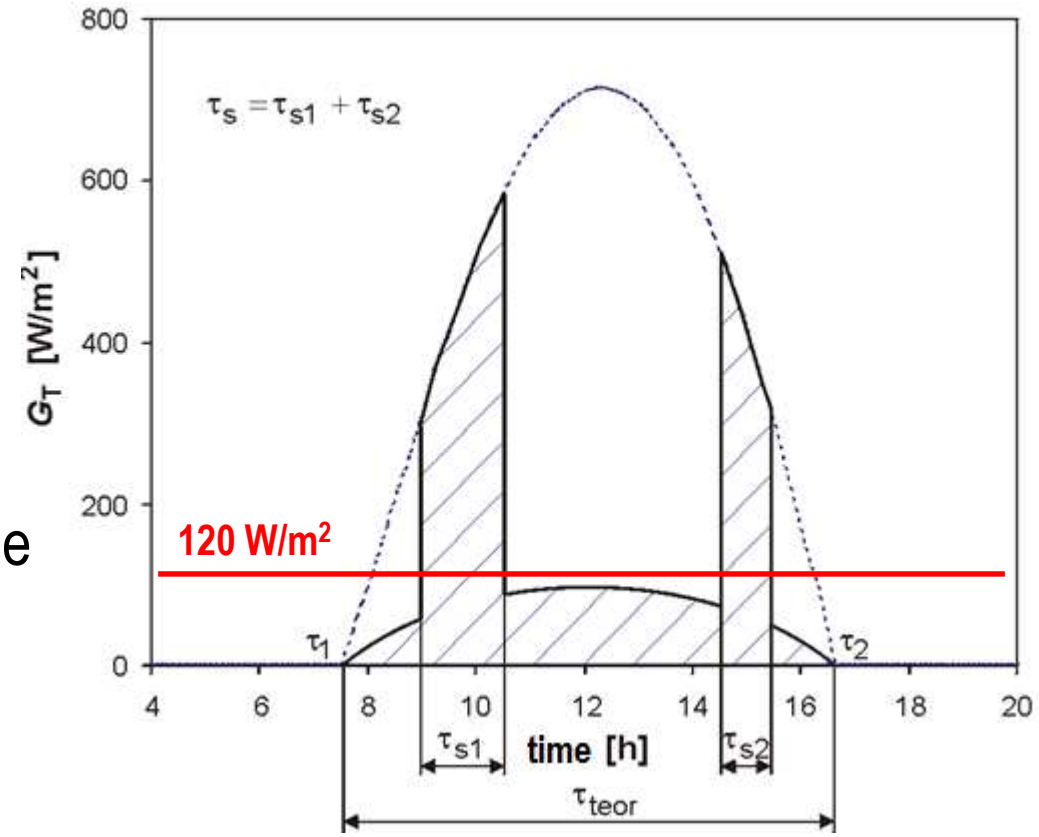
- duration of direct solar radiation > 120 W/m^2

$$\tau_s = \sum_i \tau_{s,i} \qquad [\text{h}]$$

meteo-institute (CZ) presents
the values for 22 locations

- relative period of sunshine

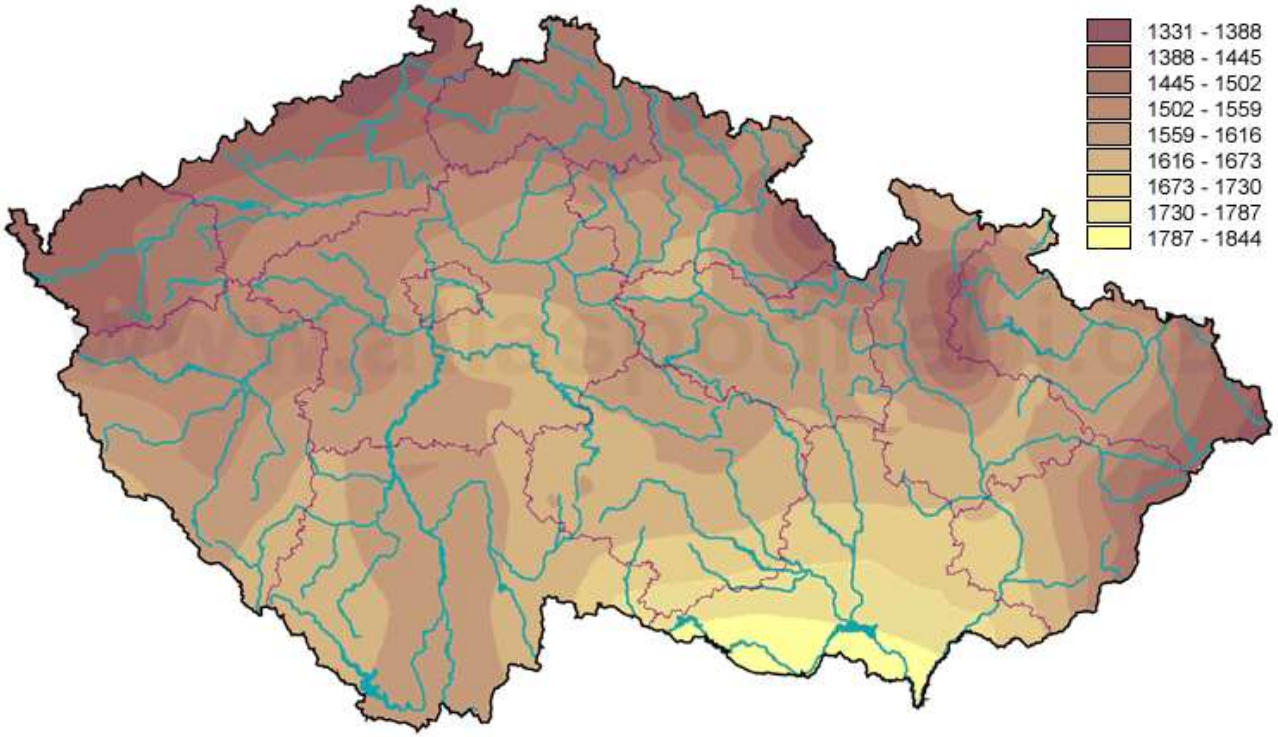
$$\tau_r = \frac{\tau_s}{\tau_t} \qquad [\text{-}]$$



Real duration of sunshine

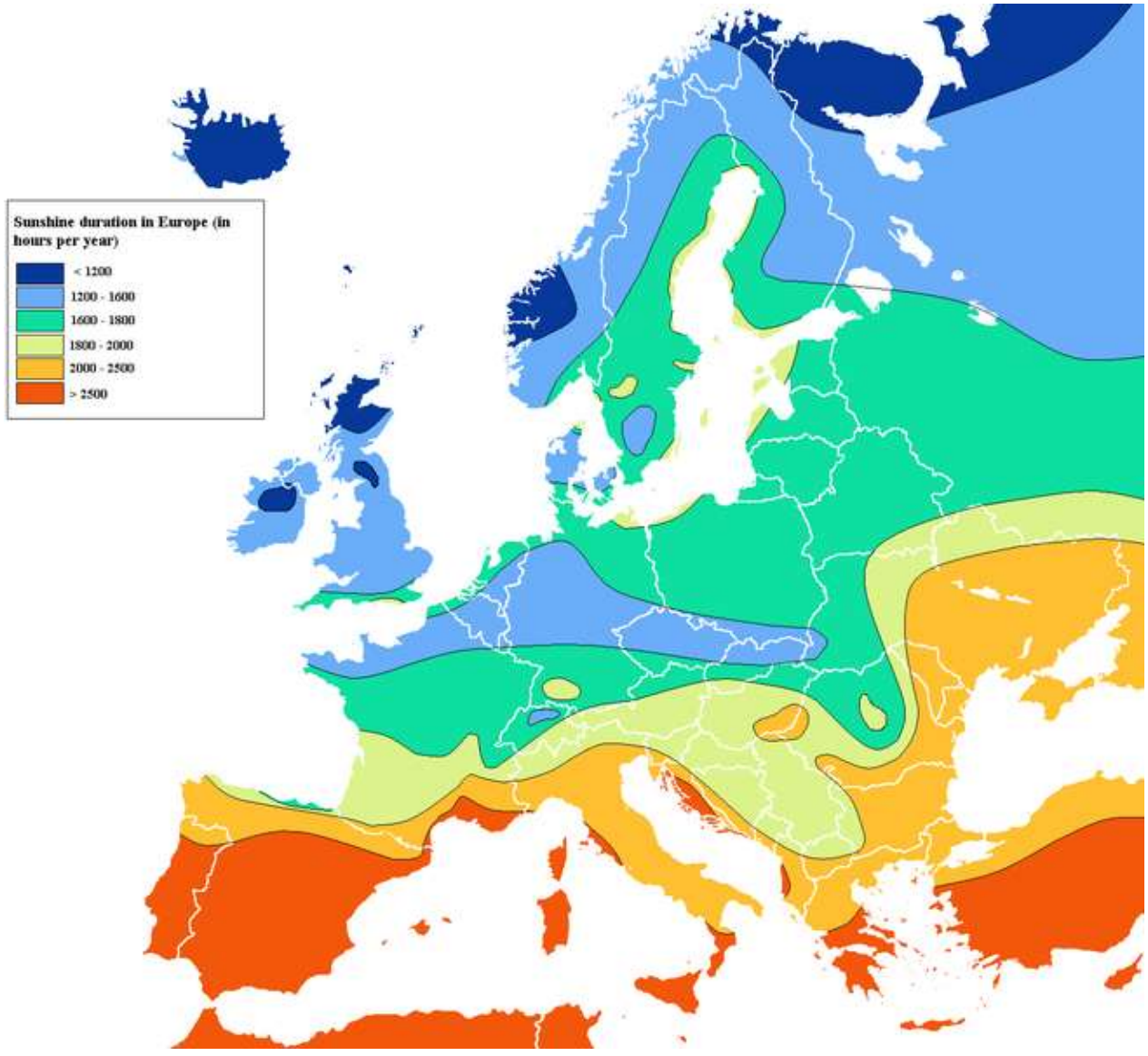
Month	Real duration of suhshine τ_s [h]			
	Praha	České Budějovice	Hradec Králové	Brno
I.	53	46	47	46
II.	90	82	77	88
III.	157	136	149	142
IV.	187	164	185	163
V.	247	207	241	232
VI.	266	226	249	258
VII.	266	238	252	270
VIII.	238	219	233	230
IX.	190	174	188	179
X.	117	108	115	116
XI.	53	55	48	56
XII.	35	36	42	30
Σ	**1 899**	**1 691**	**1 826**	**1 810**

Real duration of sunshine in CZ



1400 – 1900 h/year

Real duration of sunshine in Europe



< 1200 h

> 2500 h

Total irradiation on given plane

- daily solar irradiation

$$H_{T,day} = \tau_r \cdot H_{T,day,th} + (1-\tau_r) \cdot H_{T,day,dif} \qquad [\text{kWh/(m}^2\text{.day)}]$$

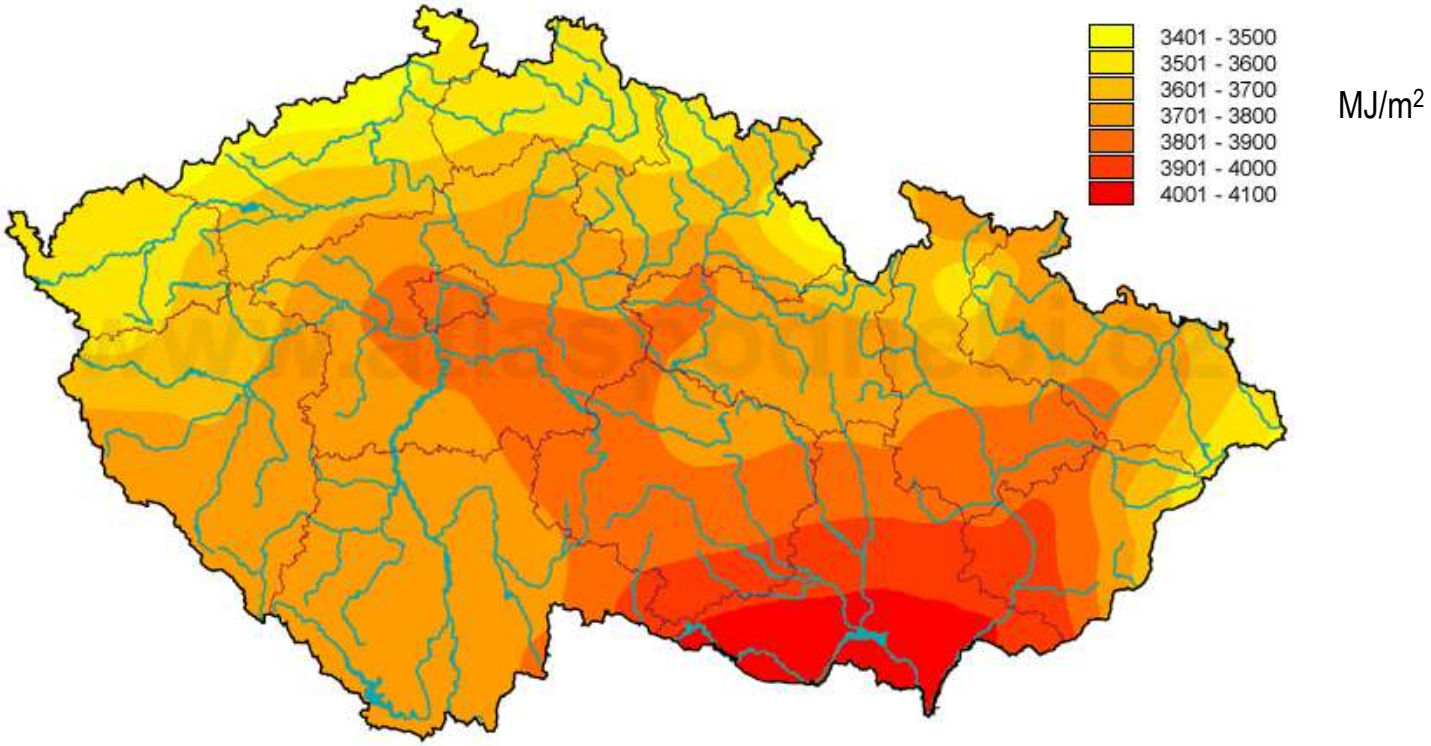
- monthly solar irradiation

$$H_{T,mon} = n \cdot H_{T,day} \qquad [\text{kWh/(m}^2\text{.mon)}]$$

- annual solar irradiation

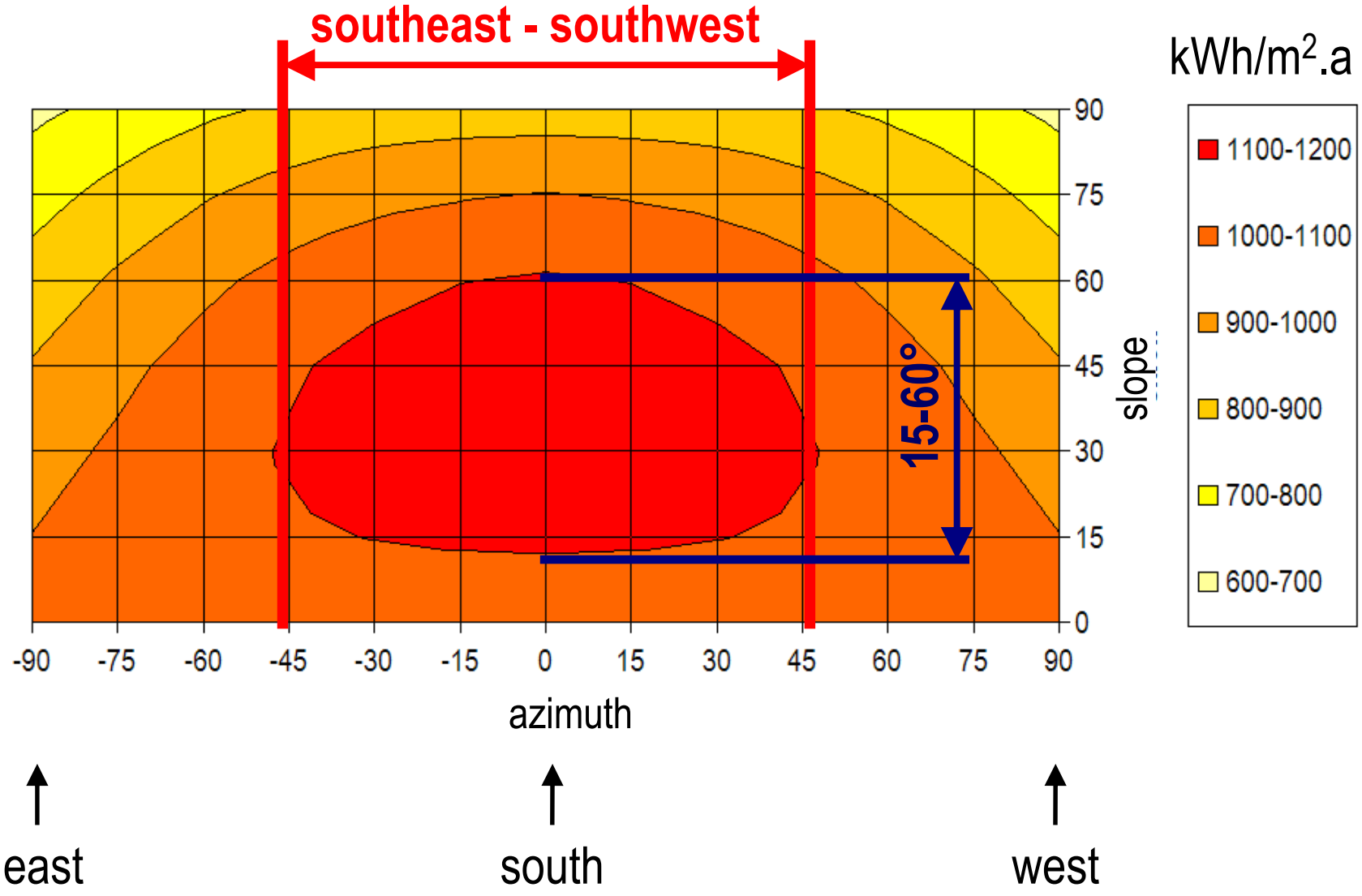
$$H_{T,year} = \sum_{I}^{XII} H_{T,mon} \qquad [\text{kWh/(m}^2\text{.year)}]$$

Annual solar irradiation in CZ



- slope 30° to 45°, south orientation: **1000 to 1200 kWh/m²**
- slope 90°, south orientation: **750 to 900 kWh/m²**

Optimum slope for Central Europe ?



Optimum slope worldwide?

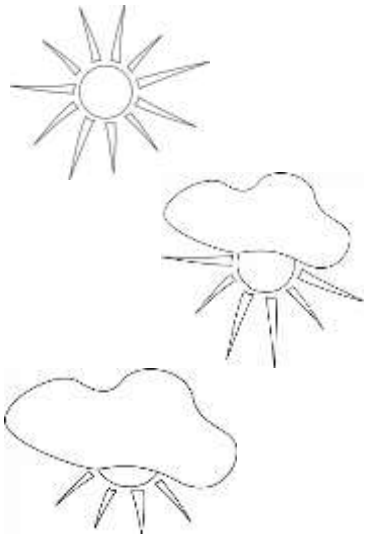
Optimum slope = latitude + (5° .. 10°)

Example:
Sydney latitude = - 33°
a) North orientation
Optimum slope 33 + (5° .. 10°) = ca 40°
b) North west orientation
Optimum slope lower … ca 20° - 30°

Solar energy: typical values

- **solar irradiance *G* (power)**

clear sky	800 to 1000 W/m^2
semibright	400 to 700 W/m^2
overcast	100 to 300 W/m^2



- **solar irradiation *H* (energy)**

winter	3 kWh/(m^2.day)
spring, autumn	5 kWh/(m^2.day)
summer	8 kWh/(m^2.day)