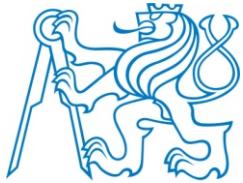




Solar energy

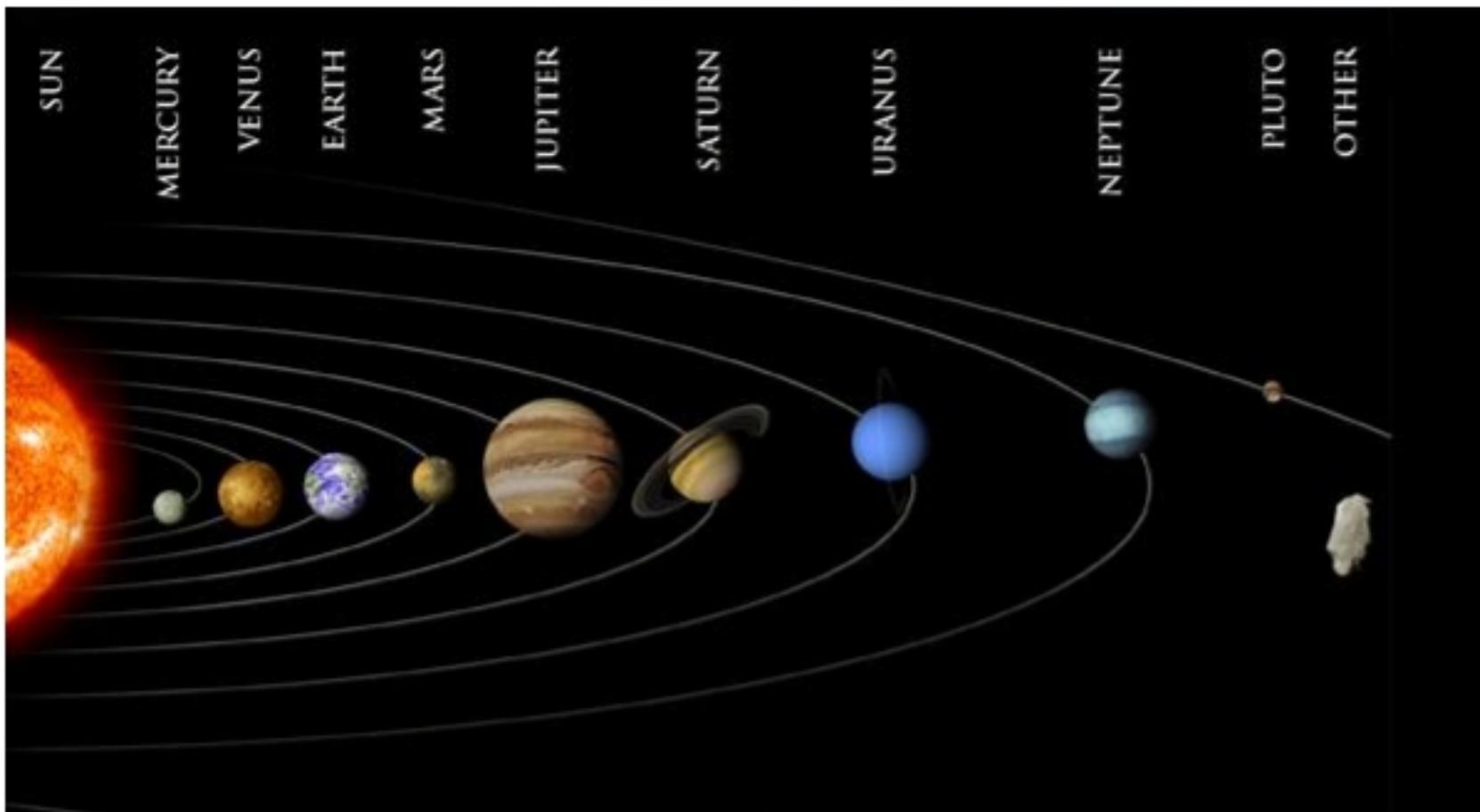
- solar radiation
- definitions
- incident solar energy

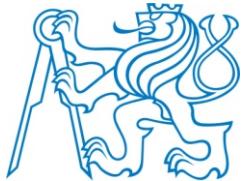




Sun

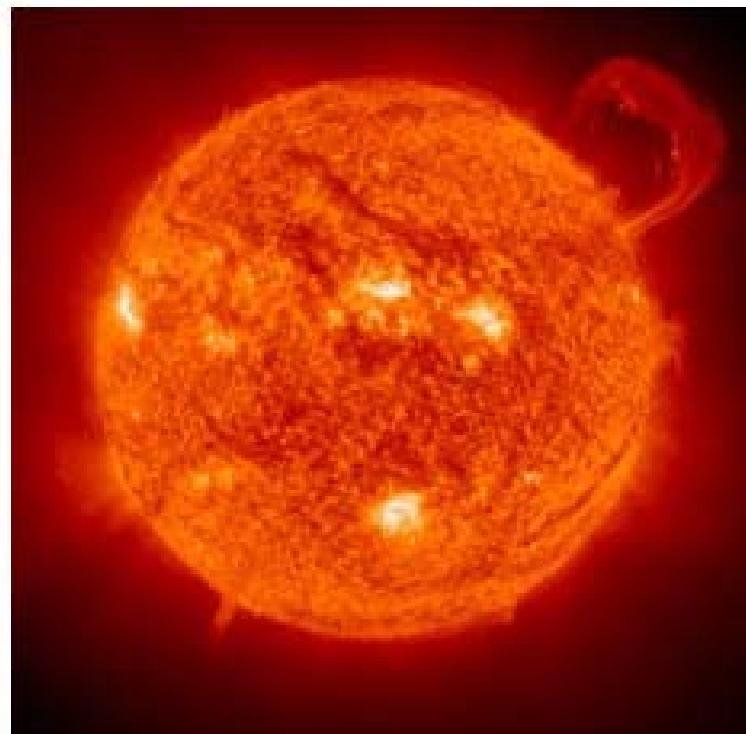
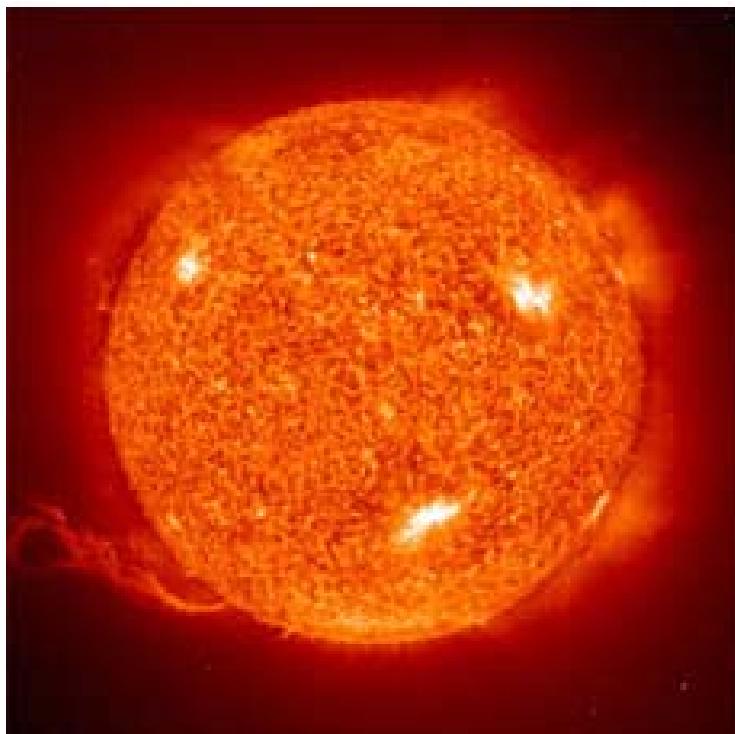
- closest star
- centre of our planetary system – solar system





Sun

- diameter 1 392 000 km 109 x larger than Earth
- weight 2×10^{30} kg 330 000 x greater than Earth
99,86 % weight of solar system
- consists of: 70 % hydrogen **H**, 28 % helium **He**, 2 % other elements





Sun

- origin of solar energy in **nuclear** reactions
- nuclear fusion takes place inside the Sun
at high temperatures **10^6 K** and pressures **10^{10} MPa**
synthesis of hydrogen nuclei (H) → helium nuclei (He)
- **564×10^9 kg/s H** transforms to **560×10^9 kg/s He**
- mass difference **4×10^9 kg/s** is radiated in the form of energy

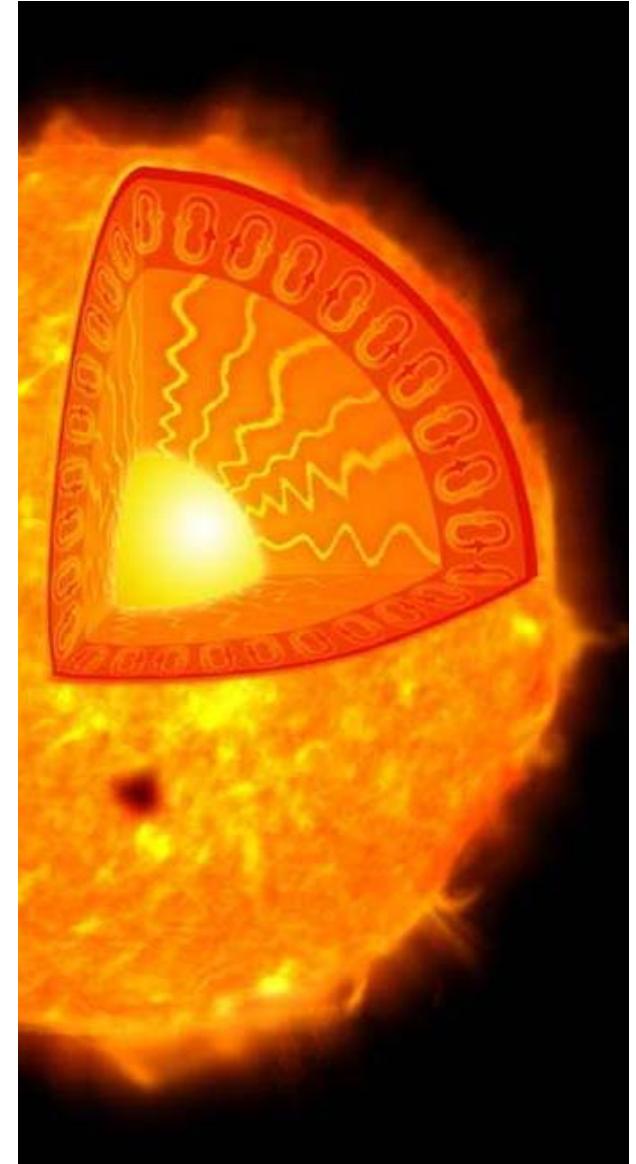
$$E = m.c^2$$

- radiative power: **$3,6 \times 10^{26}$ W**
- specific radiated power (density): **6×10^7 W/m²**



Sun

- **core** (to 23 % radius)
temperature 10^6 K, X-ray radiation
90 % of Sun's energy generated
- **radiative zone** (from 23 to 70 % radius)
temperature falls down to 130 000 K
radiative energy transfer (fotons)
- **convective zone** (from 70 to 100 % radius)
lower density, convective energy transfer
- **photosphere** (visible surface of Sun)
temperature 5800 K, solar radiation





Spectral density of radiative flux

- Sun radiates as **perfect black body** with surface temperature **5800 K**
 - spectral density of radiative solar flux (Planck's law)

$$E_c(\lambda, T) = \frac{2 \cdot \pi \cdot h \cdot c^2}{\lambda^5} \left[e^{\frac{h \cdot c}{k \cdot \lambda \cdot T}} - 1 \right]^{-1} \quad [\text{W/m}^2 \cdot \mu\text{m}]$$

$$h = 6,6256 \times 10^{-34} \text{ J.s} \quad \text{Planck's constant}$$

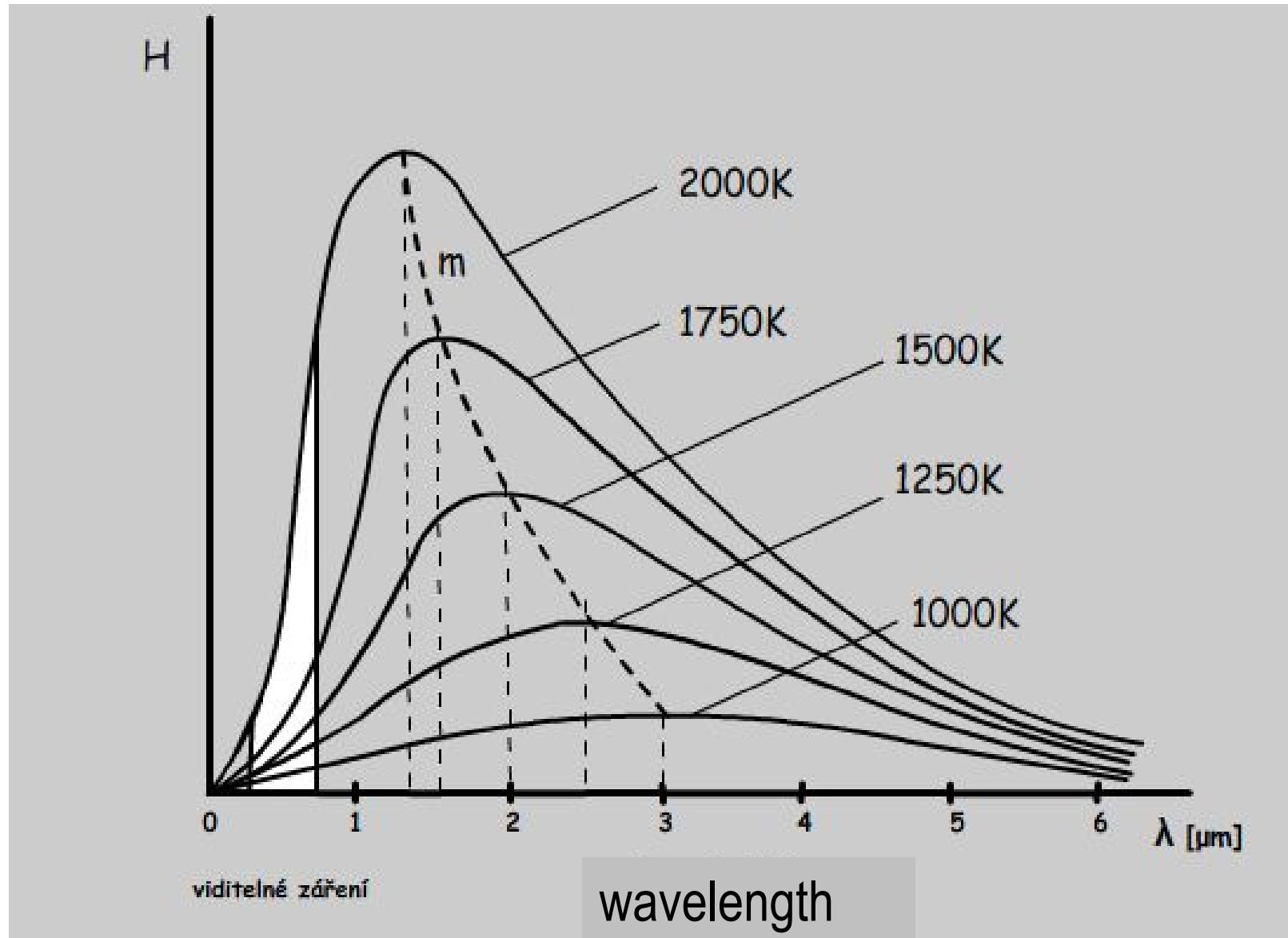
$$k = 1,3805 \times 10^{-23} \text{ J/K} \quad \text{Boltzmann's constant}$$

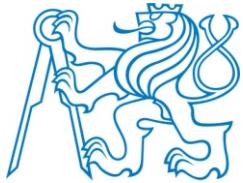
$$c = 2,9979 \times 10^8 \text{ m/s} \quad \text{light velocity in vacuum}$$

T thermodynamic surface temperature [K]

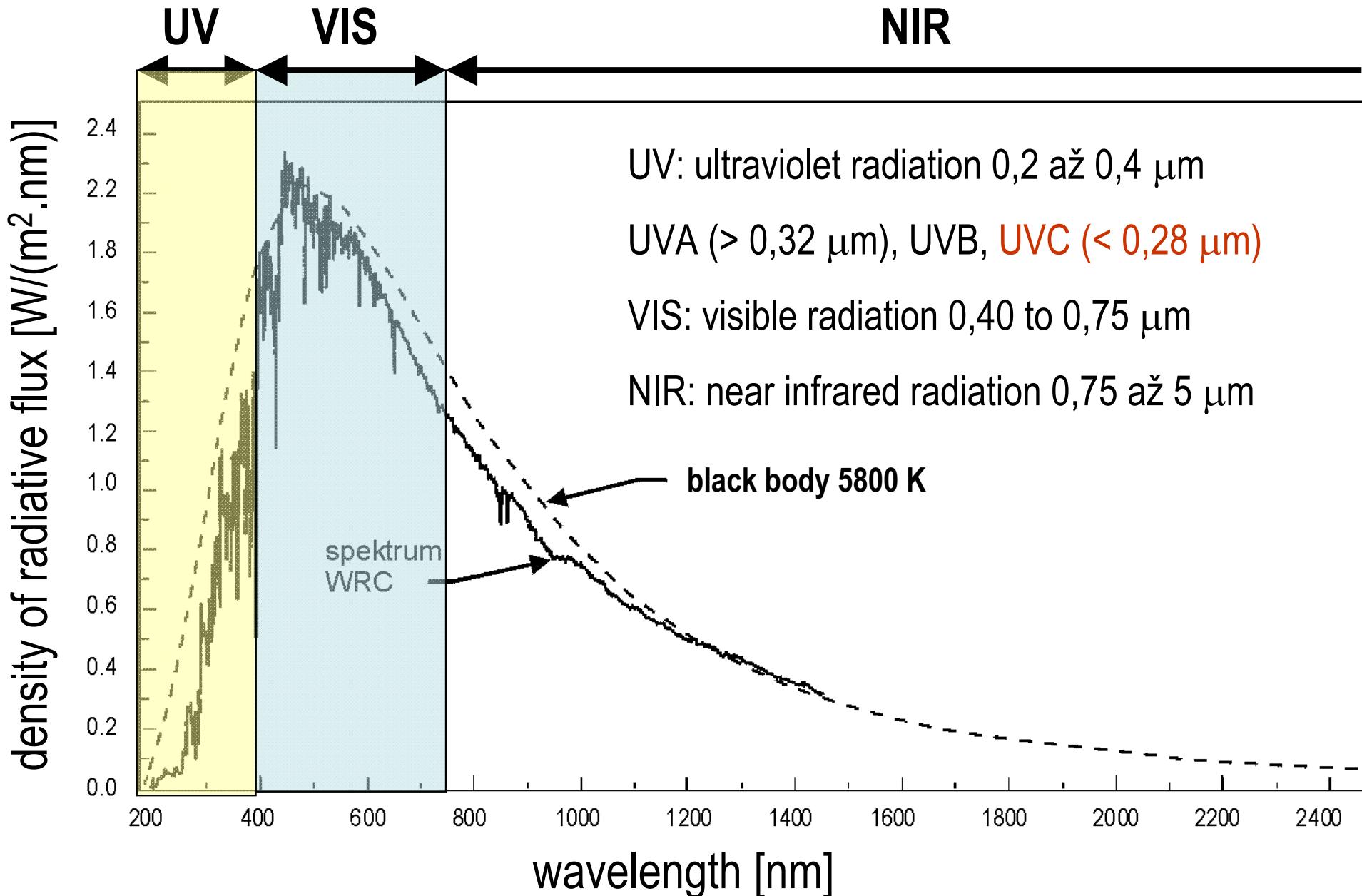


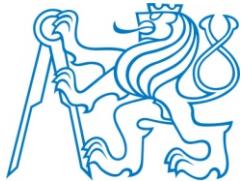
Planck's law



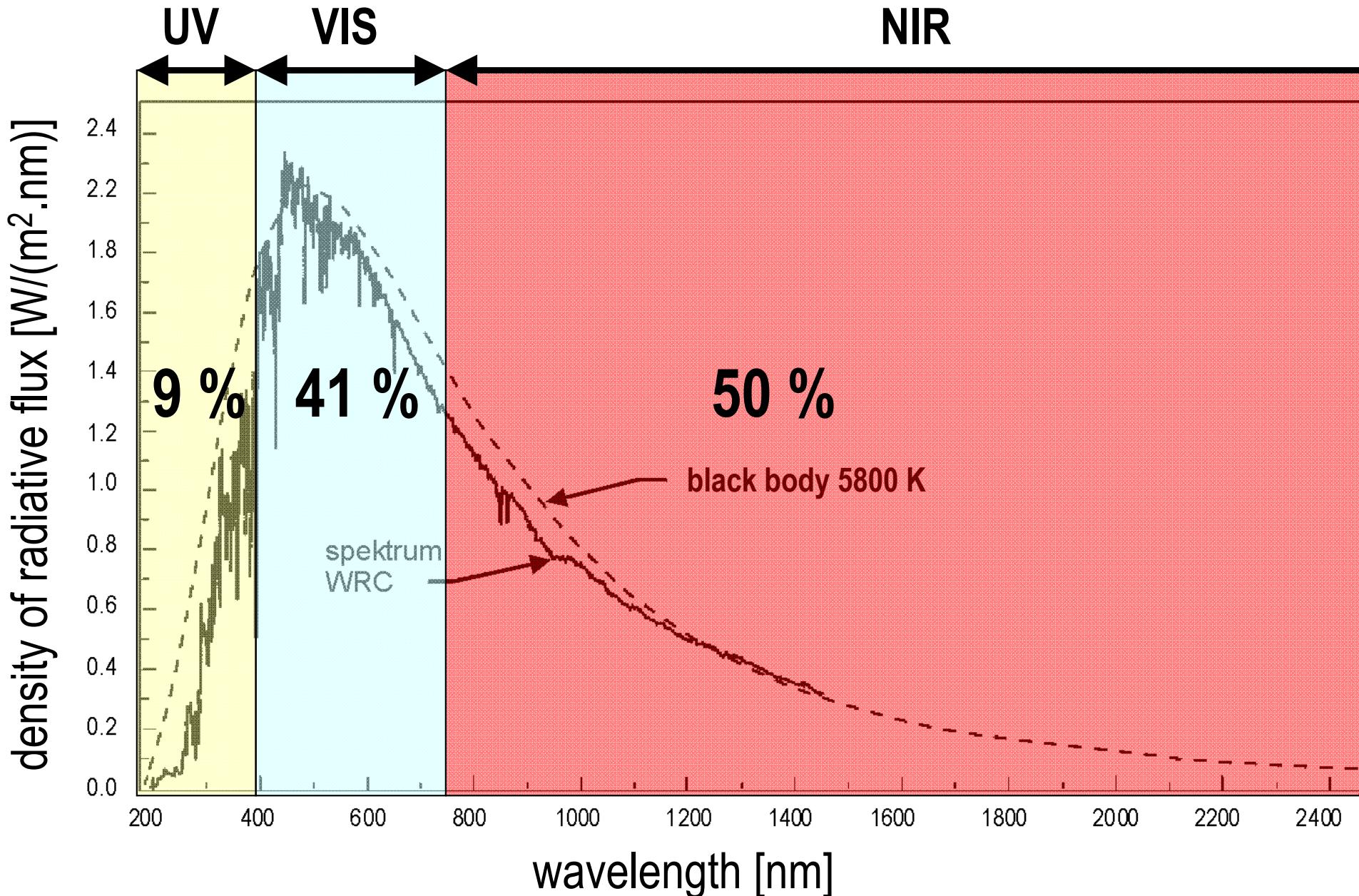


Spectral density of radiative flux





Solar energy irradiated

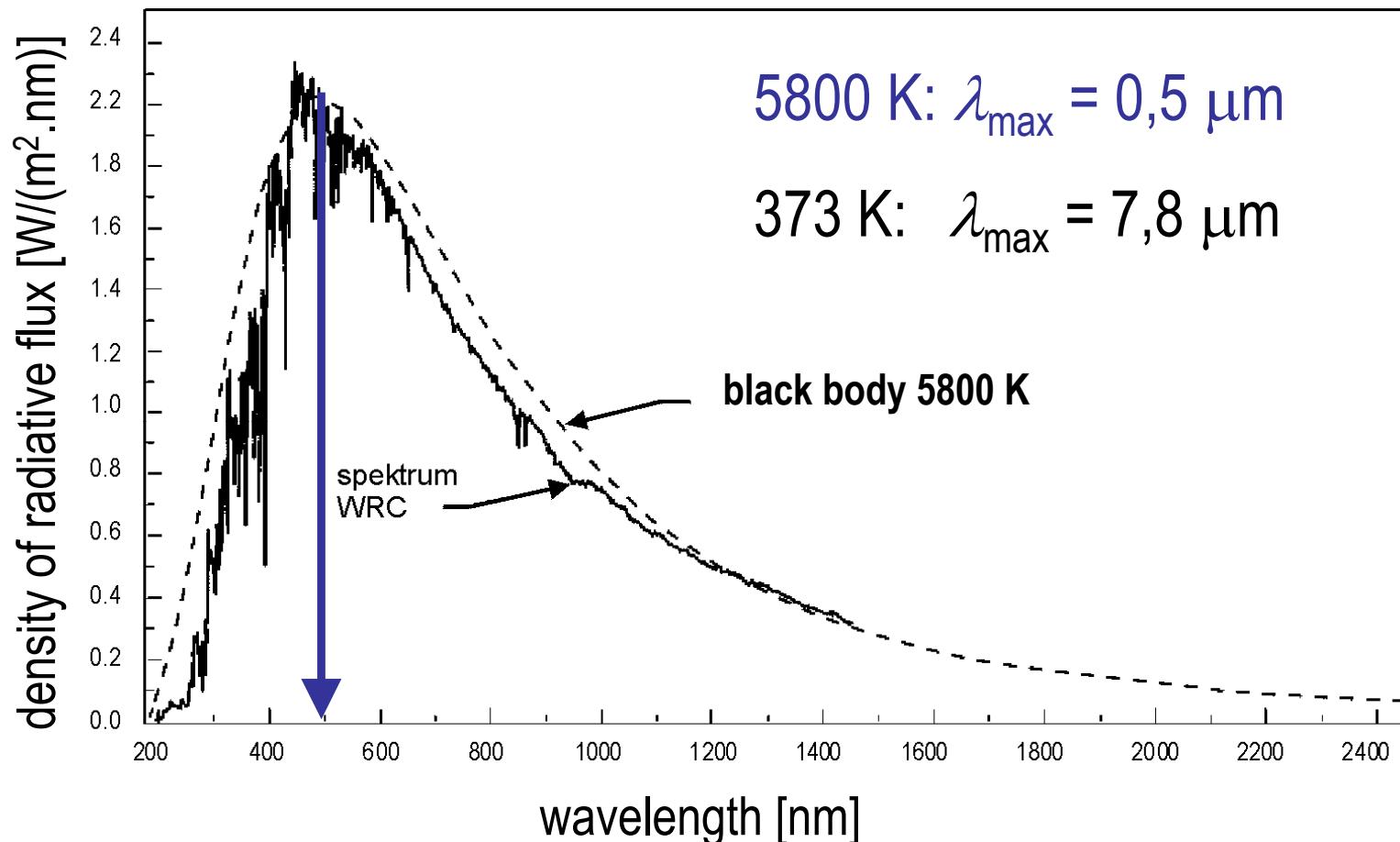




Wien's law

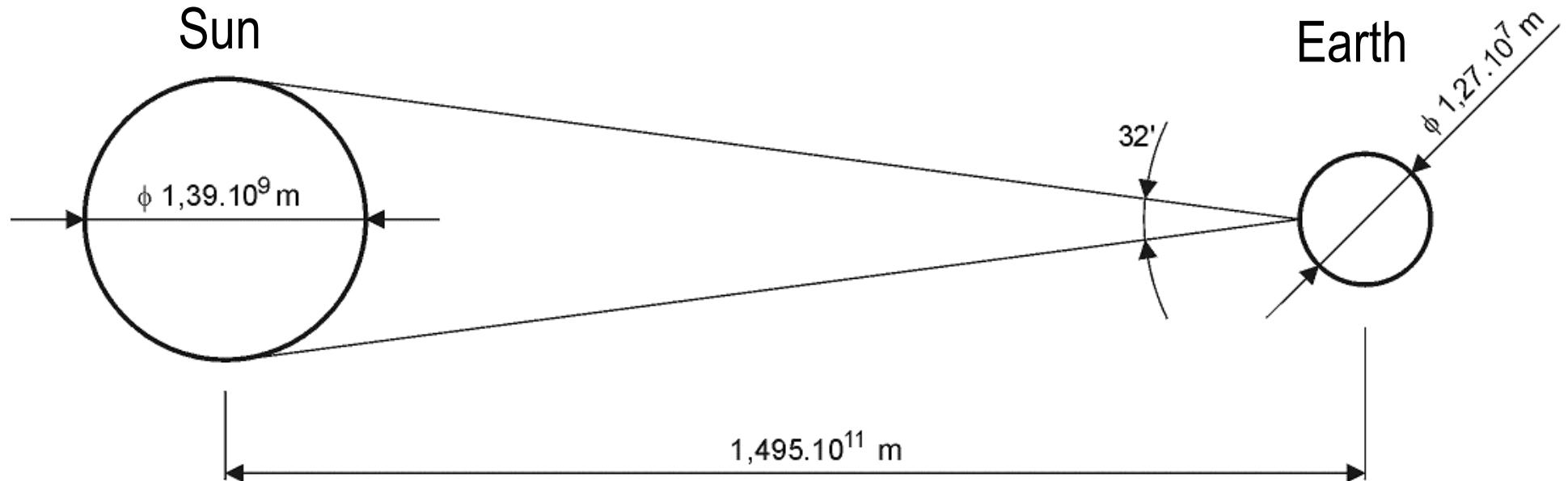
- maximum of radiative flux – seeking the extreme of Planck's function

$$\frac{\partial E_c(\lambda, T)}{\partial \lambda} = 0 \quad \rightarrow \quad \lambda_{\max} \cdot T = 2898 \quad [\mu\text{m.K}] \quad \text{Wien's law}$$





Propagation of solar energy

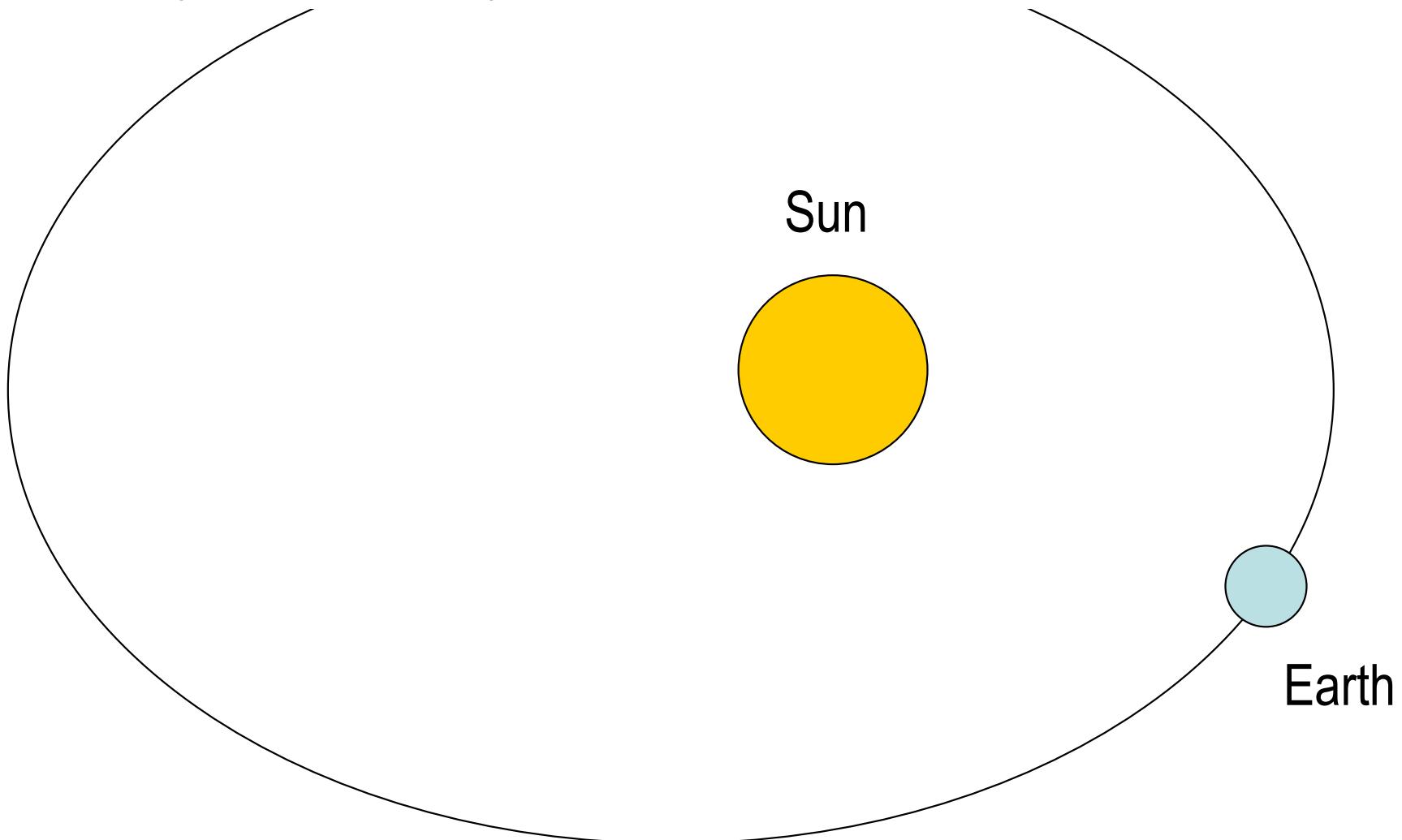


- power spreads to larger area with increasing distance from Sun
- $0,5 \times 10^{-9}$ of the irradiated Sun's power is incident on Earth
radiative flux **$1,7 \times 10^{17} \text{ W}$**
- solar „beams“ considered as parallel ($32'$)



Earth rotates around Sun

elliptic orbit (almost circular), Sun in one of focusses





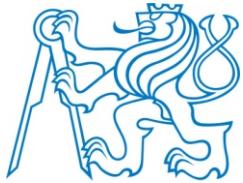
Radiative flux density out of atmosphere

- solar radiative flux incident on area unit perpendicular to direction of propagation
- changes during the year, variable distance Sun-Earth (elliptic orbit)
change of distance $\pm 1,7\%$, change of flux $\pm 3,3\%$
- value for mean distance Sun-Earth

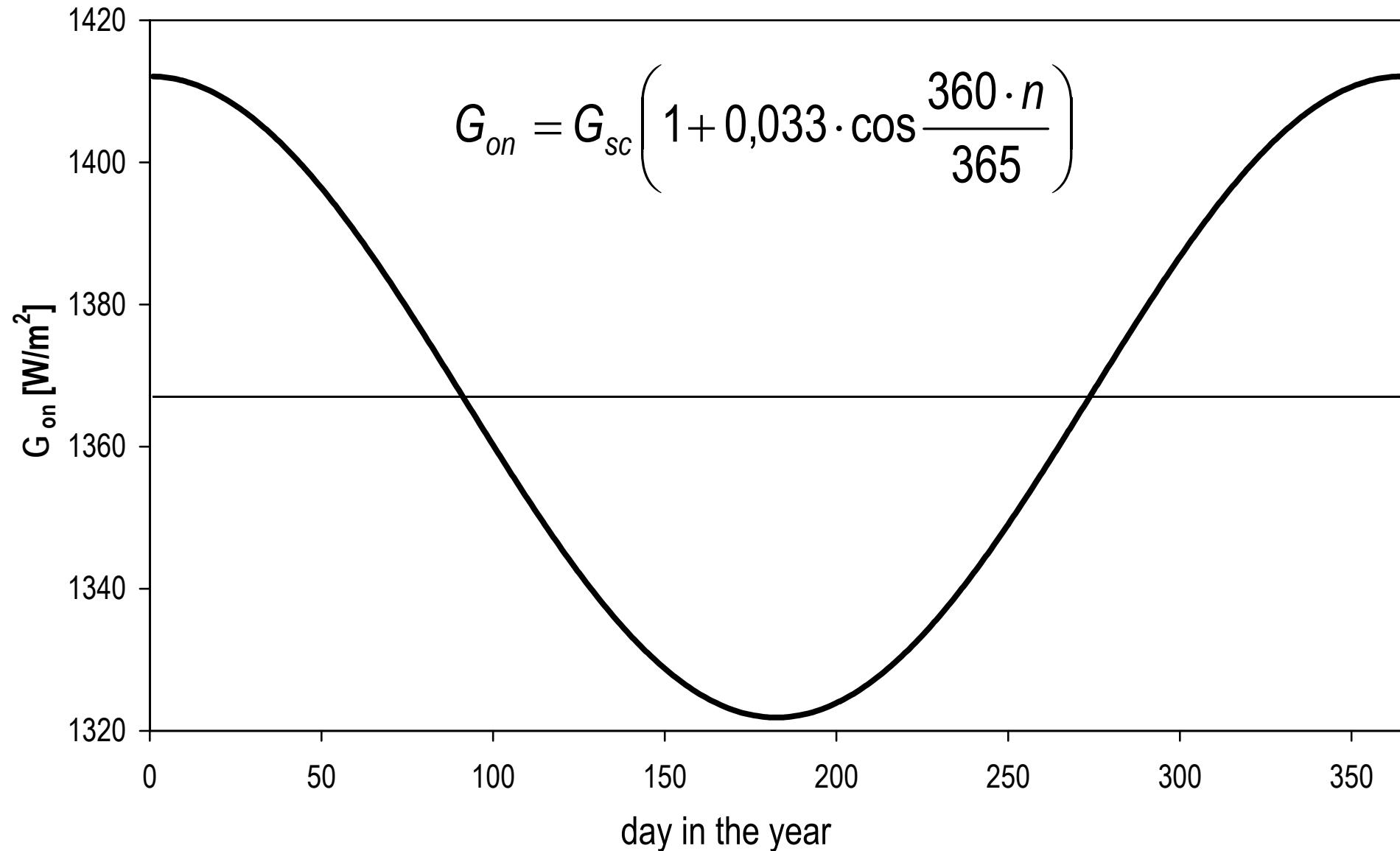
solar constant $G_{sc} = 1367 \text{ W/m}^2$ (value from WRC, 1 %)

original measurements Ch. Abbot in mountains 1322 W/m^2 , today satellites

- Merkur: 9040 W/m^2 ... Neptun: $1,5 \text{ W/m}^2$



Radiative flux density out of atmosphere



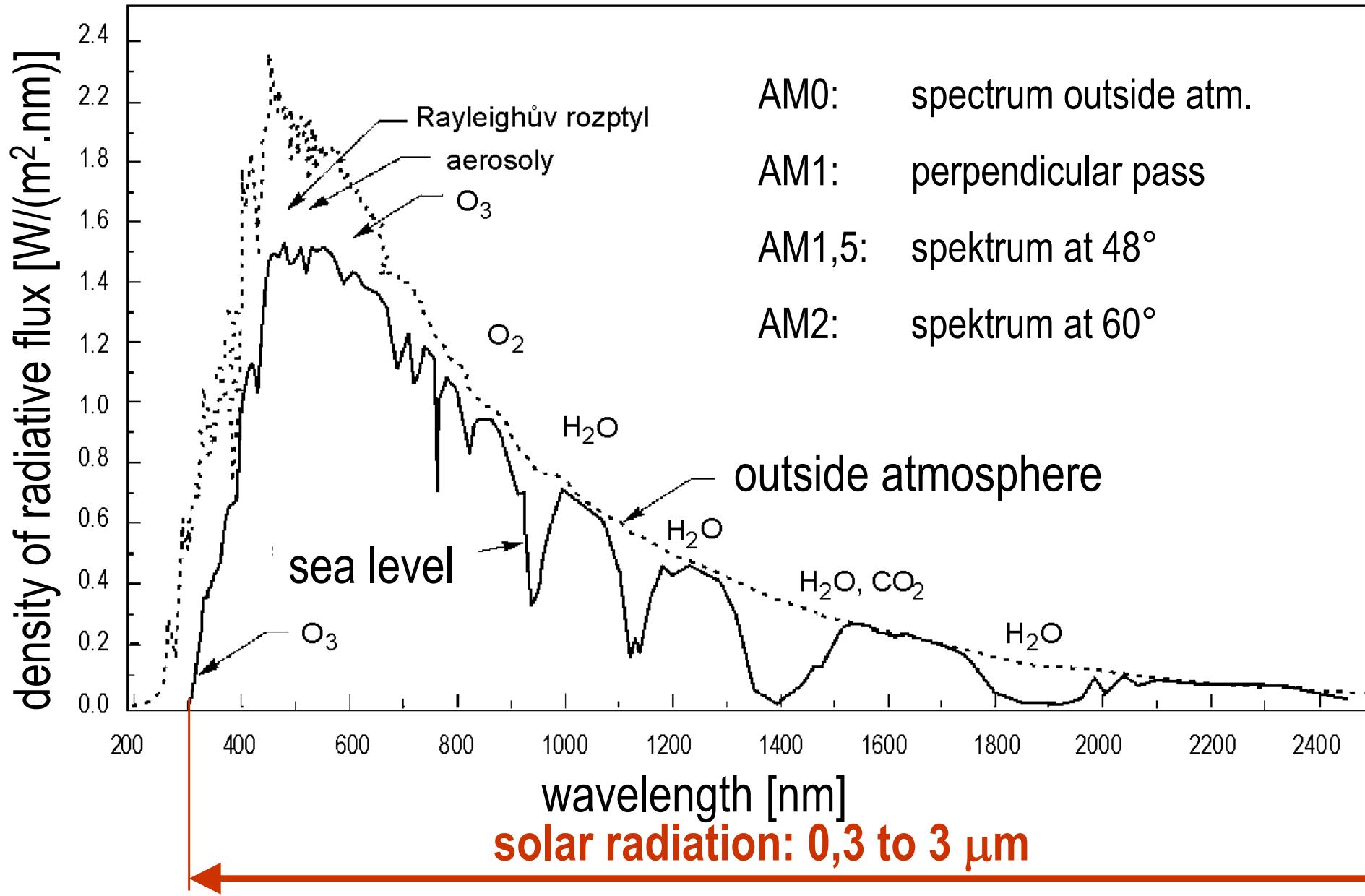


Solar radiation passing the atmosphere

- solar radiation enters the atmosphere (no definite boundary, exosphere continuously fading to interplanetary space)
- **ionosphere** (60 km)
atmospheric gases O₂, N₂ absorb **ultraviolet** and x-ray radiation, becoming ionised
- **ozonosphere** (20 až 30 km)
ozon O₃ absorbs rest of harmful **ultraviolet** radiation (UVC)
- **troposphere** (lowest layer, clouds)
water vapor, CO₂, dust, water droplets absorb **infrared** radiation



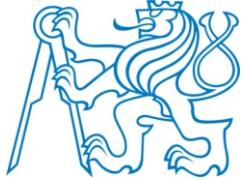
Solar radiation passing the atmosphere



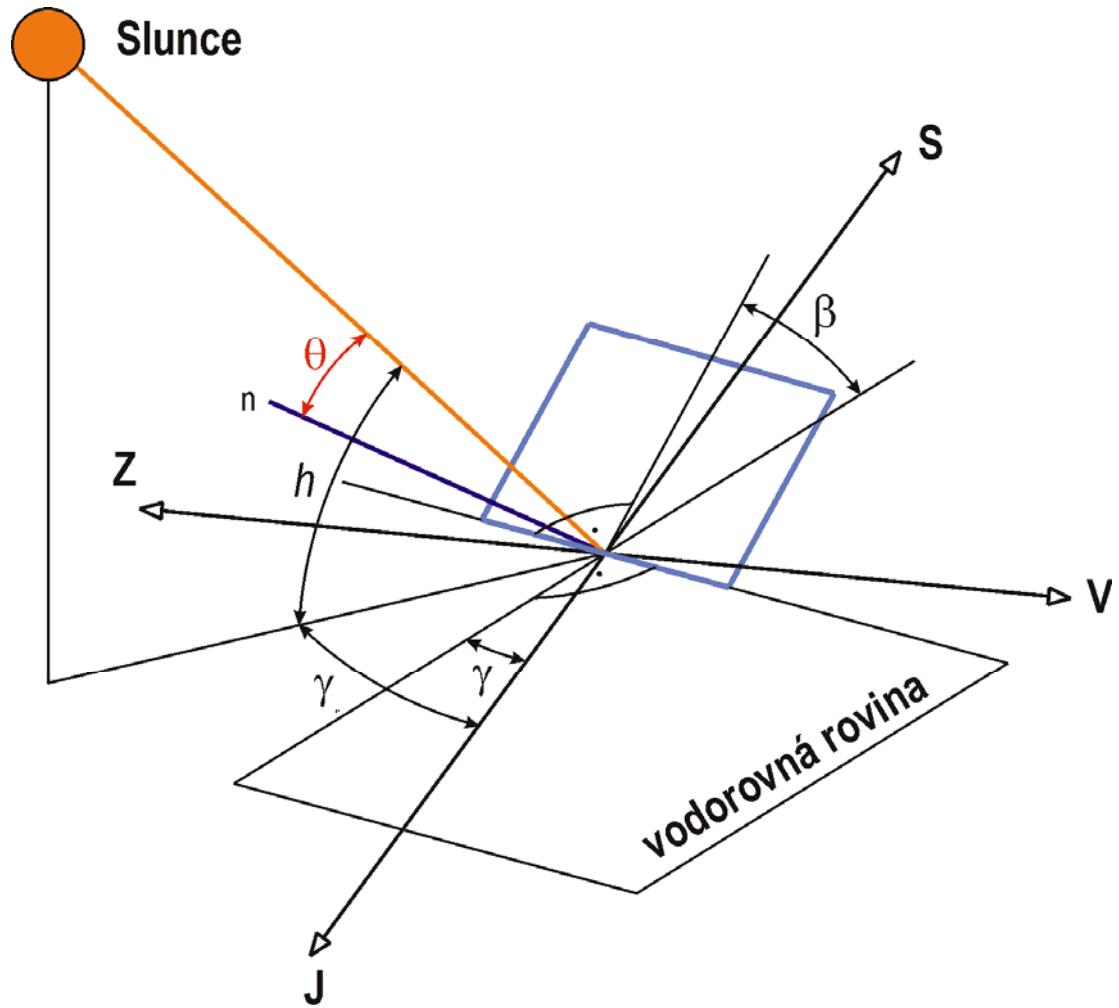


Annual balance

- reflection from atmosphere 34 %
- absorption in atmosphere 19 %
- **incident and absorbed by Earth surface** 47 %
 - absorbed at surface
 - emitted back 14 % energy of environment
 - evaporation (oceans) 23 % water energy
 - convection, winds 10 % wind energy
 - biologic reactions, photosynthesis 1 % energy of biomass



Solar geometry



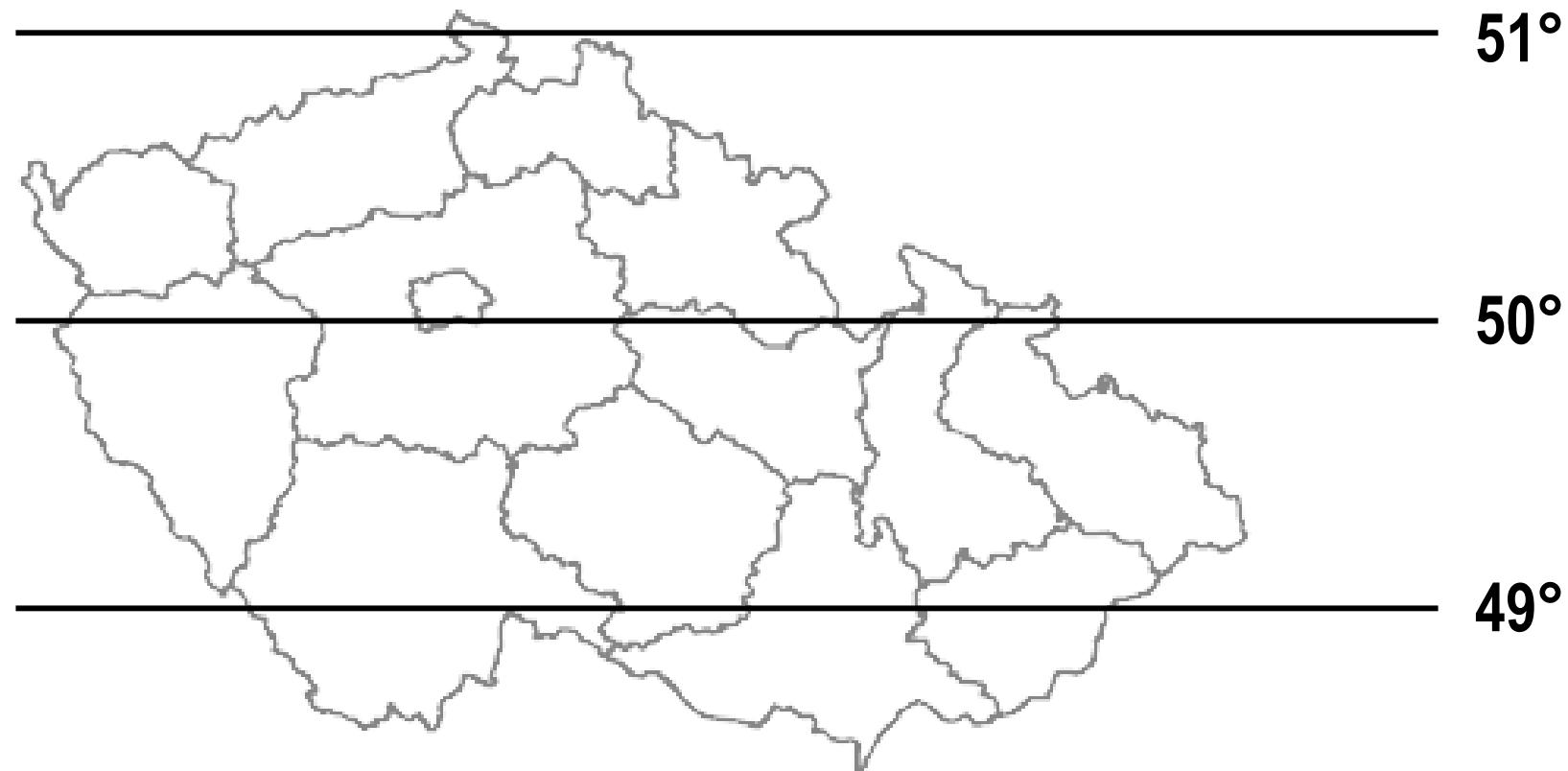
- surface slope β
 - surface azimuth γ
 - latitude ϕ
-
- time, date
 - time angel τ
 - declination δ
 - Sun height h
 - Sun azimuth γ_s
-
- incidence angle θ



Surface position

- **latitude ϕ** convention: north (+), south (-)

angle between plane of equator and a line connecting Earth centre and given place on Earth surface





Surface position



high difference between north and south in one country

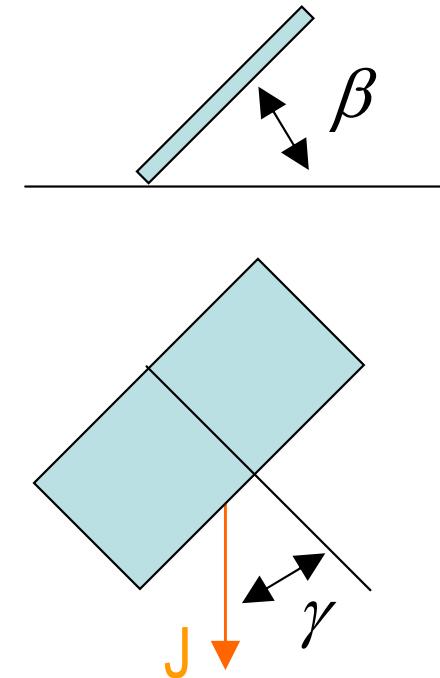
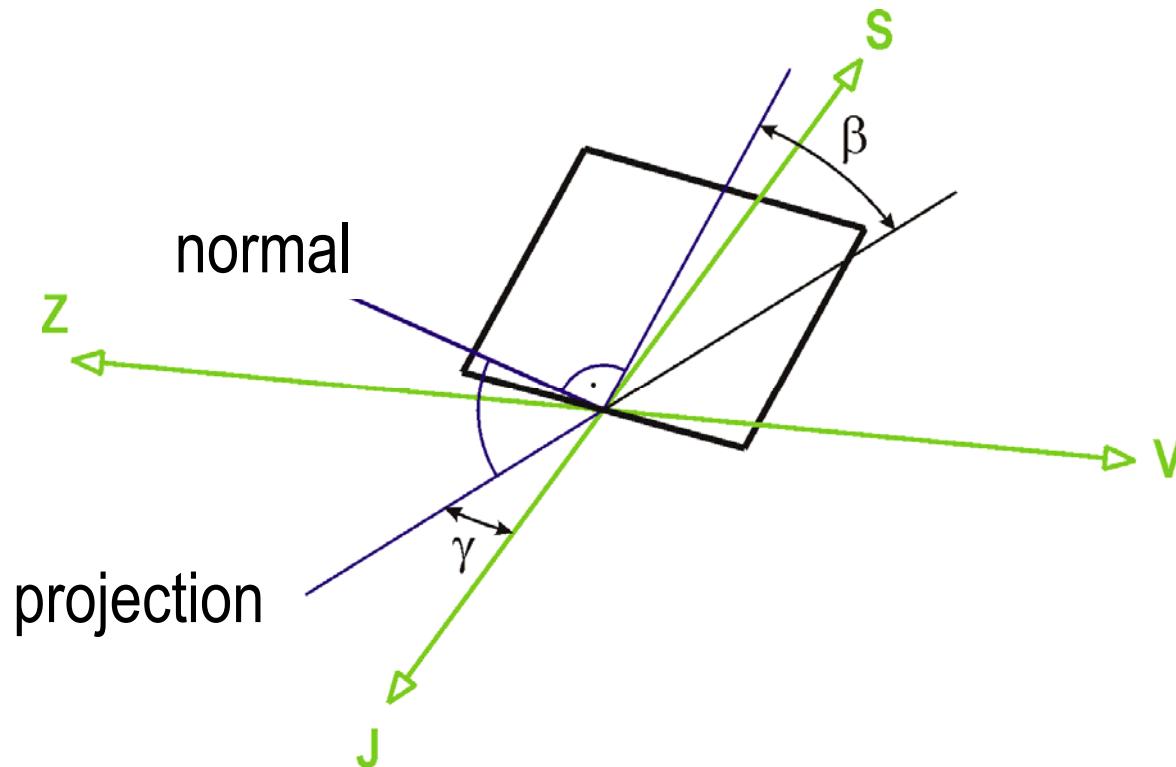
Norway: 57° to 71°

Sweden: 55 to 68°



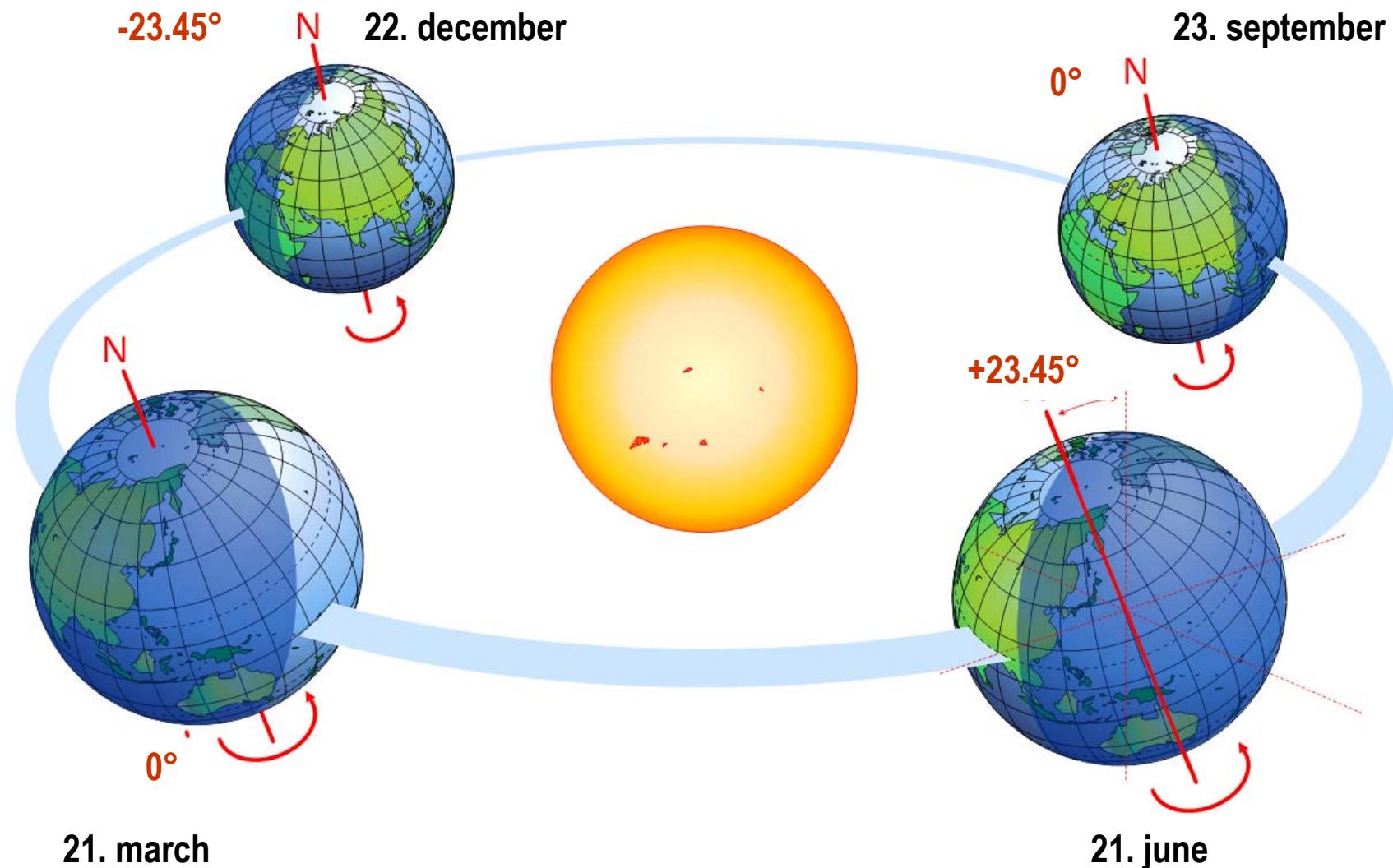
Surface orientation

- **slope angle β** convention: horizontal 0°, vertical 90°
angle between horizontal plane and surface plane
- **surface azimuth γ** convention: east (-), west (+), south (0°)
angle between projection of surface normal and local meridian (south)





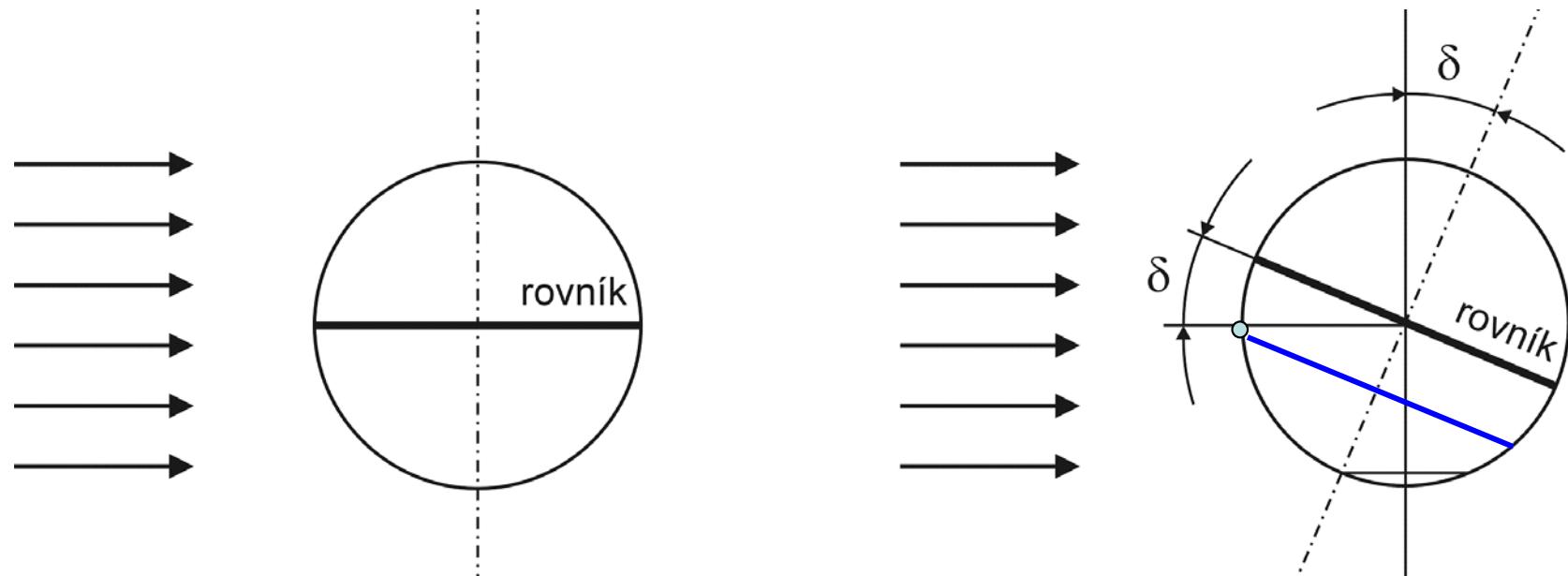
Declination





Declination δ

- tilt angle of Earth's axis due to precession movement during rotation
- angle between the line (connecting centres of Earth and Sun) and equator plane
- latitude of place on Earth where in given day in the noon the Sun is in zenith





Calculation of declination δ

- from calendar date $DD.MM.$

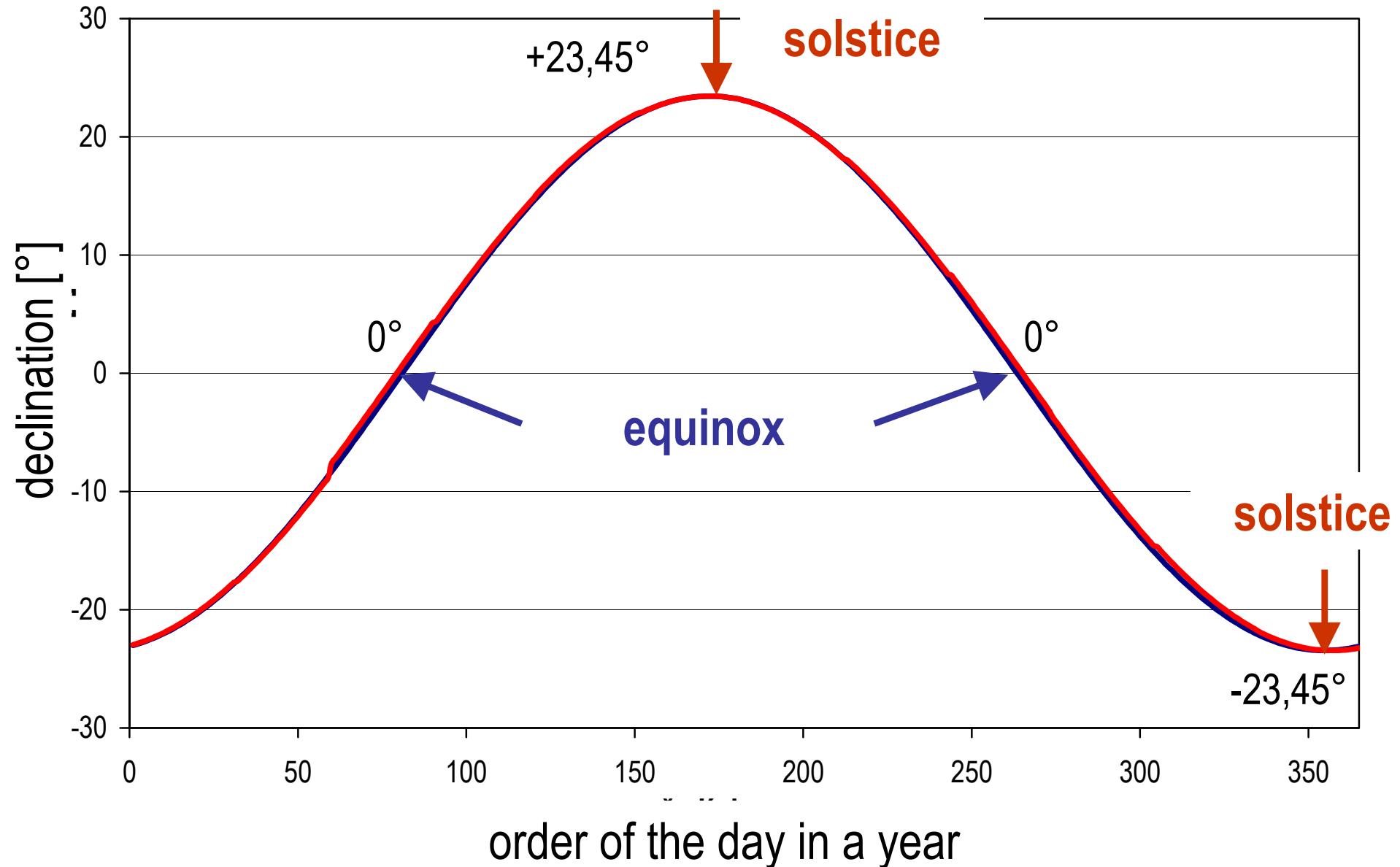
$$\delta = 23,45^\circ \sin(0,98 \cdot DD + 29,7 \cdot MM - 109^\circ)$$

- from order of the day in the year n

$$\delta = 23,45^\circ \sin\left(360 \frac{284 + n}{365}\right)$$



Calculation of declination δ





time (hour) angle τ

- angle of **virtual** translation of Sun above local meridians due to Earth rotation, **related to solar noon**
- Earth is rotating around its axis (360°) once for 24 hours
→ translation of Sun by 15° over 1 hour
- time angle is calculated from solar time ST

$$\tau = 15^\circ \cdot (ST - 12)$$

convention: before noon (-), after noon (+)



Solar time ST

- each timezone has a time related to local meridian
timezones of 1 h ~ meridians of 15°
- CET: local solar time at meridian 15° east longitude
- **solar time:** daily time defined from virtual translation of Sun
- observer at reference meridian: local time = solar time
- observer out of reference meridian: local time \neq solar time
shift up to 30 minutes

example:

solar noon

Prague $14,4^\circ$ 12:02

Brno $16,6^\circ$ 11:53

Košice $21,2^\circ$ 11:35



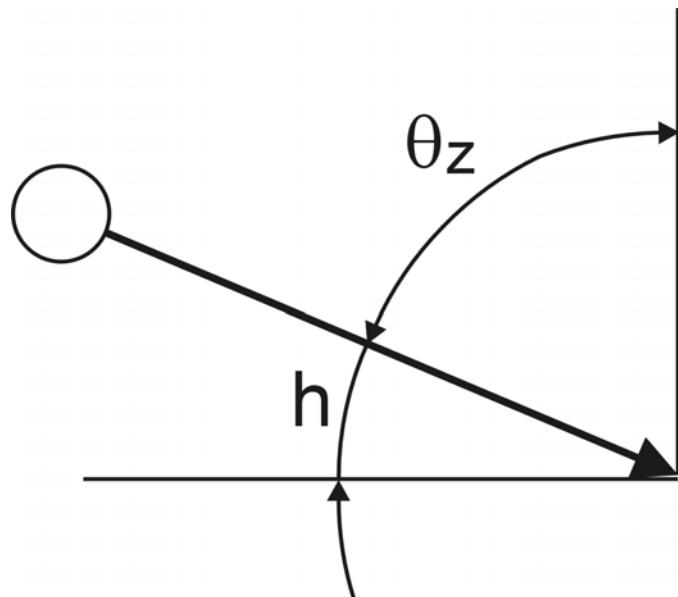
Sun altitude h

- angle between line connecting surface-Sun and horizontal

$$\sin h = \sin \delta \cdot \sin \phi + \cos \delta \cdot \cos \phi \cdot \cos \tau$$

- complement angle to 90° : zenith angle θ_z

$$\theta_z = 90^\circ - h$$



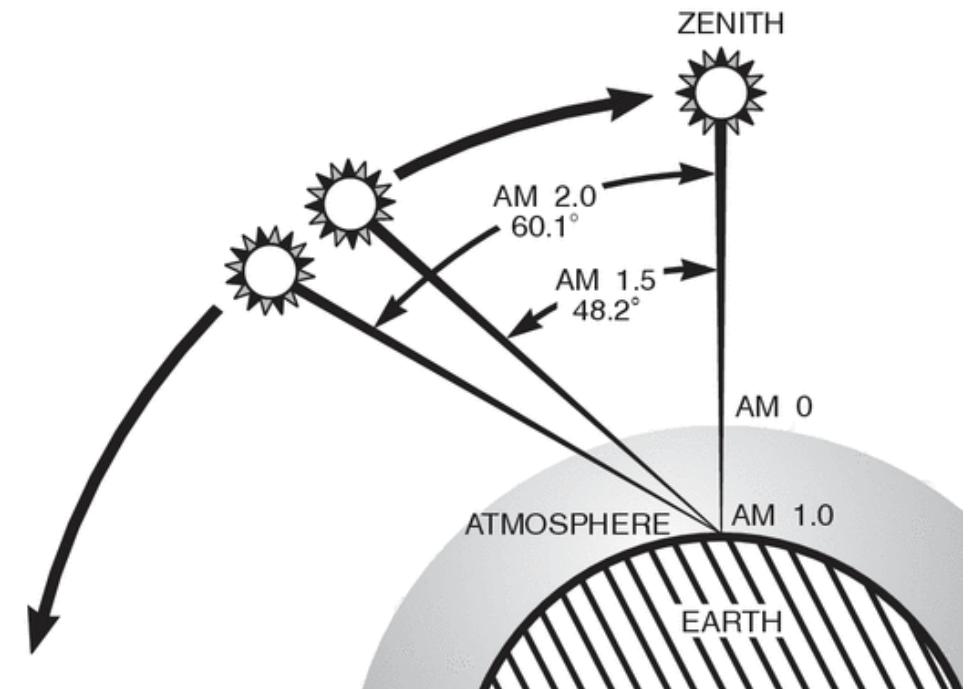


Air mass

- ratio between mass of atmosphere passed by solar radiation to mass, which would be passed if Sun is in zenith

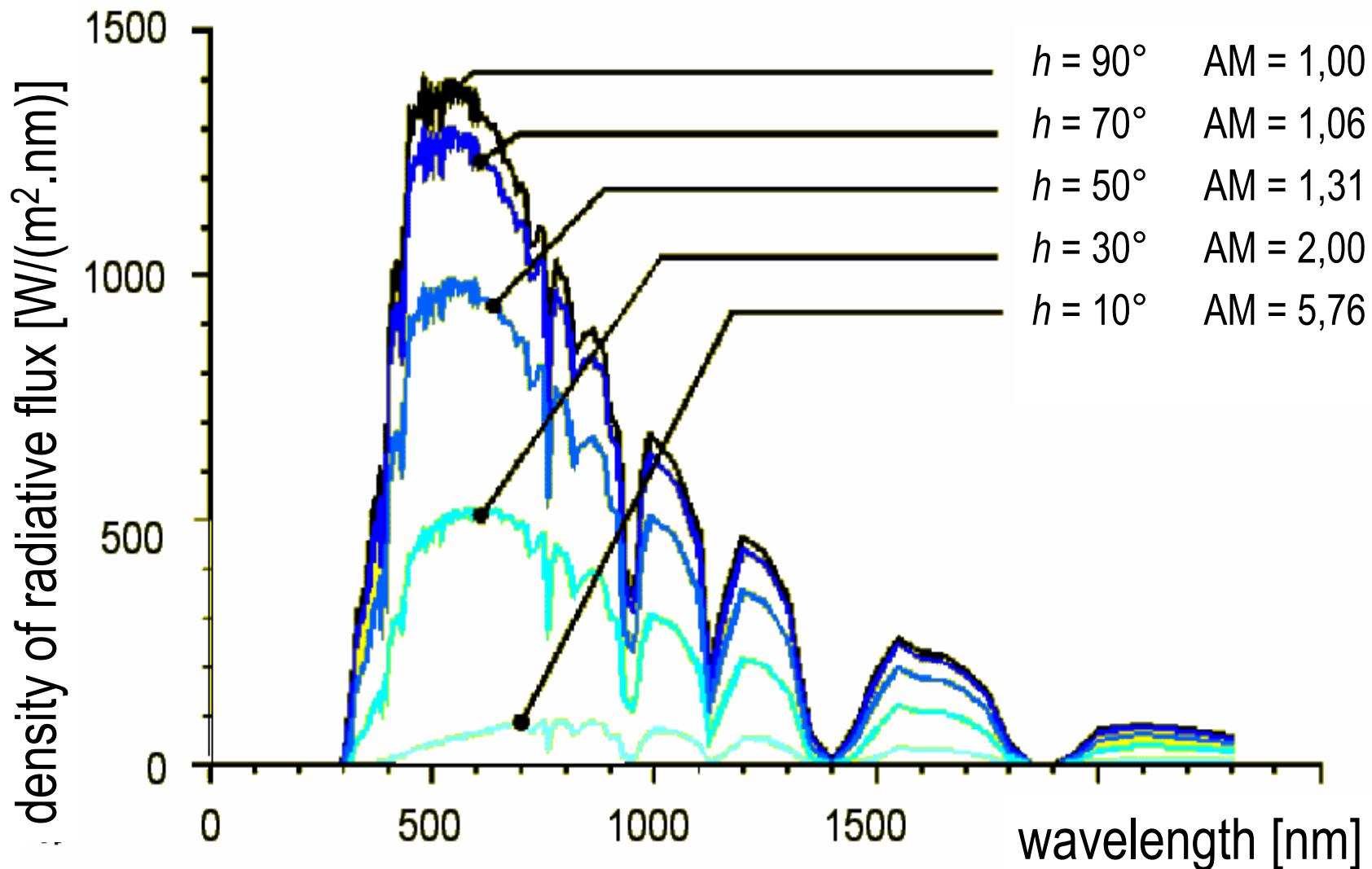
$$AM = \frac{1}{\cos \theta_z} = \frac{1}{\sin h}$$

- $AM = 0$ outside atmosphere
- $AM = 1$ zenith $h = 90^\circ$
- $AM = 1,5$ $\theta_z = 48^\circ$ $h = 42^\circ$
- $AM = 2$ $\theta_z = 60^\circ$ $h = 30^\circ$





Change of spectrum with air mass





Time of sunrise and sunset

- sunrise / sunset: altitude angle = 0°

$$\sin h = \sin \delta \cdot \sin \phi + \cos \delta \cdot \cos \phi \cdot \cos \tau = 0$$

- time angle of sunrise / sunset

$$\tau_{1,2} = \arccos(-\tan \phi \cdot \tan \delta)$$

- theoretical period of sunshine = time between sunrise and sunset

$$\tau_t = \frac{2 \cdot \tau_{1,2}}{15^\circ}$$



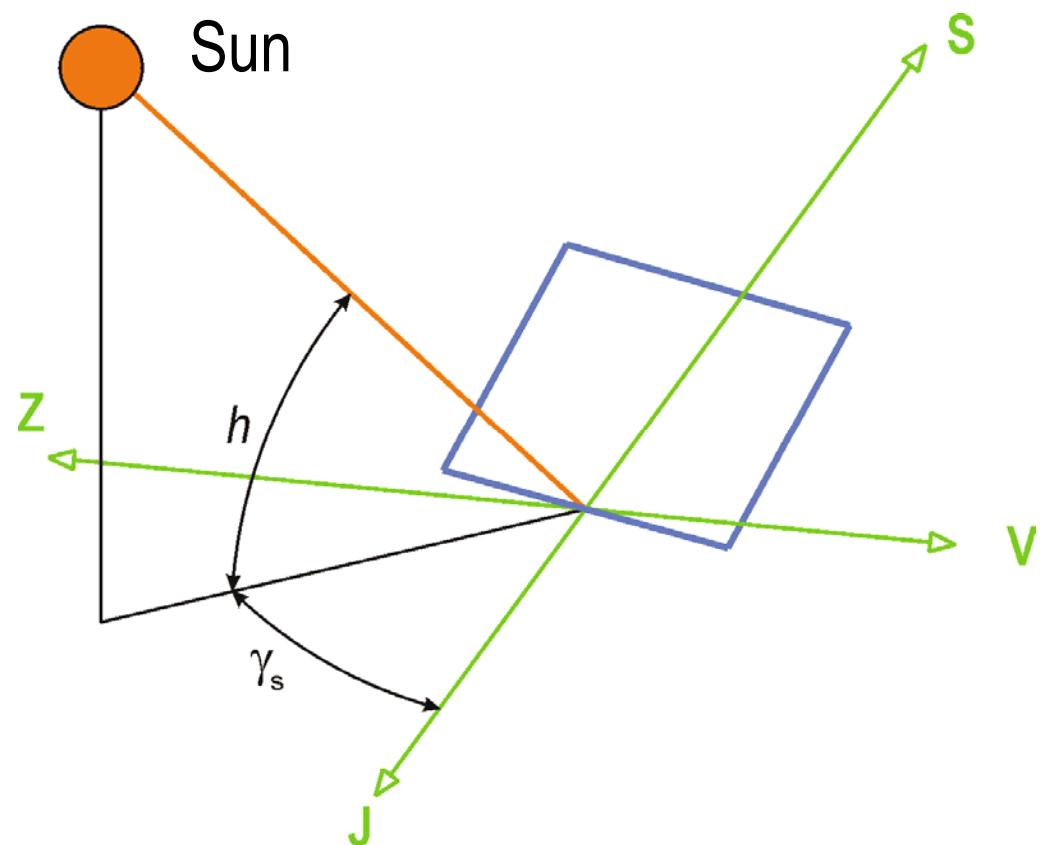
Azimuth of Sun γ_s

- angle between projection of line connecting surface-Sun and local meridian (south)

convention: measured from south

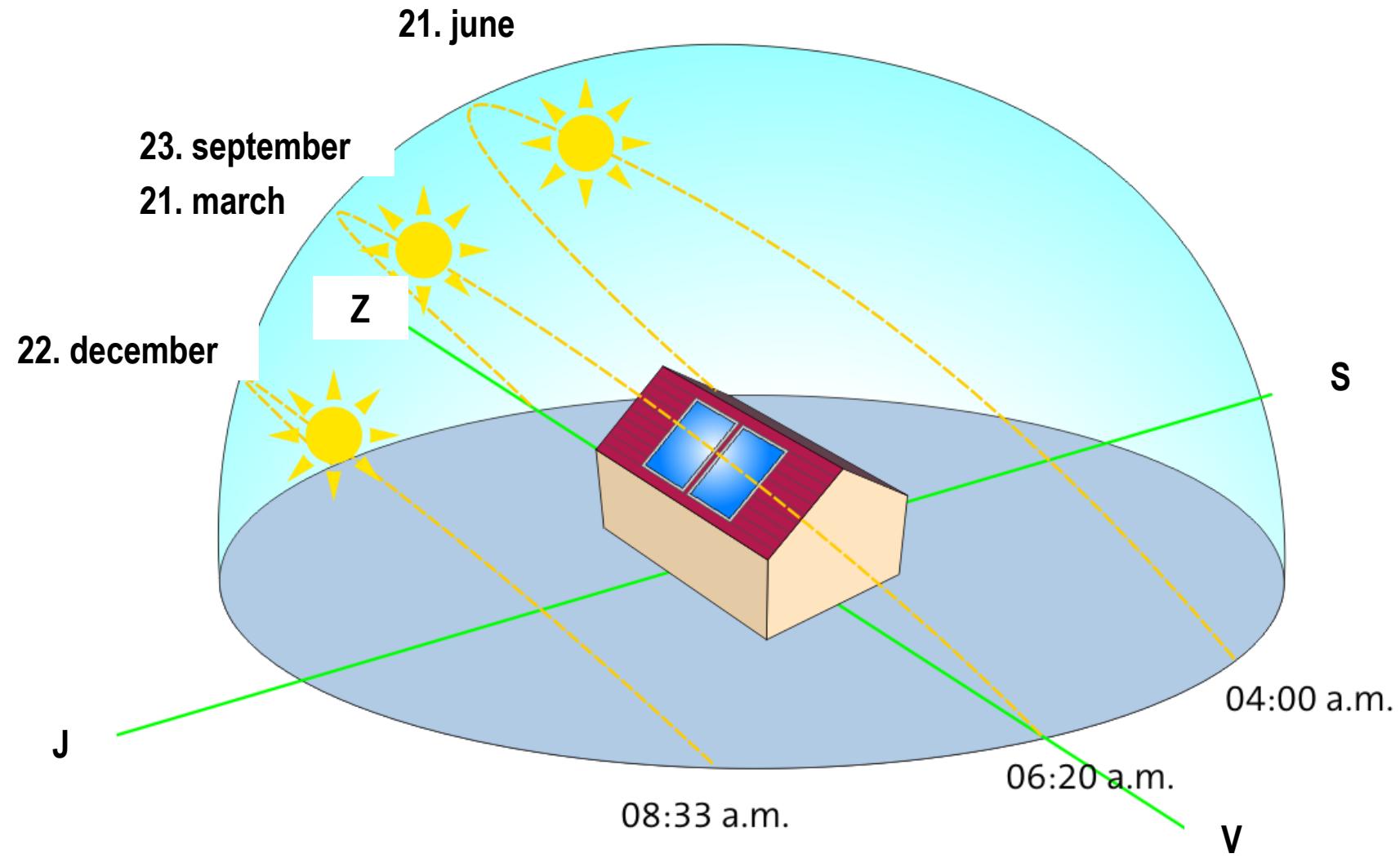
east (-), west (+)

$$\sin \gamma_s = \frac{\cos \delta}{\cos h} \sin \tau$$





Altitude and azimuth of Sun

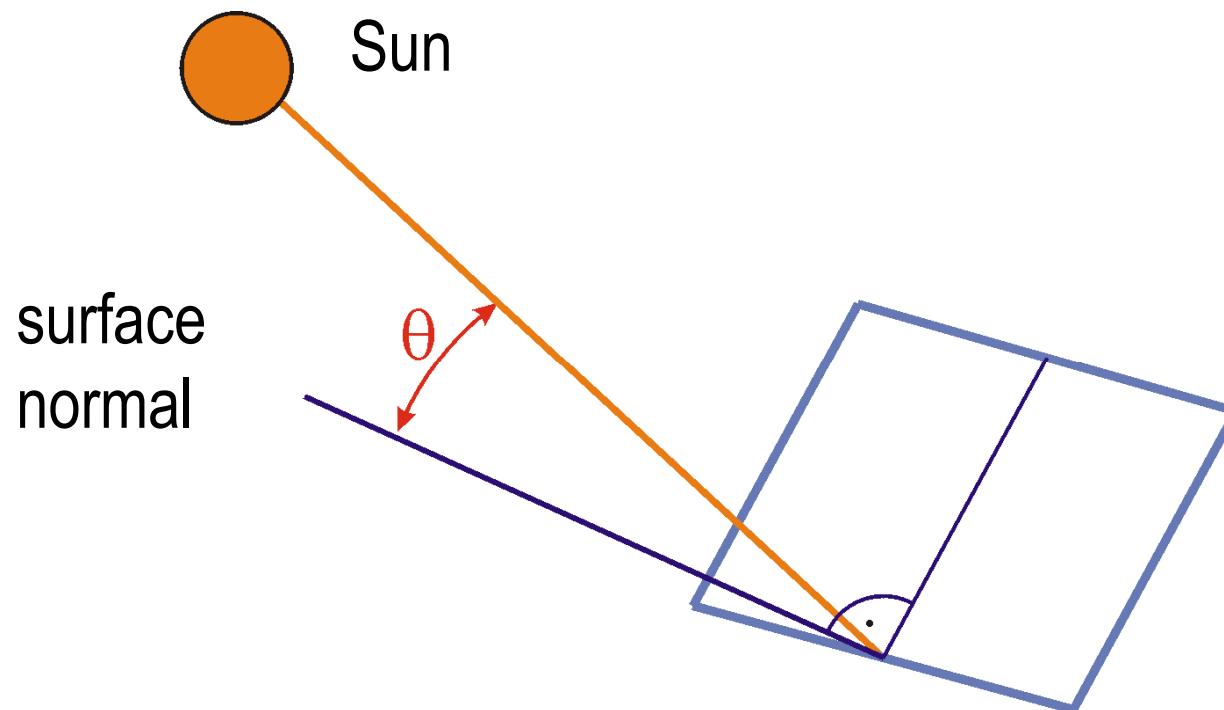




Incidence angle

- angle between line connecting surface-Sun and surface normal

$$\cos \theta = \sin h \cdot \cos \beta + \cos h \cdot \sin \beta \cdot \cos(\gamma_s - \gamma)$$





Solar radiation - definitions

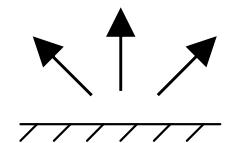
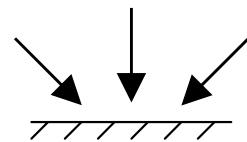
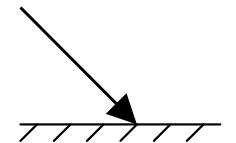
- **solar irradiance G [W/m²]** - radiative *power* incident at area unit, density of solar radiative flux
- **solar irradiation H [kWh/m², J/m²]** – density of radiative *energy*, integral of flux density per time period, e.g. hour, day, ...

$$H = \int_{\tau_1}^{\tau_2} G.d\tau$$



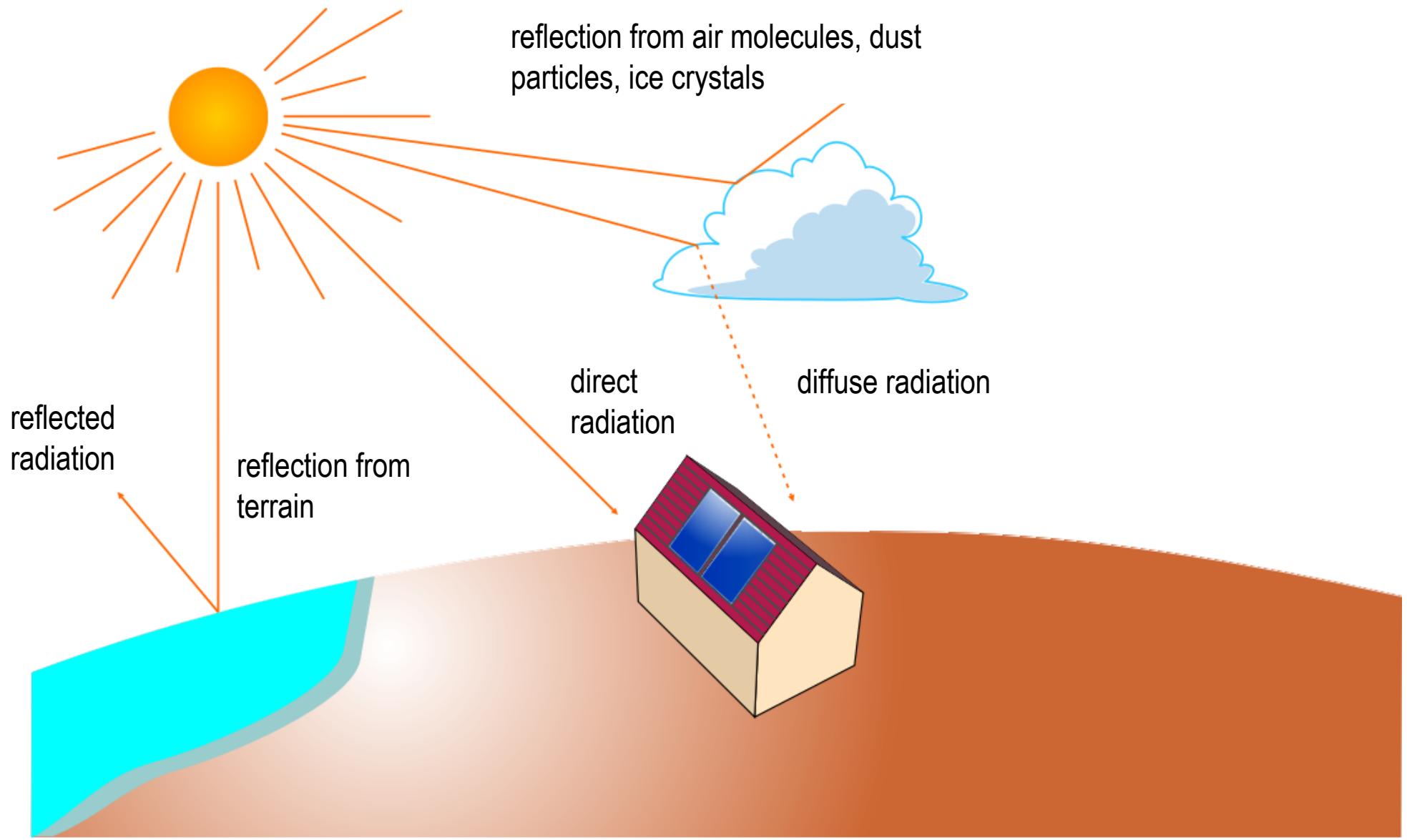
Solar radiation - definitions

- **direct** solar radiation (index „b“, beam) – without scattering in atmosphere
angle dependent, significant intensity in one direction
- **diffuse** solar radiation (index „d“, diffuse) – scattered in atmosphere
all-directions, isotropic: identical intensity in all directions
- **reflected** solar radiation (index „r“, reflected) – reflection from terrain, buildings
usual surfaces reflect diffusively – considered together with diffuse radiation





Solar radiation - definitions





Solar radiation passing the atmosphere

- direct normal solar irradiance (on surface perpendicular to direction of propagation) after passing atmosphere

$$G_{bn} = G_{on} \cdot \exp\left(-\frac{Z}{\varepsilon}\right) \quad [\text{W/m}^2]$$

G_{on} normal solar irradiance above atmosphere

Z attenuation factor

$$\varepsilon = \frac{9,38076 \cdot \left[\sinh + (0,003 + \sin^2 h)^{0,5} \right]}{2,0015 \cdot (1 - L_v \cdot 10^{-4})} + 0,91018$$

h Sun altitude

L_v elevation above sea-level [m]



Attenuation factor

- how many times the clear atmosphere should be „heavier“, to have the same transmissivity as the real polluted atmosphere
- polluted means also water vapor not only dust, emissions, etc.
- gives attenuation of solar flux when passing the real atmosphere

$$Z = \frac{\ln G_{0n} - \ln G_{bn}}{\ln G_{0n} - \ln G_{b0}}$$

G_{b0} direct irradiance after passing completely clear atmosphere (with $Z = 1$)



Attenuation factor

Month	Average monthly values for Z for locations with different environment			
	mountains	countryside	cities	industrial
I.	1,5	2,1	3,1	4,1
II.	1,6	2,2	3,2	4,3
III.	1,8	2,5	3,5	4,7
IV.	1,9	2,9	4,0	5,3
V.	2,0	3,2	4,2	5,5
VI.	2,3	3,4	4,3	5,7
VII.	2,3	3,5	4,4	5,8
VIII.	2,3	3,3	4,3	5,7
IX.	2,1	2,9	4,0	5,3
X.	1,8	2,6	3,6	4,9
XI.	1,6	2,3	3,3	4,5
XII.	1,5	2,2	3,1	4,2
annual average	1,9	2,75	3,75	5,0

simplified:

mountains $Z = 2$

countryside $Z = 3$

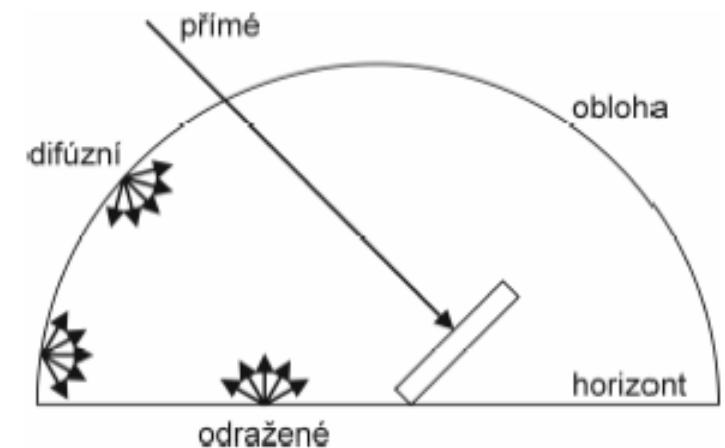
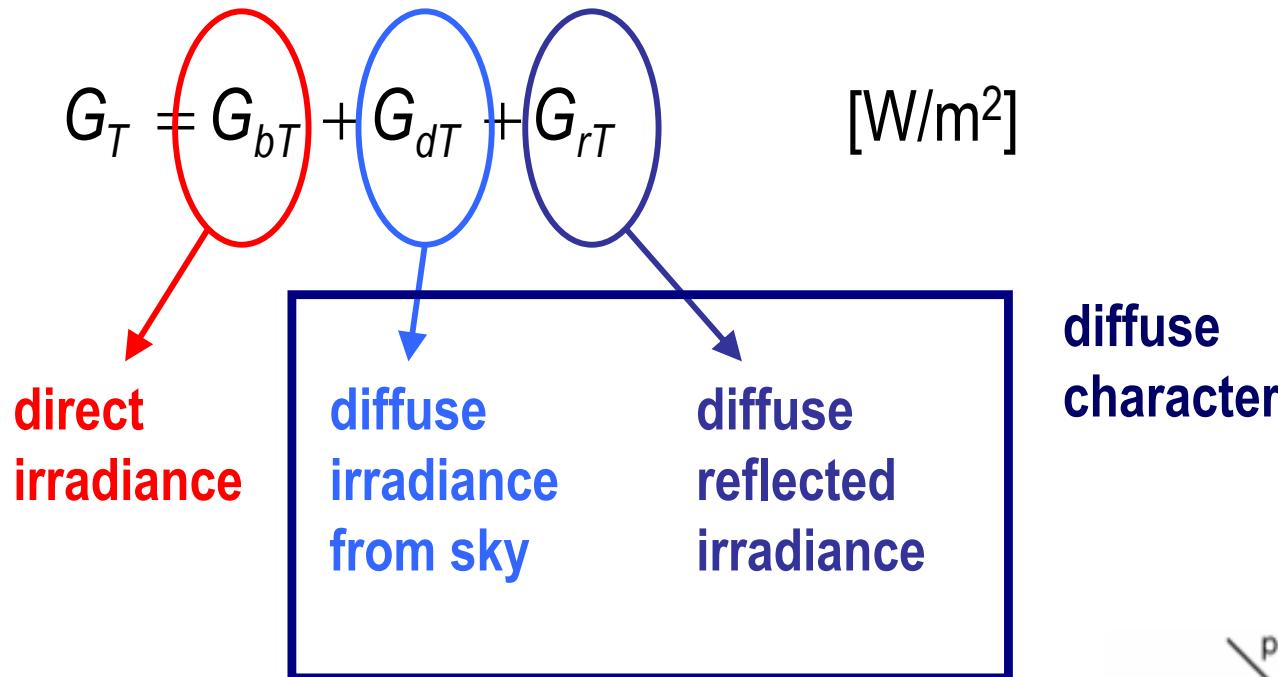
cities $Z = 4$

industrial $Z > 5$



Solar irradiance on general surface

- total solar irradiance on generally sloped and oriented surface





Solar irradiance on general surface

- direct solar irradiance on given surface

$$G_{bT} = G_{bn} \cos \theta = G_b \frac{\cos \theta}{\sin h} = G_b \frac{\cos \theta}{\cos \theta_z} \quad [\text{W/m}^2]$$

- sky diffuse solar irradiance on given surface

$$G_{dT} = \left(\frac{1 + \cos \beta}{2} \right) G_d \quad [\text{W/m}^2]$$

- reflected diffuse solar irradiance on given surface

$$G_{rT} = \rho_g \cdot \left(\frac{1 - \cos \beta}{2} \right) \cdot (G_b + G_d) \quad [\text{W/m}^2]$$



Terrain reflectance (albedo)

- ratio between reflected and incident solar irradiance
- for calculations considered $\rho_g = 0,2$

usual surfaces 0,10 až 0,15

snow 0,90

Earth albedo (planet) 0,30 (average)



Solar irradiance on horizontal plane

- direct solar irradiance on horizontal plane

$$G_b = G_{bn} \sin h \quad [\text{W/m}^2]$$

- diffuse solar irradiance on horizontal plane

$$G_d = 0,33 \cdot (G_{on} - G_{bn}) \cdot \sin h \quad [\text{W/m}^2]$$

simplified model: 1/3 of solar radiatin „lost“ in atmosphere comes to horizontal plane ($\sin h$) as a diffuse isotropic radiation



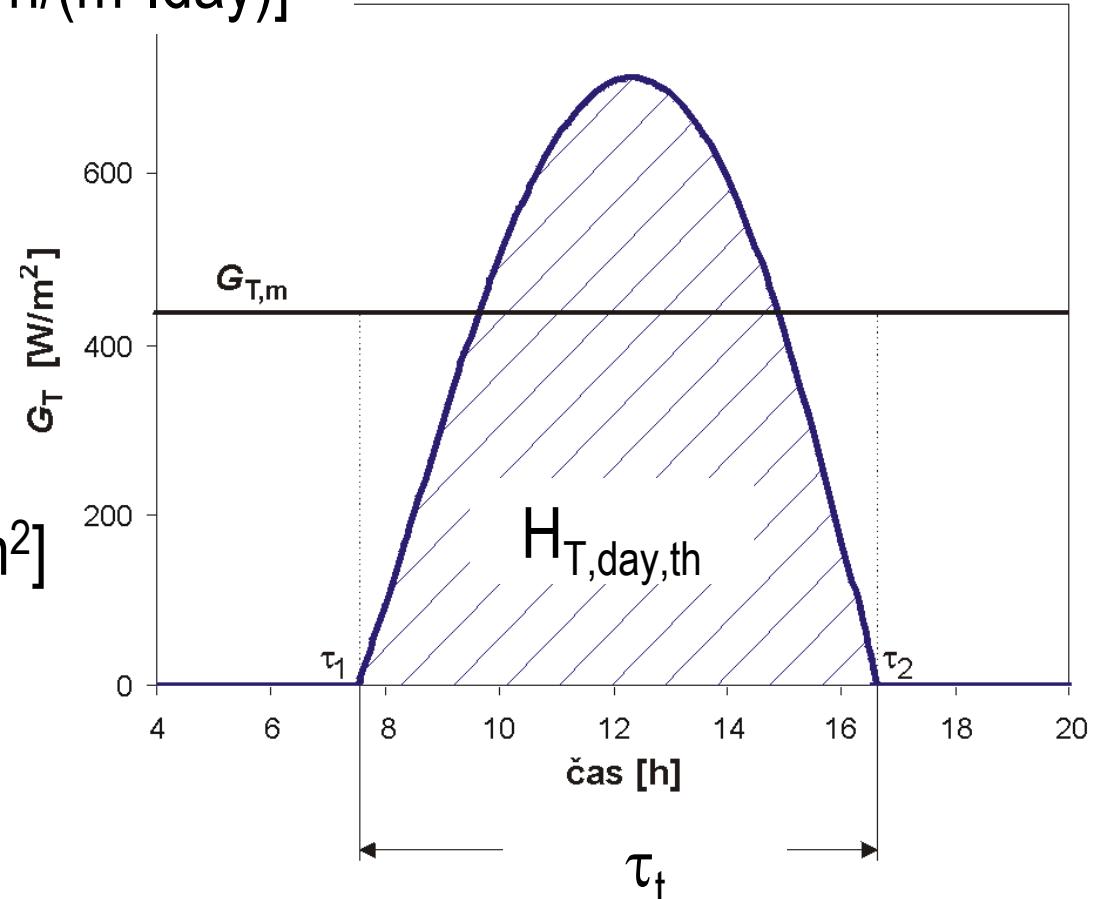
Solar irradiation on general surface

- theoretical daily solar irradiation, integration of irradiance on a plane from sunrise τ_1 to sunset τ_2

$$H_{T,day,th} = \int_{\tau_1}^{\tau_2} G_T d\tau \quad [\text{kWh}/(\text{m}^2 \cdot \text{day})]$$

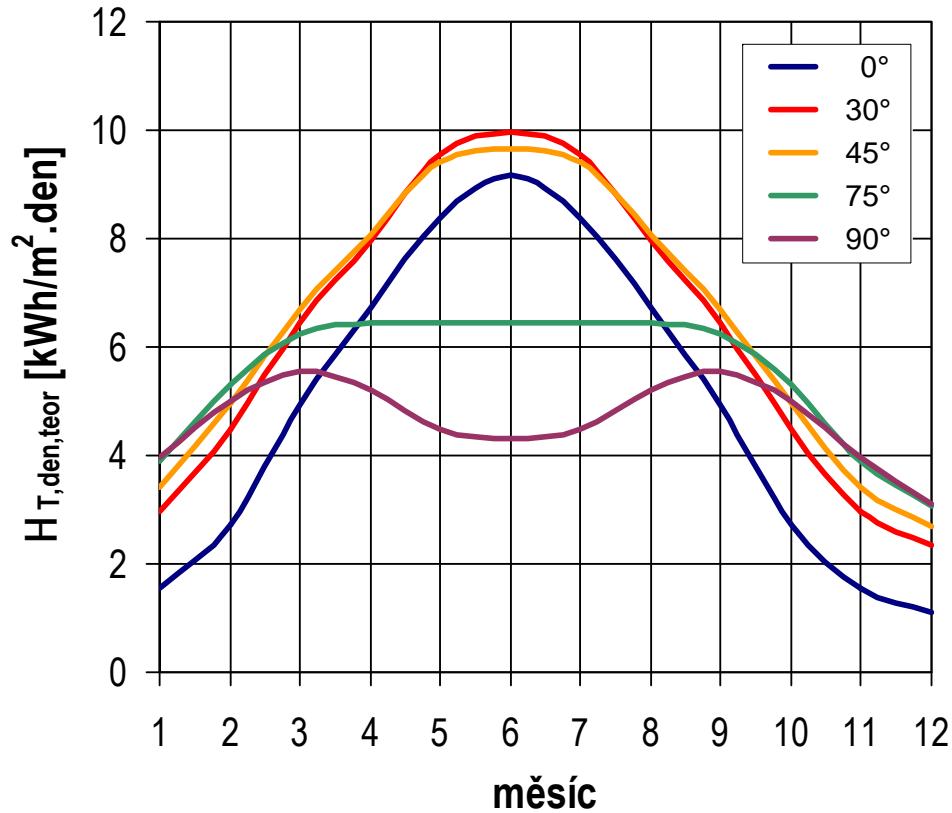
- mean daily solar irradiance

$$G_{T,m} = \frac{H_{T,day,th}}{\tau_t} \quad [\text{W/m}^2]$$





Influence of slope



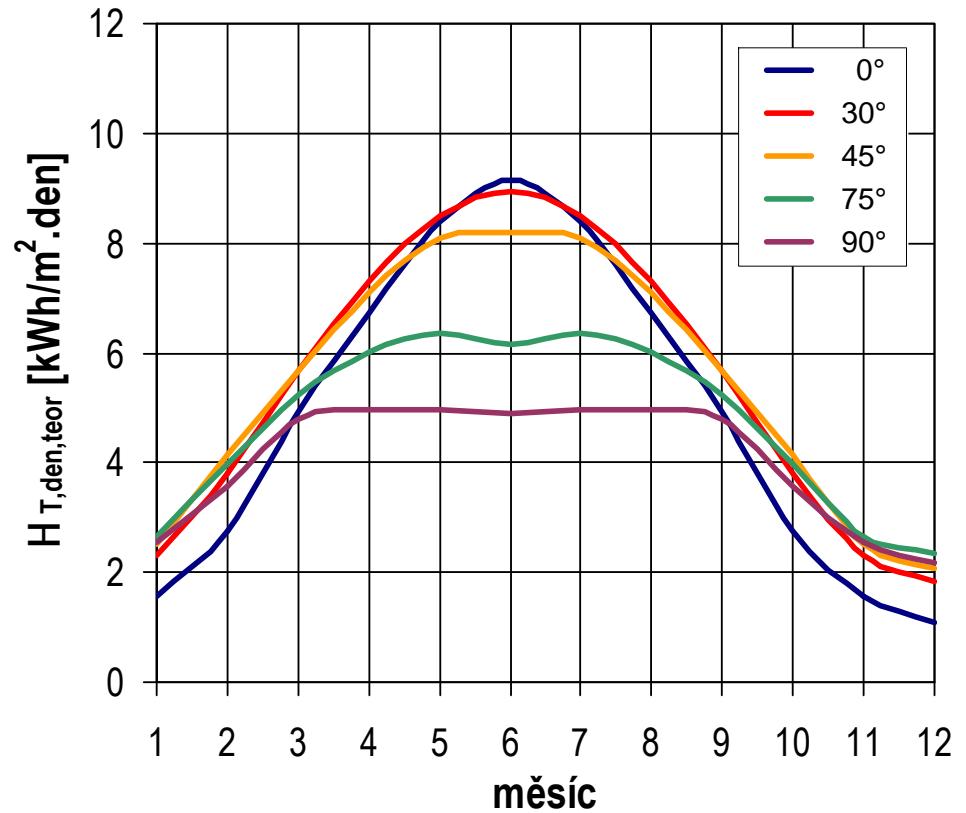
azimuth 0° (south)

optimum slope:

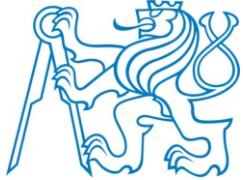
summer 20-30°

winter 75-90°

annual 35-45°



azimuth 45° (SW, SE)



Solar irradiation of general surface

- diffuse daily solar irradiation, integration of diffuse solar irradiance on a plane from sunrise τ_1 to sunset τ_2

$$H_{T,day,dif} = \int_{\tau_1}^{\tau_2} G_{dT} d\tau \quad [\text{kWh}/(\text{m}^2 \cdot \text{day})]$$

 $H_{T,day,th}$
 $H_{T,day,dif}$
 $G_{T,m}$

}

tabbed in literature for given:
slopes, azimuths, locations (attenuation factors)



Real duration of sunshine

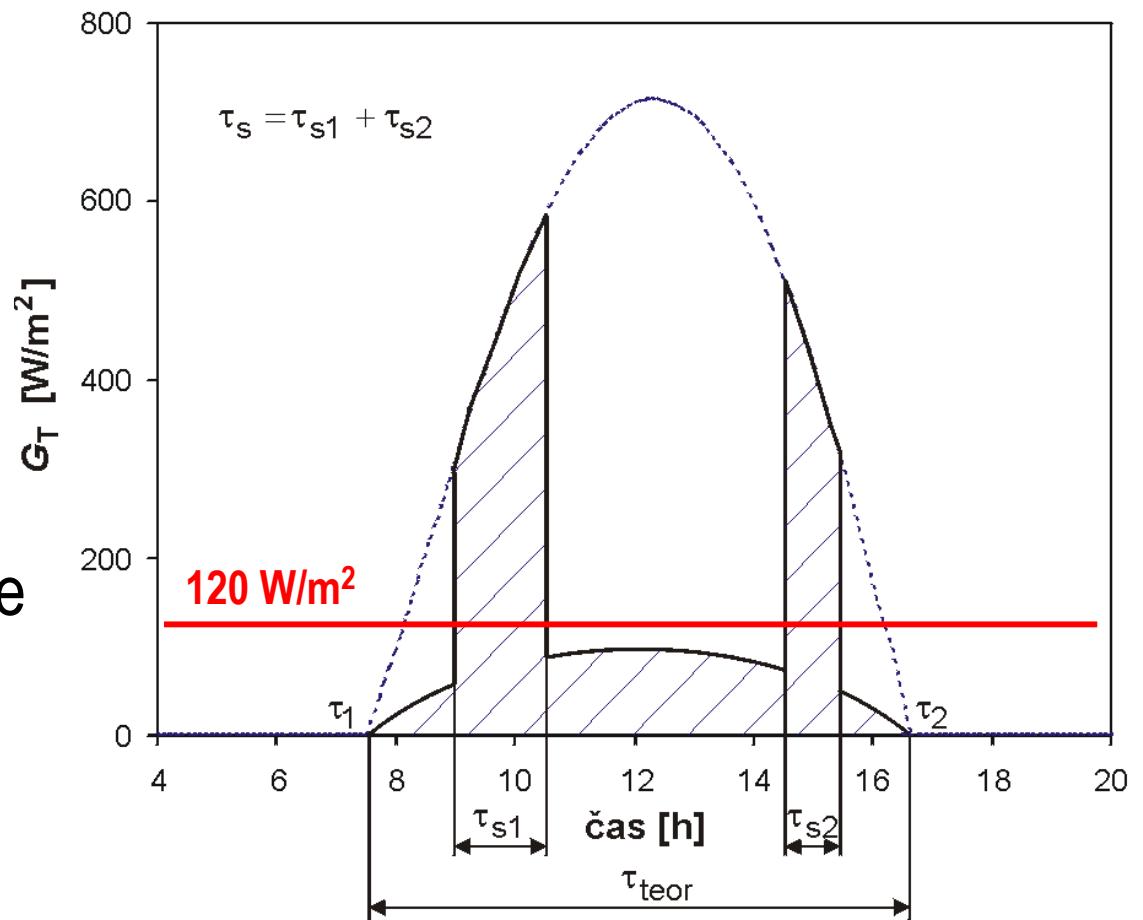
- duration of **direct** solar radiation $> 120 \text{ W/m}^2$

$$\tau_s = \sum_i \tau_{s,i} \quad [\text{h}]$$

meteo-institute (CZ)
presents the values for 22
locations

- relative period of sunshine

$$\tau_r = \frac{\tau_s}{\tau_t} \quad [-]$$



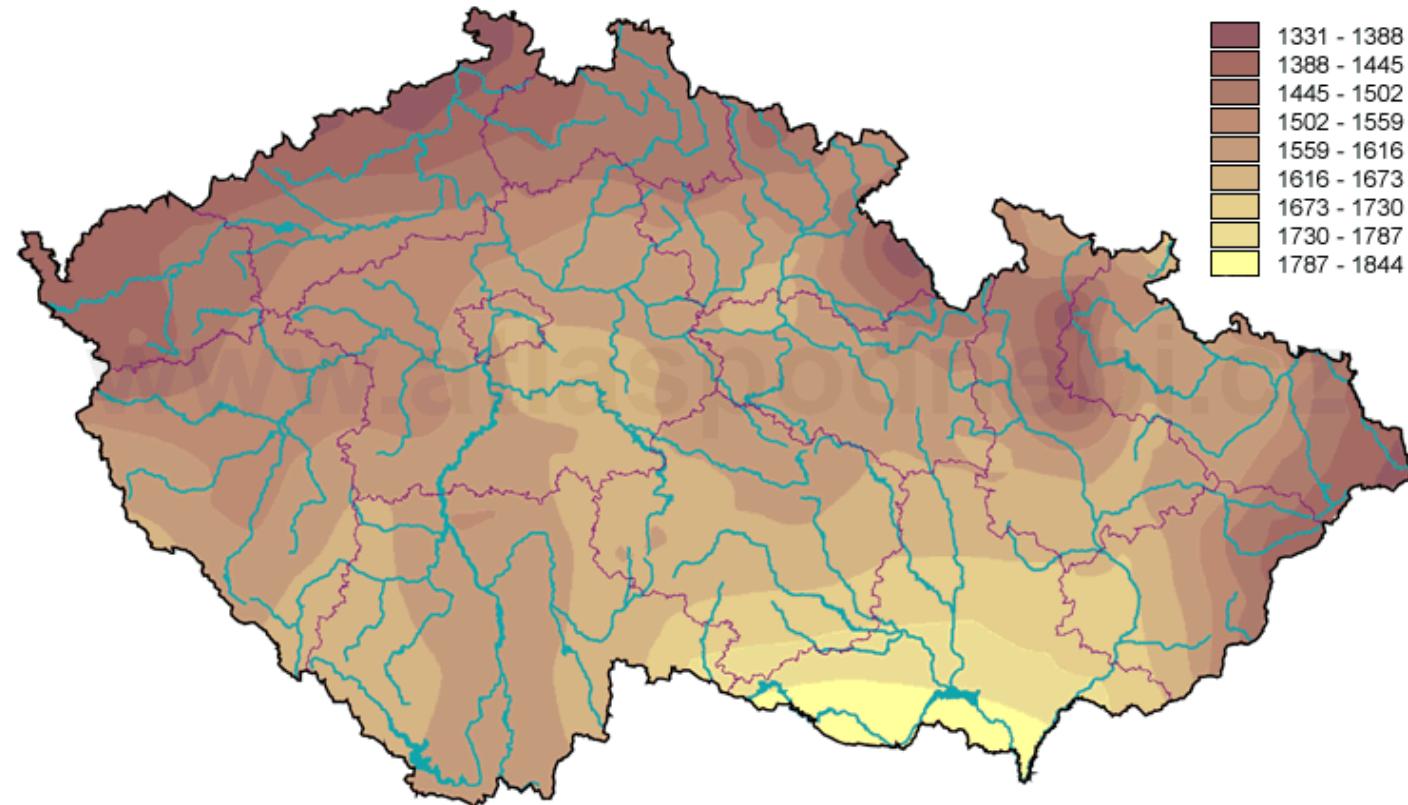


Real duration of sunshine

Month	Real duration of suhshine τ^s [h]			
	Praha	České Budějovice	Hradec Králové	Brno
I.	53	46	47	46
II.	90	82	77	88
III.	157	136	149	142
IV.	187	164	185	163
V.	247	207	241	232
VI.	266	226	249	258
VII.	266	238	252	270
VIII.	238	219	233	230
IX.	190	174	188	179
X.	117	108	115	116
XI.	53	55	48	56
XII.	35	36	42	30
Σ	1 899	1 691	1 826	1 810

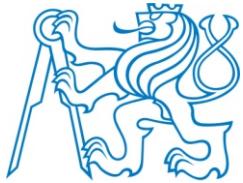


Real duration of sunshine in CZ

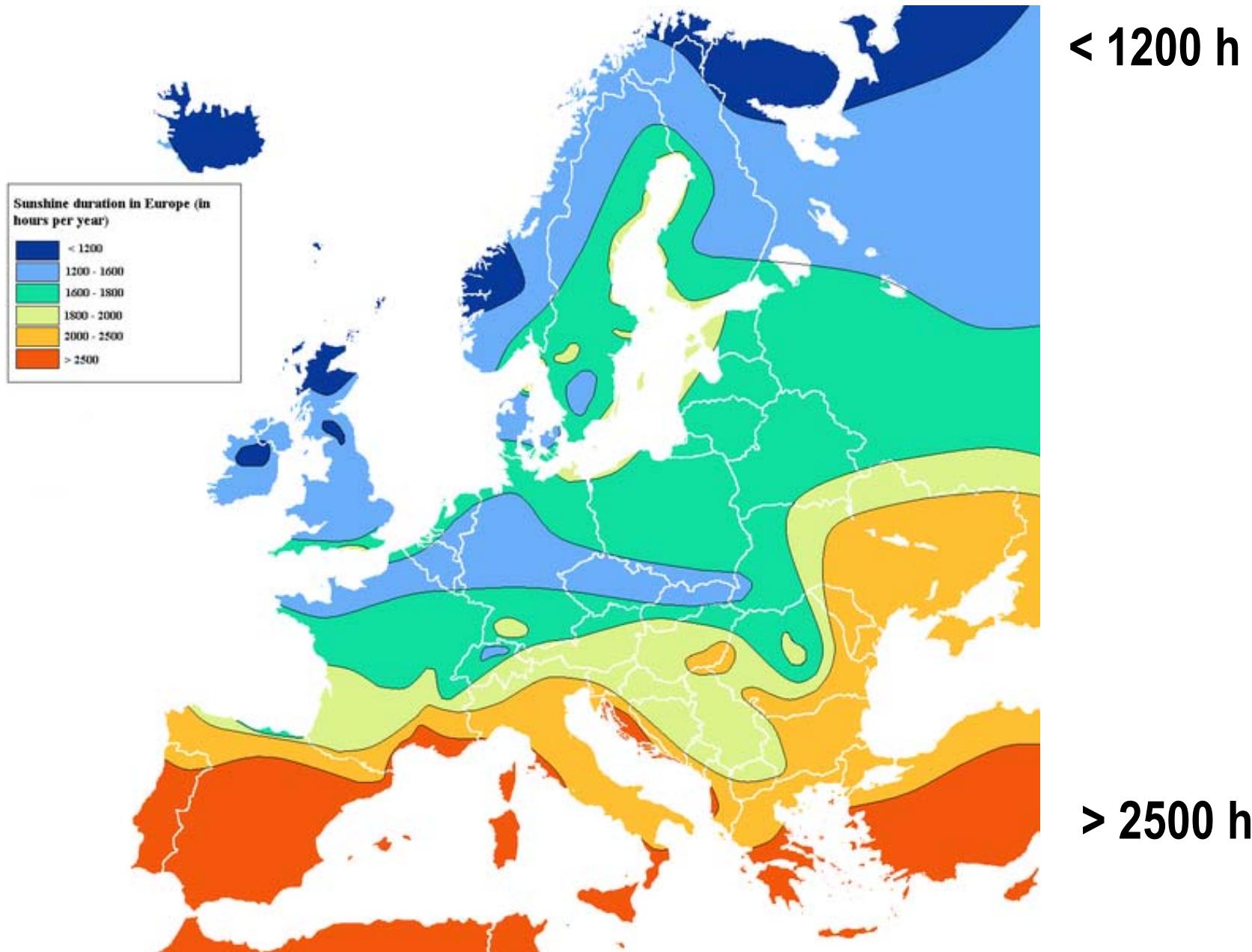


source: ČHMÚ

1400 – 1900 h/year



Real duration of sunshine in Europe





Total irradiation on given plane

- daily solar irradiation

$$H_{T,day} = \tau_r \cdot H_{T,day,th} + (1 - \tau_r) \cdot H_{T,day,dif} \quad [\text{kWh}/(\text{m}^2 \cdot \text{day})]$$

- monthly solar irradiation

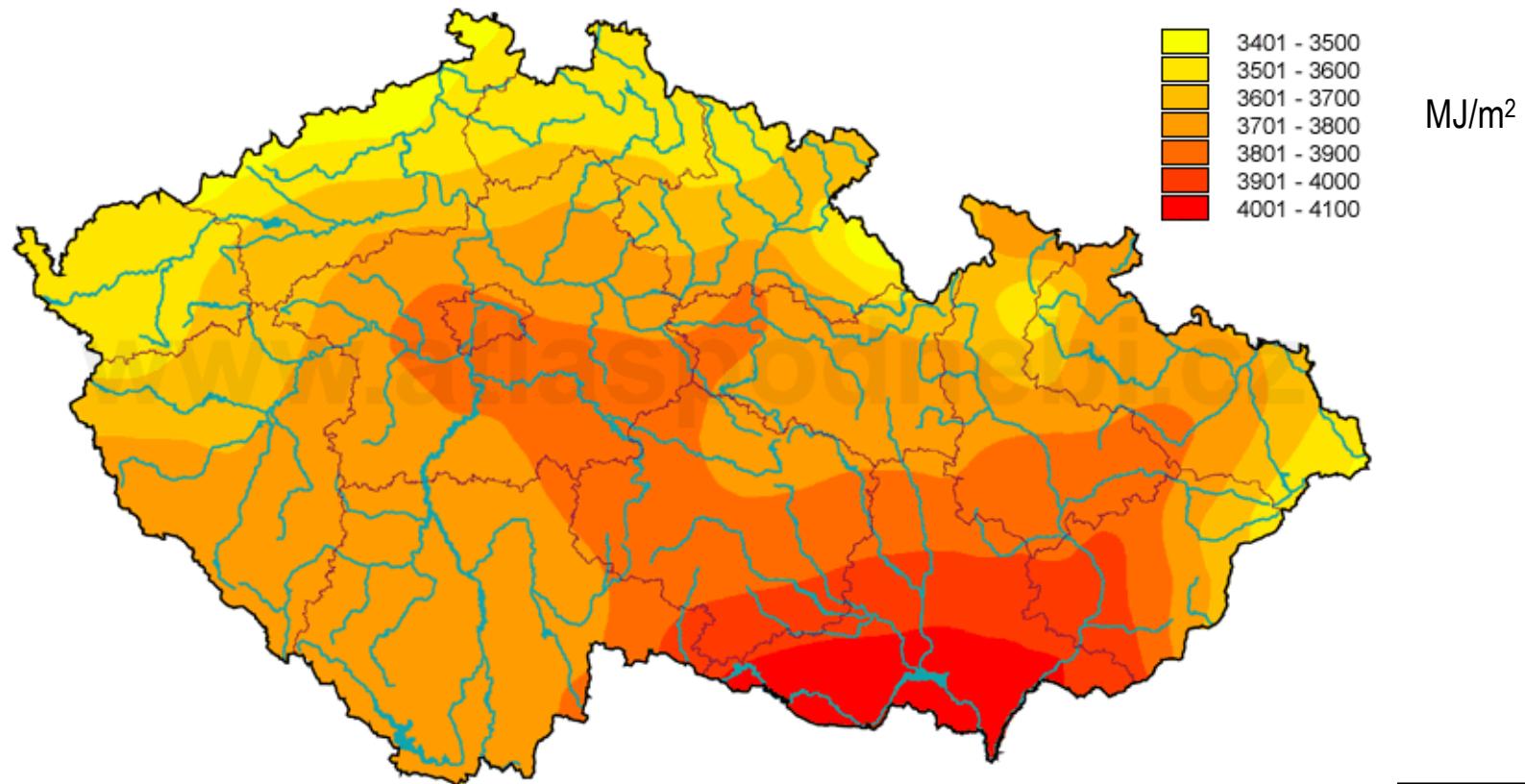
$$H_{T,mon} = n \cdot H_{T,day} \quad [\text{kWh}/(\text{m}^2 \cdot \text{mon})]$$

- annual solar irradiation

$$H_{T,year} = \sum_{I}^{XII} H_{T,mon} \quad [\text{kWh}/(\text{m}^2 \cdot \text{year})]$$



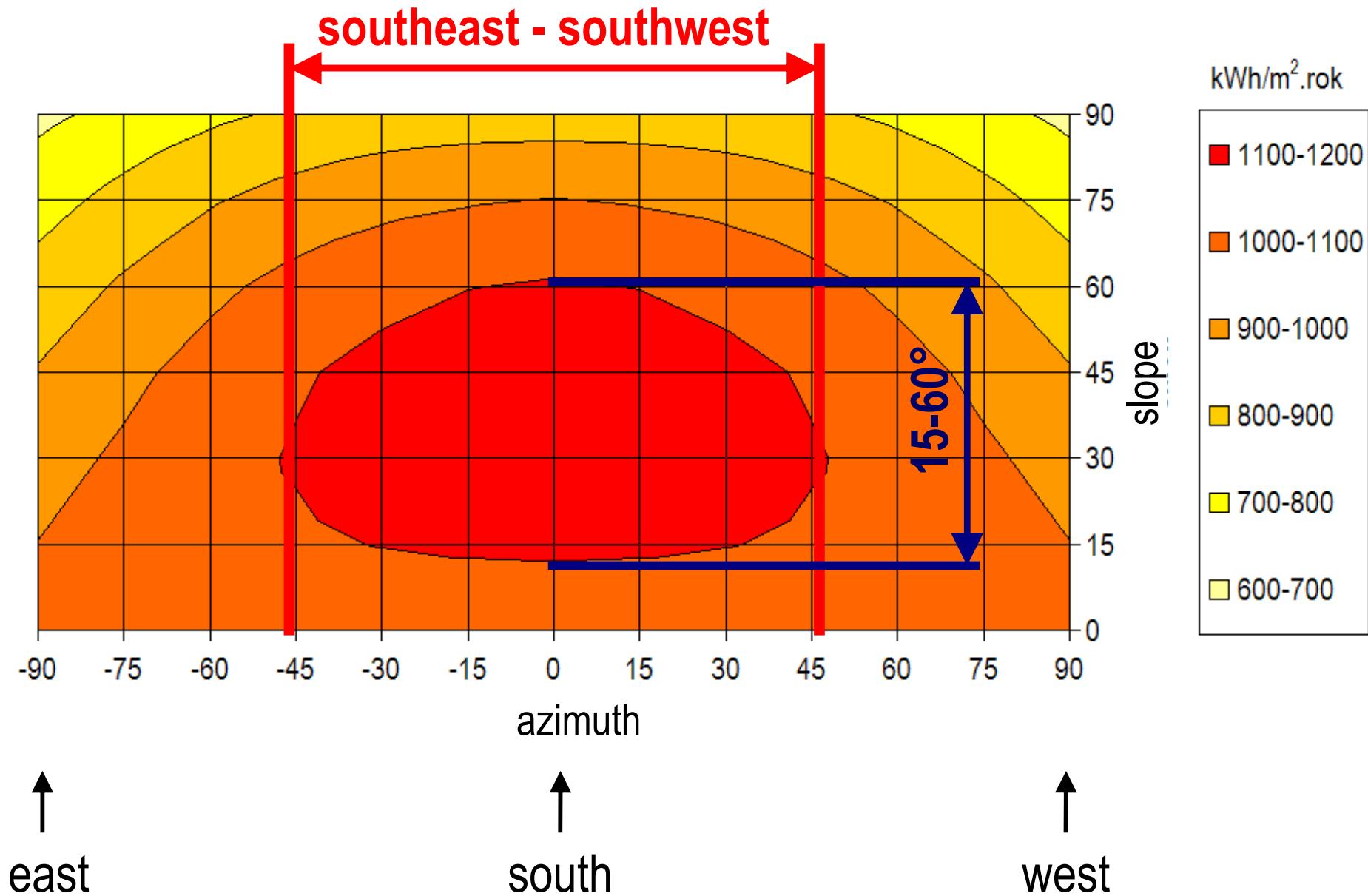
Annual solar irradiation in CZ



- slope 30 až 45°, south orientation: **1000 až 1200 kWh/m²**
- slope 90°, south orientation: **750 až 900 kWh/m²**



Optimum slope for Central Europe ?

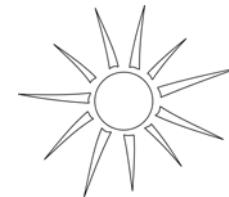




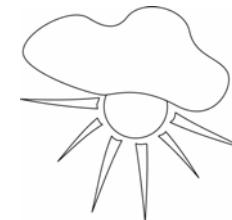
Solar energy: typical values

- **solar irradiance G (power)**

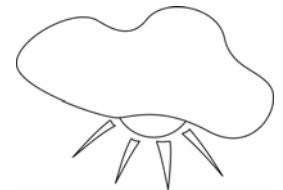
clear sky	800 to 1000 W/m ²
-----------	------------------------------



semibright	400 to 700 W/m ²
------------	-----------------------------



overcast	100 to 300 W/m ²
----------	-----------------------------



- **solar irradiation H (energy)**

winter	3 kWh/(m ² .day)
--------	-----------------------------

spring, autumn	5 kWh/(m ² .day)
----------------	-----------------------------

summer	8 kWh/(m ² .day)
--------	-----------------------------