CHAPTER 3

HEAT TRANSFER

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HEAT TRANSFER PROCESSES

HEAT is energy in transit due to a temperature difference. The thermal energy is transferred from one region to another by three modes of heat transfer: conduction, convection, and radiation. Heat transfer is among a group of energy transport phenomena that includes mass transfer (see Chapter 5), momentum transfer or fluid friction (see Chapter 2), and electrical conduction. Transport phenomena have similar rate equations, in which flux is proportional to a potential difference. In heat transfer by conduction and convection, the potential difference is the temperature difference. Heat, mass, and momentum transfer are often considered together because of their similarities and interrelationship in many common physical processes.

This chapter presents the elementary principles of single-phase heat transfer with emphasis on heating, refrigerating, and air conditioning. Boiling and condensation are discussed in Chapter 4. More specific information on heat transfer to or from buildings or refrigerated spaces can be found in Chapters 25 through 31 of this volume and in Chapter 12 of the ASHRAE Handbook—Refrigeration. Physical properties of substances can be found in Chapters 18, 22, 24, and 36 of this volume and in Chapter 8 of the ASHRAE Handbook—Refrigeration. Heat transfer equipment, including evaporators, condensers, heating and cooling coils, furnaces, and radiators, is covered in the ASHRAE Handbook—Systems and Equipment. For further information on heat transfer, see the section on Bibliography.

HEAT TRANSFER PROCESSES

Thermal Conduction. This is the mechanism of heat transfer whereby energy is transported between parts of a continuum by the transfer of kinetic energy between particles or groups of particles at the atomic level. In gases, conduction is caused by elastic collision of molecules; in liquids and electrically nonconducting solids, it is believed to be caused by longitudinal oscillations of the lattice structure. Thermal conduction in metals occurs, like electrical conduction, through the motion of free electrons. Thermal energy transfer occurs in the direction of decreasing temperature, a consequence of the second law of thermodynamics. In solid opaque bodies, thermal conduction is the significant heat transfer mechanism because no net material flows in the process and radiation is not a factor. With flowing fluids, thermal conduction dominates in the region very close to a solid boundary, where the flow is laminar and parallel to the surface and where there is no eddy motion.

Thermal Convection. This form of heat transfer involves energy transfer by fluid movement and molecular conduction (Biermeister 1983, Kays and Crawford 1980). Consider heat transfer to a fluid flowing inside a pipe. If the Reynolds number is large enough, different flow regions exist. Immediately adjacent to the wall is a laminar sublayer where heat transfer occurs by thermal conduction; outside the laminar sublayer is a transition region called the buffer layer, where both eddy mixing and conduction effects are significant; beyond the buffer layer and extending to the center of the pipe is the turbulent region, where the dominant mechanism of transfer is eddy mixing.

In most equipment, the main body of fluid is in turbulent flow, and the laminar layer exists at the solid walls only. In cases of low-velocity flow in small tubes, or with viscous liquids such as glycol (i.e., at low Reynolds numbers), the entire flow may be laminar with no transition or turbulent region.

When fluid currents are produced by external sources (for example, a blower or pump), the solid-to-fluid heat transfer is termed forced convection. If the fluid flow is generated internally by nonhomogeneous densities caused by temperature variation, the heat transfer is termed free convection or natural convection.

Thermal Radiation. In conduction and convection, heat transfer takes place through matter. In thermal radiation, there is a change in energy form from internal energy at the source to electromagnetic energy for transmission, then back to internal energy at the receiver. Whereas conduction and convection heat transfer rates are driven primarily by temperature difference and somewhat by temperature level, radiative heat transfer rates increase rapidly with temperature levels (for the same temperature difference).

Although some generalized heat transfer equations have been mathematically derived from fundamentals, they are usually obtained from correlations of experimental data. Normally, the correlations employ certain dimensionless numbers, shown in Table 1, that are derived from dimensional analysis or analogy.

STEADY-STATE CONDUCTION

For steady-state heat conduction in one dimension, the Fourier law is

\[ q = - (kA) \frac{dT}{dx} \]  \hspace{1cm} (1)

where

- \( q \) = heat flow rate, W
- \( k \) = thermal conductivity, W/m·K
- \( A \) = cross-sectional area normal to flow, m²
- \( \frac{dT}{dx} \) = temperature gradient, °C/m

Equation (1) states that the heat flow rate \( q \) in the \( x \) direction is directly proportional to the temperature gradient \( \frac{dT}{dx} \) and the cross-sectional area \( A \) normal to the heat flow. The proportionality factor is the thermal conductivity \( k \). The minus sign indicates that the heat flow is positive in the direction of decreasing temperature. Conductivity values are sometimes given in other units, but consistent units must be used in Equation (1).

3.1
3.2

Table 1 Dimensionless Numbers Commonly Used in Heat Transfer

<table>
<thead>
<tr>
<th>Name</th>
<th>Symbol</th>
<th>Valuea</th>
<th>Application</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nusselt number</td>
<td>Nu</td>
<td>$hD/k$, $hL/k$, $q^2/Dh$, or $q^2/Lh$</td>
<td>Natural or forced convection, boiling or condensing</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>Re</td>
<td>$GD/\mu$ or $\nu V L/k$</td>
<td>Forced convection, boiling or condensing</td>
</tr>
<tr>
<td>Prandtl number</td>
<td>Pr</td>
<td>$\mu c_p/k$</td>
<td>Natural or forced convection, boiling or condensing</td>
</tr>
<tr>
<td>Stanton number</td>
<td>St</td>
<td>$hD^2/c_p$</td>
<td>Forced convection</td>
</tr>
<tr>
<td>Grashof number</td>
<td>Gr</td>
<td>$L^3 \alpha g \beta \Delta T \rho$</td>
<td>Natural convection (for ideal gases)</td>
</tr>
<tr>
<td>Fourier number</td>
<td>Fo</td>
<td>$\alpha/\ell^2$</td>
<td>Unsteady-state conduction</td>
</tr>
<tr>
<td>Péclet number</td>
<td>Pe</td>
<td>$Gh \ell/\rho L/k$ or $Re Pr$</td>
<td>Forced convection (small Pr)</td>
</tr>
<tr>
<td>Graetz number</td>
<td>Gz</td>
<td>$Gh \ell^2/Pr Re D L$</td>
<td>Laminar forced convection</td>
</tr>
</tbody>
</table>

*a list of the other symbols used in this chapter appears in the section on Symbols.

Equation (1) may be integrated along a path of constant heat flow rate to obtain

$$q = k_j A_m \frac{\Delta T}{L_m}$$

where

- $A_m =$ mean cross-sectional area normal to flow, m²
- $L_m =$ mean length of heat flow path, m
- $\Delta T =$ overall temperature difference, K
- $R =$ thermal resistance, K/W

Thermal resistance $R$ is directly proportional to the mean length $L_m$ of the heat flow path and inversely proportional to the conductivity $k_j$ and the mean cross-sectional area $A_m$ normal to the flow. Equations for thermal resistances of a few common shapes are given in Table 2. Mathematical solutions to many heat conduction problems are addressed by Carslaw and Jaeger (1959). Complicated problems can be solved by graphical or numerical methods such as described by Croll and Lilley (1977), Adams and Rogers (1973), and Patankar (1980).

Analogy to Electrical Conduction. Equation (2) is analogous to Ohm’s law for electrical circuits: thermal current (heat flow) in a thermal circuit is directly proportional to the thermal potential (temperature difference) and inversely proportional to the thermal resistance. This electrical-thermal analogy can be used for heat conduction in complex shapes that resist solution by exact analytical means. The thermal circuit concept is also useful for problems involving combined conduction, convection, and radiation.

OVERALL HEAT TRANSFER

In most steady-state heat transfer problems, more than one heat transfer mode is involved. The various heat transfer coefficients may be combined into an overall coefficient so that the total heat transfer can be calculated from the terminal temperatures. The solution to this problem is much simpler if the concept of a thermal circuit is employed.

Local Overall Heat Transfer Coefficient—Resistance Method

Consider heat transfer from one fluid to another by a three-step steady-state process: from a warmer fluid to a solid wall, through the wall, then to a colder fluid. An overall heat transfer coefficient $U$ based on the difference between the bulk temperatures $t_1 - t_2$ of the two fluids is defined as follows:

$$q = UA(t_1 - t_2)$$

where $A$ is the surface area. Because Equation (3) is a definition of $U$, the surface area $A$ on which $U$ is based is arbitrary; it should always be specified in referring to $U$.

Table 2 Solutions for Some Steady-State Thermal Conduction Problems

<table>
<thead>
<tr>
<th>System</th>
<th>$R$ in Equation $q = \Delta T R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plot wall or curved wall if curvature is small (wall thickness less than 0.1 of inside diameter)</td>
<td>$R = \frac{L}{k A}$</td>
</tr>
<tr>
<td>Radial flow through a right circular cylinder</td>
<td>$R = \frac{\ln(r_f/r_i)}{2\pi k L}$</td>
</tr>
<tr>
<td>Buried cylinder</td>
<td>$R = \frac{\ln \left[ (a + \sqrt{a^2 + r_i^2})/r_i \right]}{2\pi k L}$ or $R = \frac{\cosh^{-1}(a/r_i)}{2\pi k L}$ for $L &gt; 2r_i$</td>
</tr>
<tr>
<td>Radial flow in a hollow sphere</td>
<td>$R = \frac{(1/r_f - 1/r_i)}{4\pi k}$</td>
</tr>
</tbody>
</table>

$L, r, a, \alpha =$ dimensions, m
$k =$ thermal conductivity at average material temperature, W/(m·K)
$A =$ surface area, m²

The temperature drops across each part of the heat flow path are

- $t_1 - t_{12} = q R_1$
- $t_{12} - t_{23} = q R_2$
- $t_{23} - t_2 = q R_3$

where $t_{12}$ and $t_{23}$ are the warm and cold surface temperatures of the wall, respectively, and $R_1$, $R_2$, and $R_3$ are the thermal resistances. Because the same quantity of heat flows through each thermal resistance, these equations combined yield the following:
Heat Transfer

\[ \frac{t_1 - t_2}{q} = \frac{1}{UA} = R_1 + R_2 + R_3 \]  
(4)

As shown above, the equations are analogous to those for electrical circuits; for thermal current flowing through \( n \) resistances in series, the resistances are additive:

\[ R_o = R_1 + R_2 + R_3 + \ldots + R_n \]  
(5)

Similarly, conductance is the reciprocal of resistance, and for heat flow through resistances in parallel, the conductances are additive:

\[ C = \frac{1}{R_o} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots + \frac{1}{R_n} \]  
(6)

For convection, the thermal resistance is inversely proportional to the convection coefficient \( h_c \) and the applicable surface area:

\[ R_c = \frac{1}{h_cA} \]  
(7)

The thermal resistance for radiation is written similarly to that for convection:

\[ R_r = \frac{1}{h_rA} \]  
(8)

The radiation coefficient \( h_r \) is a function of the temperatures, radiation properties, and geometrical arrangement of the enclosure and the body in question.

**Resistance Method Analysis.** Analysis by the resistance method can be illustrated by considering heat transfer from air outside to cold water inside an insulated pipe. The temperature gradients and the nature of the resistance analysis are shown in Figure 1.

Because air is sensibly transparent to radiation, some heat transfer occurs by both radiation and convection to the outer insulation surface. The mechanisms act in parallel on the air side. The total transfer then passes through the insulating layer and the pipe wall by thermal conduction, and then by convection and radiation into the cold water stream. (Radiation is not significant on the water side because liquids are sensibly opaque to radiation, although water transmits energy in the visible region.) The contact resistance between the insulation and the pipe wall is assumed negligible.

The heat transfer rate \( q_{rc} \) for a given length \( L \) of pipe may be thought of as the sum of the rates \( q_r \) and \( q_c \) flowing through the parallel resistances \( R_r \) and \( R_c \) associated with the surface radiation and convection coefficients. The total flow then proceeds through the resistance \( R_3 \) offered to thermal conduction by the insulation, through the pipe wall resistance \( R_2 \), and into the water stream through the convection resistance \( R_1 \). Note the analogy to direct-current electricity. A temperature (potential) drop is required to overcome resistances to the flow of thermal current. The total resistance to heat transfer \( R_o \) is the sum of the individual resistances:

\[ R_o = R_1 + R_2 + R_3 + R_s \]  
(9)

where the resultant parallel resistance \( R_s \) is obtained from

\[ \frac{1}{R_s} = \frac{1}{R_1} + \frac{1}{R_2} \]  
(10)

If the individual resistances can be evaluated, the total resistance can be obtained from this relation. The heat transfer rate for the length of pipe \( L \) can be established by

\[ q_{rc} = \frac{l_c - t}{R_o} \]  
(11)

For a unit length of the pipe, the heat transfer rate is

\[ \frac{q_{rc}}{L} = \frac{l_c - t}{R_o L} \]  
(12)

The temperature drop \( \Delta T \) through each individual resistance may then be calculated from the relation:

\[ \Delta T_n = R_n q_{rc} \]  
(13)

where \( n \) = 1, 2, and 3.

**Mean Temperature Difference**

When heat is exchanged between two fluids flowing through a heat exchanger, the local temperature difference \( \Delta T \) varies along the flow path. Heat transfer may be calculated using

\[ q = UA \Delta T \]  
(14)

where \( U \) is the overall uniform coefficient of heat transfer from fluid to fluid, \( A \) is the area associated with the coefficient \( U \), and \( \Delta T \) is the appropriate mean temperature difference.

For parallel flow or countercurrent exchangers and for any exchanger in which one fluid temperature is substantially constant, the mean temperature difference is

\[ \Delta T = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)} = \frac{\Delta T_1 - \Delta T_2}{2.3 \log(\Delta T_1 / \Delta T_2)} \]  
(15)

where \( \Delta T_1 \) and \( \Delta T_2 \) are the temperature differences between the fluids at each end of the heat exchanger, \( \Delta T \) is called the logarithmic mean temperature difference. For the special case of \( \Delta T_1 = \Delta T_2 \) (possible only with a countercurrent heat exchanger with equal capacities), which leads to an indeterminate form of Equation (15), \( \Delta T = \Delta T_1 = \Delta T_2 \).

Equation (15) for \( \Delta T \) is true only if the overall coefficient and the specific heat of the fluids are constant through the heat exchanger, and any heat losses occur (often well-approximated in practice). Parker et al. (1969) give a procedure for cases with variable overall coefficient \( U \).
Calculations using Equation (14) and $\Delta T_{in}$ are convenient when terminal temperatures are known. In many cases, however, the temperatures of the fluids leaving the exchanger are not known. To avoid trial-and-error calculations, an alternate method involves the use of three nondimensional parameters, defined as follows:

1. **Exchanger Heat Transfer Effectiveness $\varepsilon$**

   \[
   \varepsilon = \begin{cases} 
   \frac{(t_{hi} - t_{ho})}{(t_{hi} - t_{ci})} & \text{when } C_b = C_{min} \\
   \frac{(t_{co} - t_{ci})}{(t_{hi} - t_{ci})} & \text{when } C_c = C_{min} 
   \end{cases} 
   \]  
   \[ (16) \]

   where
   
   $C_b = \frac{(m_c)_{b}}{C_{bat}}$ is hot fluid capacity rate, W/K
   
   $C_c = \frac{(m_c)_{c}}{C_{cat}}$ is cold fluid capacity rate, W/K
   
   $C_{min}$ is smaller of capacity rates $C_b$ and $C_c$
   
   $t_{hi}$ is terminal temperature of hot fluid, °C
   
   $t_{ci}$ is terminal temperature of cold fluid, °C

2. **Number of Exchanger Heat Transfer Units (NTU)**

   \[ NTU = \frac{A U_{air}}{C_{min}} = \frac{1}{C_{min}} \int_A U \, dA \]  
   \[ (17) \]

   where $A$ is the area used to define overall coefficient $U$.

3. **Capacity Rate Ratio $Z$**

   \[ Z = \frac{C_{min}}{C_{max}} \]  
   \[ (18) \]

For a given exchanger, the heat transfer effectiveness can generally be expressed for a given exchanger as a function of the number of transfer units and the capacity ratio:

\[ \varepsilon = f(NTU, Z, \text{flow arrangement}) \]  
\[ (19) \]

The effectiveness is independent of the temperatures in the exchanger. For any exchanger in which the capacity rate ratio $Z$ is zero (where one fluid undergoes a phase change; e.g., in a condenser or evaporator), the effectiveness is

\[ \varepsilon = 1 - \exp\{ -\text{NTU} \} \]  
\[ (20) \]

Heat transferred can be determined from

\[ q = C_{b}(t_{hi} - t_{ho}) = C_{c}(t_{co} - t_{ci}) \]  
\[ (21) \]

Combining Equations (16) and (21) produces an expression for heat transfer rate in terms of entering fluid temperatures:

\[ q = \varepsilon C_{min}(t_{hi} - t_{ci}) \]  
\[ (22) \]

The proper mean temperature difference for Equation (14) is then given by

\[ \Delta T_{in} = \frac{(t_{hi} - t_{ci})\varepsilon}{NTU} \]  
\[ (23) \]

The effectiveness for parallel flow exchangers is

\[ \varepsilon = \frac{1 - \exp\{ -\text{NTU}(1 + Z) \}}{1 + Z} \]  
\[ (24) \]

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For $Z = 1$,

\[ \varepsilon = \frac{1 - \exp\{ -2 \text{ NTU} \}}{2} \]  
\[ (25) \]

The effectiveness for counterflow exchangers is

\[ \varepsilon = \frac{1 - \exp\{ -\text{NTU}(1 - Z) \}}{1 - Z \exp\{ -\text{NTU}(1 - Z) \}} \]  
\[ (26) \]

\[ \varepsilon = \frac{\text{NTU}}{1 + \text{NTU}} \text{ for } Z = 1 \]  
\[ (27) \]

Incropera and DeWitt (1996) and Kays and London (1984) show the relations of $\varepsilon$, NTU, and $Z$ for other flow arrangements. These authors and Afgan and Schlunder (1974) present graphical representations for convenience.

**TRANSPORT HEAT FLOW**

Often, the heat transfer and temperature distribution under unsteady-state (varying with time) conditions must be known. Examples are (1) cold storage temperature variations on starting or stopping a refrigeration unit; (2) variation of external air temperature and solar irradiation affecting the heat load of a cold storage room or wall temperatures; (3) the time required to freeze a given material under certain conditions in a storage room; (4) quick freezing of objects by direct immersion in brines; and (5) sudden heating or cooling of fluids and solids from one temperature to a different temperature.

The equations describing transient temperature distribution and heat transfer are presented in this section. Numerical methods are the simplest means of solving these equations because numerical data are easy to obtain. However, with some numerical solutions and off-the-shelf software, the physics that drives the energy transport can be lost. Thus, analytical solution techniques are also included in this section.

The fundamental equation for unsteady-state conduction in solids or fluids in which there is no substantial motion is

\[ \frac{\partial T}{\partial t} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \]  
\[ (28) \]

where thermal diffusivity $\alpha$ is the ratio $k/c_p \rho$; $k$ is thermal conductivity; $\rho$, density; and $c_p$, specific heat. If $\alpha$ is large (high conductivity, low density and specific heat, or both), heat will diffuse faster.

One of the most elementary transient heat transfer models predicts the rate of temperature change of a body or material being held at constant volume with uniform temperature, such as a well-stirred reservoir of fluid whose temperature is changing because of a net rate of heat gain or loss:

\[ q_{net} = (M c_p) \frac{dT}{dt} \]  
\[ (29) \]

where $M$ is the mass of the body, $c_p$ is the specific heat at constant pressure, and $q_{net}$ is the net heat transfer rate to the substance (heat transfer into the substance is positive, and heat transfer out of the substance is negative). Equation (29) is applicable when the pressure around the substance is constant; if the volume of the substance is constant, $c_p$ should be replaced by the constant volume specific heat $c_v$. It should be noted that with the density of solids and liquids being almost constant, the two specific heats are almost equal. The term $q_{net}$ may include heat transfer by conduction, convection, or radiation and is the difference between the heat transfer rates into and out of the body.
Heat Transfer

From Equations (28) and (29), it is possible to derive expressions for temperature and heat flow variations at different instants and different locations. Most common cases have been solved and presented in graphical forms (Jakob 1957, Schneider 1964, Myers 1971). In other cases, it is simpler to use numerical methods (Croft and Lilley 1977, Patankar 1980). When convective boundary conditions are required in the solution of Equations (28) and (29), it values based on steady-state correlations are often used. However, this approach may not be valid when rapid transients are involved.

Estimating Cooling Times

Cooling times for materials can be estimated (McAdams 1954) by Gurnie-Lurie charts (Figures 2, 3, and 4), which are graphical solutions for the heating or cooling of infinite slabs, infinite cylinders, and spheres. These charts assume an initial uniform temperature distribution and no change of phase. They apply to a body exposed to a constant temperature fluid with a constant surface convection coefficient of \( h \).

Using Figures 2, 3, and 4, it is possible to estimate both the temperature at any point and the average temperature in a homogeneous mass of material as a function of time in a cooling process. It is possible to estimate cooling times for rectangular-shaped solids, cubes, cylinders, and spheres.

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From the point of view of heat transfer, a cylinder insulated on its ends behaves like a cylinder of infinite length, and a rectangular solid insulated so that only two parallel faces allow heat transfer behaves like an infinite slab. A thin slab or a long, thin cylinder may be considered infinite objects.

Consider a slab having insulated edges being cooled. If the cooling time is the time required for the center of the slab to reach a temperature of \( t_c \), the cooling time can be calculated as follows:

1. Evaluate the temperature ratio \( \left( \frac{t_c - t_f}{t_i - t_f} \right) \).

where

- \( t_c \) = temperature of cooling medium
- \( t_i \) = initial temperature of product
- \( t_f \) = final temperature of product at center

Note that in Figures 2, 3, and 4, the temperature ratio \( \left( \frac{t_c - t_f}{t_i - t_f} \right) \) is designated as \( Y \) to simplify the equations.

2. Determine the radius ratio \( r/r_m \) designated as \( n \) in Figures 2, 3, and 4.

where

- \( r \) = distance from centerline
- \( r_m \) = half thickness of slab

3. Evaluate the resistance ratio \( k/\alpha r_m \) designated as \( m \) in Figures 2, 3, and 4.

where

- \( k \) = thermal conductivity of material
- \( \alpha \) = heat transfer coefficient

4. From Figure 2 for infinite slabs, select the appropriate value of \( k/\alpha cp r_m^2 \) designated as \( F_p \) in Figures 2, 3, and 4.

where

- \( \tau \) = time elapsed
- \( \alpha \) = specific heat
- \( \rho \) = density

5. Determine \( r_m \) from the value of \( k/\alpha cp r_m^2 \).

Multidimensional Temperature Distribution

The solution for semi-infinite slabs and cylinders (shown in Figures 2, 3, and 4) can be used to find the temperatures in finite rectangular solids or cylinders.
Fig. 3 Transient Temperatures for Infinite Cylinder

The temperature in the finite object can be calculated from the temperature ratio \( \Phi \) of the infinite objects that intersect to form the finite object. The product of the temperature ratios of the infinite objects is the temperature ratio of the finite object; for example, for the finite cylinder of Figure 5,

\[
Y_{fc} = Y_{i3} Y_{ic}
\]

(30)

where

- \( Y_{fc} \) = temperature ratio of finite cylinder
- \( Y_{i3} \) = temperature ratio of infinite slab
- \( Y_{ic} \) = temperature ratio of infinite cylinder

For a finite rectangular solid,

\[
Y_{fxs} = (Y_{i1})_1 (Y_{i2})_2 (Y_{i3})_3
\]

(31)

where \( Y_{i3} \) = temperature ratio of finite rectangular solid, and subscripts 1, 2, and 3 designate three infinite slabs. The convective heat transfer coefficients associated with one pair of parallel surfaces need not be equal to the coefficient associated with another pair. However, the temperature of the fluid adjacent to every surface should be the same. In evaluating the resistance ratio and the Fourier number \( Fo \), the appropriate values of the heat transfer coefficient and the characteristic dimension should be used.

Heat Exchanger Transients

Determination of the transient behavior of heat exchangers is becoming increasingly important in evaluating the dynamic behavior of heating and air-conditioning systems. Many studies of the transient behavior of counterflow and parallel flow heat exchangers have been conducted; some are listed in the section on Bibliography.

THERMAL RADIATION

Radiation, one of the basic mechanisms for energy transfer between different temperature regions, is distinguished from conduction and convection in that it does not depend on an intermediate
material as a carrier of energy but rather is impeded by the presence of material between the regions. The radiation energy transfer process is the consequence of energy-carrying electromagnetic waves that are emitted by atoms and molecules due to changes in their energy content. The amount and characteristics of radiant energy emitted by a quantity of material depend on the nature of the material, its microscopic arrangement, and its absolute temperature. Although rate of energy emission is independent of the surroundings, the net energy transfer rate depends on the temperatures and spatial relationships of the surface and its surroundings.

**Blackbody Radiation**

The rate of thermal radiant energy emitted by a surface depends on its absolute temperature. A surface is called **black** if it can absorb all incident radiation. The total energy emitted per unit time per unit area of black surface $W_b$ to the hemispherical region above it is given by the **Stefan-Boltzmann law**.

$$W_b = \sigma T^4$$ (32)
where \( W_b \) is the total rate of energy emission per unit area, and \( \sigma \) is the Stefan-Boltzmann constant \( [5.670 \times 10^{-8} \text{ W/(m}^2\cdot\text{K}^4)] \).

The heat radiated by a body comprises electromagnetic waves of many different frequencies or wavelengths. Planck showed that the spectral distribution of the energy radiated by a blackbody is

\[
W_b(\lambda) = \frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1}
\]

(33)

where

- \( W_b(\lambda) \) is monochromatic emissive power of blackbody, W/m\(^2\)
- \( \lambda \) is wavelength, \( \mu \text{m} \)
- \( T \) is temperature, K
- \( C_1 \) is first Planck’s law constant \( = 3.742 \times 10^{36} \text{ W} \cdot \text{m}^{-2} \)
- \( C_2 \) is second Planck’s law constant \( = 0.014388 \text{ m} \cdot \text{K} \)

\( W_b(\lambda) \) is the monochromatic emissive power, defined as the energy emitted per unit time per unit surface area at wavelength \( \lambda \) per unit wavelength interval around \( \lambda \); that is, the energy emitted per unit time per unit surface area in the interval \( d\lambda \) is equal to \( W_{b(\lambda)}d\lambda \).

The Stefan-Boltzmann equation can be obtained by integrating Planck’s equation:

\[
W_b = \sigma T^4 = \int_0^\infty W_b(\lambda) d\lambda
\]

(34)

Wien showed that the wavelength of maximum emissive power multiplied by the absolute temperature is a constant:

\[
\lambda_{\text{max}} T = 2898 \mu \text{m} \cdot \text{K}
\]

(35)

where \( \lambda_{\text{max}} \) is the wavelength at which the monochromatic emissive power is a maximum and not the maximum wavelength. Equation (35) is known as Wien’s displacement law. According to this law, the maximum spectral emissive power is displaced to shorter wavelengths with increasing temperature, such that significant emission eventually occurs over the entire visible spectrum as shorter wavelengths become more prominent. For additional details, see Incropera and DeWitt (1996).

### Actual Radiation

Substances and surfaces diverge variously from the Stefan-Boltzmann and Planck laws. \( W_b \) and \( W_{b(\lambda)} \) are the maximum emissive powers at a surface temperature. Actual surfaces emit and absorb less than these maximums and are called nonblack. The emissive power of a nonblack surface at temperature \( T \) radiating to the hemispherical region above it is written as

\[
W = \varepsilon \sigma T^4
\]

(36)

where \( \varepsilon \) is known as the hemispherical emittance. The term emittance conforms to physical and electrical terminology; the suffix “ance” denotes a property of a piece of material as it exists. The ending “ivity” denotes a property of the bulk material independent of geometry or surface condition. Thus, emittance, reflectance, absorptance, and transmittance refer to actual pieces of material. Emissivity, reflectivity, absorptivity, and transmissivity refer to the properties of materials that are optically smooth and thick enough to be opaque.

The emittance is a function of the material, the condition of its surface, and the temperature of the surface. Table 3 lists selected values; Siegel and Howell (1981) and Modest (1995) have more extensive lists.

The monochromatic emissive power of a nonblack surface is similarly written as

\[
W_{\lambda} = \varepsilon_{b(\lambda)} W_{b(\lambda)} = \varepsilon_{b(\lambda)} \left( \frac{C_1 \lambda^{-5}}{e^{C_2/\lambda T} - 1} \right)
\]

(37)

where \( \varepsilon_{b(\lambda)} \) is the monochromatic hemispherical emittance. The relationship between \( \varepsilon \) and \( \varepsilon_{b(\lambda)} \) is given by

\[
W = \varepsilon \sigma T^4 = \int_0^\infty W_{b(\lambda)} d\lambda = \int_0^\infty \varepsilon_{b(\lambda)} W_{b(\lambda)} d\lambda
\]

or

\[
\varepsilon = \frac{1}{\sigma T^4} \int_0^\infty \varepsilon_{b(\lambda)} W_{b(\lambda)} d\lambda
\]

(38)

If \( \varepsilon_{b(\lambda)} \) does not depend on \( \lambda \), then, from Equation (38), \( \varepsilon = \varepsilon_{\lambda} \). Surfaces with this characteristic are called gray. Gray surface characteristics are often assumed in calculations. Several classes of surfaces approximate this condition in some regions of the spectrum. The simplicity is desirable, but care must be exercised, especially if
temperatures are high. Assumption of grayness is sometimes made because of the absence of information relating $\epsilon$ and $\lambda$.

When radiant energy falls on a surface, it can be absorbed, reflected, or transmitted through the material. Therefore, from the first law of thermodynamics,

$$\alpha + \tau + \rho = 1$$  \hspace{1cm} \text{(39)}

where

- $\alpha = \text{fraction of incident radiation absorbed or absorptance}$
- $\tau = \text{fraction of incident radiation transmitted or transmittance}$
- $\rho = \text{fraction of incident radiation reflected or reflectance}$

If the material is opaque, as most solids are in the infrared, $\tau = 0$ and $\alpha + \rho = 1$. For a black surface, $\alpha = 1$, $\rho = 0$, and $\tau = 0$. Platinum black and gold black are as black as any actual surface and have absorptances of about 98% in the infrared. Any desired degree of blackness can be simulated by a small hole in a large enclosure. Consider a ray of radiant energy entering the opening. It will undergo many internal reflections and be almost completely absorbed before it has a reasonable probability of passing back out of the opening.

Certain flat black paints also exhibit emittances of 98% over a wide range of conditions. They provide a much more durable surface than gold or platinum black and are frequently used on radiation instruments and as standard reference in emittance or reflectance measurements.

Kirchhoff’s law relates emittance and absorptance of any opaque surface from thermodynamic considerations; it states that for any surface where the incident radiation is independent of angle or where the surface is diffuse, $\epsilon = \alpha$. If the surface is gray, or the incident radiation is from a black surface at the same temperature, then $\tau = \tau$ as well, but many surfaces are not gray. For most surfaces listed in Table 3, absorptance for solar radiation is different from emittance for low-temperature radiation. This is because the wavelength distributions are different in the two cases, and $\epsilon$ varies with wavelength.

The foregoing discussion relates to total hemispherical radiation from surfaces. Energy distribution over the hemispherical region above the surface also has an important effect on the rate of heat transfer in various geometric arrangements.

Lambert’s law states that the emissive power of radiant energy over a hemispherical surface above the emitting surface varies as the cosine of the angle between the normal to the radiating surface and the line joining the radiating surface to the point of the hemispherical surface. This radiation is diffuse radiation. The Lambert emissive power variation is equivalent to assuming that radiation from a surface in a direction other than normal occurs as if it came from an equivalent area with the same emissive power (per unit area) as the original surface. The equivalent area is obtained by projecting the original area onto a plane normal to the direction of radiation. Black surfaces obey the Lambert law exactly. The law is approximate for many actual radiation and reflection processes, especially those involving rough surfaces and nonmetallic materials. Most radiation analyses are based on the assumption of gray diffuse radiation and reflection.

In estimating heat transfer rates between surfaces of different geometries, radiation characteristics, and orientations, it is usually assumed that:

- All surfaces are gray or black
- Radiation and reflection are diffuse
- Properties are uniform over the surfaces
- Absorptance equals emittance and is independent of the temperature of the source of incident radiation
- The material in the space between the radiating surfaces neither emits nor absorbs radiation

These assumptions greatly simplify problems, although results must be considered approximate.

### Angle Factor

The distribution of radiation from a surface among the surfaces it irradiates is indicated by a quantity variously called an interception, a view, a configuration, a shape factor, or an angle factor. In terms of two surfaces $i$ and $j$, the angle factor $F_{ij}$ from surface $i$ to surface $j$ is the ratio of the radiant energy leaving surface $i$ and directly reaching surface $j$ to the total radiant energy leaving surface $i$. The angle factor from $j$ to $i$ is similarly defined, merely by interchanging the roles of $i$ and $j$. This second angle factor is not, in general, numerically equal to the first. However, the reciprocity relation $F_{ij}A_i = F_{ji}A_j$, where $A$ is the surface area, is always valid. Note that a concave surface may “see itself” ($F_{ii} \neq 0$), and that if $n$ surfaces form an enclosure,

$$\sum_{j=1}^n F_{ij} = 1$$  \hspace{1cm} \text{(40)}

The angle factor $F_{12}$ between two surfaces is

$$F_{12} = \frac{1}{A_1 A_2} \int \frac{\cos \phi_1 \cos \phi_2}{r^2} \, dA_1 \, dA_2$$  \hspace{1cm} \text{(41)}

where $dA_1$ and $dA_2$ are elemental areas of the two surfaces, $r$ is the distance between $dA_1$ and $dA_2$, and $\phi_1$ and $\phi_2$ are the angles between the respective normals to $dA_1$ and $dA_2$ and the connecting line $r$. Numerical, graphical, and mechanical techniques can solve this equation (Stieglitz and Howell 1981, Modest 1993). Numerical values of the angle factor for common geometries are given in Figure 6.

### Calculation of Radiant Exchange Between Surfaces Separated by Nonabsorbing Media

A surface radiates energy at a rate independent of its surroundings and absorbs and reflects incident energy at a rate dependent on its surface condition. The net energy exchange per unit area is denoted by $q$ or $q_n$ for unit area $A$. It is the rate of emission of the surface minus the total rate of absorption at the surface from all radiant effects in its surroundings, possibly including the return of some of its own emission by reflection off its surroundings. The rate at which energy must be supplied to the surface by other exchange processes if its temperature is to remain constant is $q$; therefore, to define $q$, the total radiant surroundings (in effect, an enclosure) must be specified.

Several methods have been developed to solve certain problems. To calculate the radiation exchange at each surface of an enclosure of $n$ opaque surfaces by simple, general equations convenient for machine calculation, two terms must be defined:

- $G$ = irradiation; total radiation incident on surface per unit time and per unit area
- $J$ = radiosity; total radiation that leaves surface per unit time and per unit area

The radiosity is the sum of the energy emitted and the energy reflected:

$$J = \epsilon W_k + p G$$  \hspace{1cm} \text{(42)}

Because the transmittance is zero, the reflectance is

$$p = 1 - \alpha = 1 - \epsilon$$

Thus,

$$J = \epsilon W_k + (1 - \epsilon) G$$  \hspace{1cm} \text{(43)}

The net energy lost by a surface is the difference between the radiosity and the irradiation:
\[ q/A = J - G = \varepsilon W_b + (1 - \varepsilon)G - G \]  
(44)

Substituting for \( G \) in terms of \( J \) from Equation (43),

\[ q = W_b \frac{J}{(1 - \varepsilon)/\varepsilon A} \]  
(45)

Consider an enclosure of \( n \) isothermal surfaces with areas of \( A_1, A_2, \ldots, A_n \), emissivities of \( \varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n \), and reflectances of \( \rho_1, \rho_2, \ldots, \rho_n \) respectively.

The irradiation of surface \( i \) is the sum of the radiation incident on it from all \( n \) surfaces:

\[ G_i A_i = \sum_{j=1}^{n} F_{ij} J_j A_j = \sum_{j=1}^{n} F_{ij} J_j A_i \]

or

\[ G_i = \sum_{j=1}^{n} F_{ij} J_j \]

Substituting in Equation (44) yields the following simultaneous equations when each of the \( n \) surfaces is considered:

\[ J_i = \varepsilon_i W_b + (1 - \varepsilon_i) \sum_{j=1}^{n} F_{ij} J_j \quad i = 1, 2, \ldots, n \]  
(46)

Equation (46) can be solved manually for the unknown \( J_i \)s if the number of surfaces is small. The solution for more complex enclosures requires a computer.

Once the radiosities (\( J_i \)s) are known, the net radiant energy lost by each surface is determined from Equation (45) as

\[ q_i = \frac{W_{bi} - J_i}{(1 - \varepsilon_i)/\varepsilon_i A_i} \]

If the surface is black, Equation (45) becomes indeterminate, and an alternate expression must be used, such as
Heat Transfer

\[ q_i = \sum_{j=1}^{n} j A_j F_{ij} - j A_j F_{ji} \]

or

\[ q_i = \sum_{j=1}^{n} F_{ij} A_j (J_i - J_j) \]  \hspace{1cm} (47)

since

\[ F_{ij} A_j = F_{ji} A_j \]

All diffuse radiation processes are included in the aforementioned enclosure method, and surfaces with special characteristics are assigned consistent properties. An opening is treated as an equivalent surface area \( A_s \) with a reflectance of zero. If energy enters the enclosure diffusely through the opening, \( A_s \) is assigned an equivalent temperature; otherwise, its temperature is taken as zero. If the loss through the opening is desired, \( q_i \) is found. A window in the enclosure is assigned its actual properties.

A surface in radiant balance is one for which radiant emission is balanced by radiant absorption; heat is neither removed from nor supplied to the surface. Radiating surfaces (insulated surfaces with \( q_{sur} = 0 \)), can be treated in Equation (46) as being perfectly reflective (i.e., \( \varepsilon = 0 \)). The equilibrium temperature of such a surface can be found from

\[ T_k = \left( \frac{I_s}{\sigma} \right)^{0.25} \]

once Equation (46) has been solved for the radiosity.

Use of angle factors and radiation properties as defined assumes that the surfaces are diffuse radiators—a good assumption for most nonmetals in the infrared region, but a poor assumption for highly polished metals. Subdividing the surfaces and considering the variation of radiation properties with angle of incidence improves the approximation but increases the work required for a solution.

Radiation in Gases

Elementary gases such as oxygen, nitrogen, hydrogen, and helium are essentially transparent to thermal radiation. Their absorption and emission bands are confined mainly to the ultraviolet region of the spectrum. The gaseous vapors of most compounds, however, have absorption bands in the infrared region. Carbon monoxide, carbon dioxide, water vapor, sulfur dioxide, ammonia, acid vapors, and organic vapors absorb and emit significant amounts of energy.

Radiation exchange by opaque solids is considered a surface phenomenon. Radiant energy does, however, penetrate the surface of all materials. The absorption coefficient gives the rate of exponential attenuation of the energy. Metals have large absorption coefficients, and radiant energy penetrates less than 100 nm at most. Absorption coefficients for nonmetals are lower. Radiation may be considered a surface phenomenon unless the material is transparent. Gases have small absorption coefficients, so the path length of radiation through gas becomes very significant.

Beer's law states that the attenuation of radiant energy in a gas is a function of the product of the partial pressure of the gas and the path length. The monochromatic absorbance of a body of gas of thickness \( L \) is then given by

\[ \alpha_{3L} = 1 - e^{-\alpha 3L} \]  \hspace{1cm} (48)

Because absorption occurs in discrete wavelengths, the absorbances must be summed over the spectral region corresponding to the temperature of the blackbody radiation passing through the gas.

<table>
<thead>
<tr>
<th>Table 4</th>
<th>Emissivity of CO₂ and Water Vapor in Air at 24°C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Path Length, m</td>
<td>CO₂, % by Volume</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------------</td>
</tr>
<tr>
<td>0.1</td>
<td>0.03</td>
</tr>
<tr>
<td>0.3</td>
<td>0.09</td>
</tr>
<tr>
<td>1.0</td>
<td>0.16</td>
</tr>
<tr>
<td>10</td>
<td>0.23</td>
</tr>
<tr>
<td>50</td>
<td>0.47</td>
</tr>
<tr>
<td>100</td>
<td>0.22</td>
</tr>
</tbody>
</table>

The monochromatic absorption coefficient \( \alpha_{3L} \) is also a function of temperature and pressure of the gas; therefore, detailed treatment of gas radiation is quite complex.

Estimated emittance for carbon dioxide and water vapor in air at 24°C is a function of concentration and path length (Table 4). The values are for a hemispherically shaped body of gas radiating to an element of area at the center of the hemispherical. Among others, Modest (1993), Siegel and Howell (1981), and Hotzel and Sarofim (1967) describe geometrical calculations in their texts on radiation transfer. Generally, at low values of \( p_L \), the mean path length \( L \) (or equivalent hemispherical radius for a gas body radiating to its surrounding surfaces) is four times the mean hydraulic radius of the enclosure. A room with a dimensional ratio of 1:1:1 has a mean path length of 0.89 times the shortest dimension when considering radiation to all walls. For a room with a dimensional ratio of 1:2:6, the mean path length for gas radiating to all surfaces is 1.2 times the shortest dimension. The mean path length for radiation to the 2 by 6 face is 1.18 times the shortest dimension. These values are for cases where the partial pressure of the gas times the mean path length approaches zero (\( p_L = 0 \)). The factor decreases with increasing values of \( p_L \).

For average rooms with approximately 2.4 m ceilings and relative humidity ranging from 10 to 75% at 24°C, the effective path length for carbon dioxide radiation is about 85% of the ceiling height, or 2.0 m. The effective path length for water vapor is about 93% of the ceiling height, or 2.3 m. The effective emittance of the water vapor and carbon dioxide radiating to the walls, ceiling, and floor of a room 4.9 m by 14.6 m with 2.4 m ceilings is in the following tabulation.

<table>
<thead>
<tr>
<th>Relative Humidity, %</th>
<th>( T_k )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.10</td>
</tr>
<tr>
<td>50</td>
<td>0.19</td>
</tr>
<tr>
<td>75</td>
<td>0.22</td>
</tr>
</tbody>
</table>

The radiation heat transfer from the gas to the walls is then

\[ q = \alpha A_k F_k (T_k - T_w^4) \]  \hspace{1cm} (49)

The examples in Table 4 and the preceding text indicate the importance of gas radiation in environmental heat transfer problems. Gas radiation in large furnaces is the dominant mode of heat transfer, and many additional factors must be considered. Increased pressure broadens the spectral bands, and interaction of different radiating species prohibits simple summation of the emittance factors for the individual species. Departures from blackbody conditions necessitate separate calculations of the emittance and absorptance. McAdams (1954) and Hotzel and Sarofim (1967) give more complete treatments of gas radiation.

**NATURAL CONVECTION**

Heat transfer involving motion in a fluid due to the difference in density and the action of gravity is called **natural convection** or **free convection**. Heat transfer coefficients associated with gases for natural convection are generally much lower than those for forced convection, and it is therefore important not to ignore radiation in calculating the total heat loss or gain. Radiant transfer may be
Table 5  Natural Convection Heat Transfer Coefficients

<table>
<thead>
<tr>
<th>Characteristic length ( L )</th>
<th>Nu = ( c(Gr\ Pr)^n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical plates or pipes</td>
<td>( h = c \left( \frac{L^3 \rho \mu \delta}{\mu^2} \right)^{1/2} \frac{(\mu/L \rho)^{1/2}}{k} )</td>
</tr>
<tr>
<td>Horizontal plates</td>
<td>( L = \text{height} )</td>
</tr>
<tr>
<td>Horizontal pipes</td>
<td>( L = \text{length} )</td>
</tr>
<tr>
<td>Spheres</td>
<td>( L = \text{diameter} )</td>
</tr>
<tr>
<td>Rectangular block, with horizontal length ( L_h ) and vertical length ( L_v )</td>
<td>( 1/L = (1/L_h) + (1/L_v) )</td>
</tr>
</tbody>
</table>

II. Planes and pipes

(a) Laminar range, when \( Gr\ Pr \) is between \( 10^7 \) and \( 10^8 \)

(b) Turbulent range, when \( Gr\ Pr \) is between \( 10^8 \) and \( 10^{12} \)

\( Nu = 0.56(Gr\ Pr)^{0.25} \) \hspace{1cm} (3)

\( Nu = 0.13(Gr\ Pr)^{0.33} \) \hspace{1cm} (4)

III. Wires

For horizontal or vertical wires, use \( L = \text{diameter} \), for \( Gr\ Pr \) between \( 10^{-7} \) and 1

\( Nu = (Gr\ Pr)^{0.1} \) \hspace{1cm} (5)

IV. With air

\( Gr\ Pr = 1.6 \times 10^8 \rho \Delta t \) \hspace{1cm} (at 21°C, \( L \) in m, \( \Delta t \) in K)

(a) Horizontal cylinders

Small cylinder, laminar range

\( h = 1.32(\Delta t/L)^{0.25} \) \hspace{1cm} (6)

Large cylinder, turbulent range

\( h = 1.24(\Delta t/L)^{1/2} \) \hspace{1cm} (7)

(b) Vertical plates

Small plates, laminar range

\( h = 1.42(\Delta t/L)^{0.25} \) \hspace{1cm} (8)

Large plates, turbulent range

\( h = 1.31(\Delta t/L)^{1/2} \) \hspace{1cm} (9)

(c) Horizontal plates, facing upward when heated or downward when cooled

Small plates, laminar range

\( h = 1.32(\Delta t/L)^{0.25} \) \hspace{1cm} (10)

Large plates, turbulent range

\( h = 1.52(\Delta t/L)^{1/2} \) \hspace{1cm} (11)

(d) Horizontal plates, facing downward when heated or upward when cooled

Small plates

\( h = 0.59(\Delta t/L)^{0.25} \) \hspace{1cm} (12)

A region that has \( Gr\ Pr \) less than for laminar natural convection, for which the exponent \( n \) gradually diminishes from 0.25 to lower values

Note that for wires, the \( Gr\ Pr \) is likely to be very small, so that the exponent \( n \) is 0.1 [Equation (5) in Table 5].

To calculate the natural-convection heat transfer coefficient, determine \( Gr\ Pr \) to find whether the boundary layer is laminar or turbulent; then apply the appropriate equation from Table 5. The correct characteristic length indicated in the table must be used. Because the exponent \( n \) is 0.33 for a turbulent boundary layer, the characteristic length cancels out in Equation (2) in Table 5, and the heat transfer coefficient is independent of the characteristic length, as seen in Equations (7), (9), and (11) in Table 5. Turbulence occurs when length or temperature difference is large. Because the length of a pipe is generally greater than its diameter, the heat transfer coefficient for vertical pipes is larger than for horizontal pipes.

Convection from horizontal plates facing downward when heated (or upward when cooled) is a special case. Because the hot air is above the colder air, theoretically no convection should occur. Some convection is caused, however, by secondary influences such as temperature differences on the edges of the plate. As an approximation, a coefficient of somewhat less than half the coefficient for a heated horizontal plate facing upward can be used.

Because air is often the heat transport fluid, simplified equations for air are given in Table 5. Other information on natural convection is available in the section on Bibliography under Heat Transfer, General.

Observed differences in the comparison of recent experimental and numerical results with existing correlations for natural convective heat transfer coefficients indicate that caution should be used when applying coefficients for (isolated) vertical plates to vertical surfaces in enclosed spaces (buildings). Bauman et al. (1983) and
Heat Transfer

Altmayer et al. (1983) developed improved correlations for calculating natural convective heat transfer from vertical surfaces in rooms under certain temperature boundary conditions.

Natural convection can affect the heat transfer coefficient in the presence of weak forced convection. As the forced-convection effect (i.e., the Reynolds number) increases, "mixed convection" (superimposed forced-on-free convection) gives way to the pure forced-convection regime. In these cases, other sources describing combined free and forced convection should be consulted, since the heat transfer coefficient in the mixed-convection region is often larger than that calculated based on the natural- or forced-convection calculation alone. Metius and Eckert (1964) summarize natural-, mixed-, and forced-convection regimes for vertical and horizontal tubes. Figure 7 shows the approximate limits for horizontal tubes. Other studies are described by Grigull et al. (1982).

FORCED CONVECTION

Forced air coolers and heaters, forced air- or water-cooled condensers and evaporators, and liquid suction heat exchangers are examples of equipment that transfer heat primarily by forced convection.

When fluid flows over a flat plate, a boundary layer forms adjacent to the plate. The velocity of the fluid at the plate surface is zero and increases to its maximum free stream value just past the edge of the boundary layer (Figure 8). Boundary layer formation is important because the temperature change from plate to fluid (thermal resistance) is concentrated here. Where the boundary layer is thick, thermal resistance is great and the heat transfer coefficient is small. Flow within the boundary layer immediately downstream from the leading edge is laminar and is known as laminar forced convection. As flow proceeds along the plate, the laminar boundary layer increases in thickness to a critical value. Then, turbulent eddies develop within the boundary layer, except for a thin laminar sublayer adjacent to the plate.

The boundary layer beyond this point is a turbulent boundary layer, and the flow is turbulent forced convection. The region between the breakdown of the laminar boundary layer and the establishment of the turbulent boundary layer is the transition region. Because the turbulent eddies greatly enhance heat transport into the mainstream, the heat transfer coefficient begins to increase rapidly through the transition region. For a flat plate with a smooth leading edge, the turbulent boundary layer starts at Reynolds numbers, based on distance from the leading edge, of about 500,000 to 500,000. In blunt-edged plates, it can start at much smaller Reynolds numbers.

For long tubes or channels of small hydraulic diameter, at sufficiently low flow velocity, the laminar boundary layers on each wall grow until they meet. Beyond this point, the velocity distribution does not change, and no transition to turbulent flow takes place. This is called fully developed laminar flow. For tubes of large diameter or at higher velocities, transition to turbulence takes place and fully developed turbulent flow is established (Figure 9). Therefore, the length dimension that determines the critical Reynolds number is the hydraulic diameter of the channel. For smooth circular tubes, flow is laminar for Reynolds numbers below 2100 and turbulent above 10,000.

Table 6 lists various forced-convection correlations. In the generalized, dimensionless formula of Equation (1) in Table 6, heat transfer is determined by flow conditions and by the fluid properties, as indicated by the Reynolds number and the Prandtl number. This equation can be modified to Equation (4) in Table 6 to get the heat transfer factor $f$. The heat transfer factor is related to the friction factor $f$ by the interrelationship of the transport of momentum and heat; it is approximately $f/2$ for turbulent flow in straight ducts. These factors are plotted in Figure 10.
Table 6  Equations for Forced Convection

<table>
<thead>
<tr>
<th>Description</th>
<th>Author</th>
<th>Page</th>
<th>Eq. No.</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>I. Generalized correlations</td>
<td>Jakob</td>
<td>491</td>
<td>(23-36)</td>
<td>( \frac{hD}{k} = \left( \frac{GD}{\mu} \right)^{0.2} \left( \frac{c_p}{k} \right)^{0.2} ) (1)</td>
</tr>
<tr>
<td>(a) Turbulent flow inside tubes</td>
<td>McAdams</td>
<td>219</td>
<td>(9-10a)</td>
<td>( \frac{hD}{k} = 0.022 \left( \frac{GD}{\mu} \right)^{0.8} \left( \frac{c_p}{k} \right)^{0.4} ) (2)</td>
</tr>
<tr>
<td>(1) Using fluid properties based on bulk temperature ( t )</td>
<td>McAdams</td>
<td>219</td>
<td>(9-10c)</td>
<td>( \frac{h}{c_p G \left( \frac{k}{k} \right)} = \frac{0.023}{(GD/\mu)^{0.2}} ) (3)</td>
</tr>
<tr>
<td>(2) Same as (1), except ( \mu ) at surface temperature ( t_s )</td>
<td>McAdams</td>
<td>219</td>
<td>(9-10b)</td>
<td>( \frac{h}{c_p G \left( \frac{k}{k} \right)} = \frac{0.023}{(GD/\mu)^{0.2}} ) (4)</td>
</tr>
<tr>
<td>(3) Using fluid properties based on film temperature ( t_f = 0.5(t_f + t) ), except ( c_p ) in Stantion modulus</td>
<td>McAdams</td>
<td>219</td>
<td>(9-10b)</td>
<td></td>
</tr>
<tr>
<td>(4) For viscous fluids (viscosities higher than twice water), using viscosity ( \mu ) at bulk temperature ( t_b ) and ( \mu_s ) at surface temperature ( t_s )</td>
<td>Jakob</td>
<td>547</td>
<td>(26-12)</td>
<td>( \frac{hD}{k} = 0.027 \left( \frac{GD}{\mu} \right)^{0.8} \left( \frac{c_p}{k} \right)^{0.14} ) (5)</td>
</tr>
<tr>
<td>(b) Laminar flow inside tubes</td>
<td>Jakob</td>
<td>544</td>
<td>(26-5)</td>
<td>( \frac{hD}{k} = 1.86 \left( \frac{GD}{\mu} \right)^{0.8} \left( \frac{c_p}{k} \right)^{0.14} ) (6)</td>
</tr>
<tr>
<td>(1) For large ( D ) or high ( \Delta t ), the effect of natural convection should be included</td>
<td>Jakob</td>
<td>544</td>
<td>(26-5)</td>
<td></td>
</tr>
<tr>
<td>(2) For very long tubes</td>
<td>McAdams</td>
<td>242</td>
<td>(9-32c)</td>
<td>( \frac{h}{c_p G \left( \frac{k}{k} \right)} = \frac{0.023}{(G D / \mu)^{0.2}} ) (7)</td>
</tr>
<tr>
<td>(c) Annular spaces, turbulent flow</td>
<td>McAdams</td>
<td>242</td>
<td>(9-32c)</td>
<td></td>
</tr>
<tr>
<td>All fluid properties at bulk temperature ( \mu ), except ( \mu_s ) at surface temperature ( t_s )</td>
<td>McAdams</td>
<td>242</td>
<td>(9-32c)</td>
<td></td>
</tr>
<tr>
<td>II. Simplified equations for gases, turbulent flow inside tubes [( [K ) in W/(m²·K), ( c_p ) in kJ/(kg·K), ( G ) in kg/(m²·s), ( D ) in m]</td>
<td>McAdams</td>
<td>228</td>
<td>(9-19)</td>
<td>( h = 3.031 \left( c_p G / D^{0.2} / \mu \right) ) (8)</td>
</tr>
<tr>
<td>(a) Most common gases, turbulent flow (assuming ( \mu = 18.8 \mu_{ls} ) and ( \mu_s / \mu = 0.78 ))</td>
<td>McAdams</td>
<td>228</td>
<td>(9-19)</td>
<td></td>
</tr>
<tr>
<td>(b) Air at ordinary temperatures</td>
<td>McAdams</td>
<td>228</td>
<td>(9-19)</td>
<td>( h = 155.2 \left( c_p G / D^{0.2} / \mu \right) ) (9)</td>
</tr>
<tr>
<td>(c) Fluorinated hydrocarbon refrigerant gas at ordinary pressures</td>
<td>McAdams</td>
<td>228</td>
<td>(9-19)</td>
<td>( h = 155.2 \left( c_p G / D^{0.2} / \mu \right) ) (10)</td>
</tr>
<tr>
<td>(d) Ammonia gas at approximately 65°C, 2 MPa</td>
<td>McAdams</td>
<td>228</td>
<td>(9-19)</td>
<td>( h = 6.633 \left( c_p G / D^{0.2} / \mu \right) ) (11)</td>
</tr>
<tr>
<td>At ~18°C, 165 kPa (gage)</td>
<td>McAdams</td>
<td>228</td>
<td>(9-19)</td>
<td>( h = 5.333 \left( c_p G / D^{0.2} / \mu \right) ) (12)</td>
</tr>
<tr>
<td>III. Simplified equations for liquids, turbulent flow inside tubes [( [L ) in W/(m²·K), ( G ) in kg/(m²·s), ( V ) in m/s, ( D ) in m, ( t ) in °C, ( \mu ) in N·s/m²]</td>
<td>McAdams</td>
<td>228</td>
<td>(9-19)</td>
<td></td>
</tr>
<tr>
<td>(a) Water at ordinary temperatures, ( 4 ) to 93°C</td>
<td>McAdams</td>
<td>228</td>
<td>(9-19)</td>
<td>( h = 1057 \left( 1.352 + 0.0198 t \right) ) (13)</td>
</tr>
<tr>
<td>(b) Fluorinated hydrocarbon refrigerant liquid</td>
<td>McAdams</td>
<td>228</td>
<td>(9-19)</td>
<td>( h = 155.2 \left( c_p G / D^{0.2} / \mu \right) ) (14)</td>
</tr>
<tr>
<td>(c) Ammonia liquid at approximately 38°C</td>
<td>McAdams</td>
<td>228</td>
<td>(9-19)</td>
<td>( h = 13.75 \left( c_p G / D^{0.2} / \mu \right) ) (15)</td>
</tr>
<tr>
<td>(d) Oil heating, approximate equation</td>
<td>Brown and Marco</td>
<td>146</td>
<td>(7-15)</td>
<td>( h = 0.0041 V / \mu^{0.63} ) (16)</td>
</tr>
<tr>
<td>(e) Oil cooling, approximate equation</td>
<td>Brown and Marco</td>
<td>146</td>
<td>(7-15)</td>
<td>( h = 0.0035 V / \mu^{0.63} ) (17)</td>
</tr>
<tr>
<td>IV. Simplified equations for air</td>
<td>McAdams</td>
<td>249</td>
<td>(9-42)</td>
<td>( h' = 7.2 V^{0.78} ) (18)</td>
</tr>
<tr>
<td>(a) Vertical plane surfaces, ( V = 5 ) to 30 m/s (room temperature)³</td>
<td>McAdams</td>
<td>249</td>
<td>(9-42)</td>
<td>( h' = 5.62 + 3.9 V ) (19)</td>
</tr>
<tr>
<td>(b) Vertical plane surfaces, ( V &lt; 5 ) m/s (room temperature)³</td>
<td>McAdams</td>
<td>249</td>
<td>(9-42)</td>
<td></td>
</tr>
<tr>
<td>(c) Single cylinder cross flow (film temperature ( = 93°C ) ( 1000 &lt; GD &lt; 50000 )</td>
<td>McAdams</td>
<td>249</td>
<td>(9-42)</td>
<td>( h = 4.83 \left( G / D^{0.4} \right) ) (20)</td>
</tr>
<tr>
<td>(d) Single sphere ( 17 &lt; GD &lt; 7000 )</td>
<td>McAdams</td>
<td>249</td>
<td>(9-42)</td>
<td>( h = 0.43 \left( GD \right)^{0.6} ) (21)</td>
</tr>
<tr>
<td>V. Gases flowing normal to pipes (dimensionless)</td>
<td>McAdams</td>
<td>260</td>
<td>(10-3)</td>
<td>( \frac{hD}{k_f} = 0.32 + 0.43 \left( \frac{GD}{\mu} \right)^{0.52} ) (22)</td>
</tr>
<tr>
<td>(a) Single cylinder Re from 0.1 to 1000</td>
<td>McAdams</td>
<td>260</td>
<td>(10-3)</td>
<td>( \frac{hD}{k_f} = 0.24 \left( \frac{GD}{\mu} \right)^{0.6} ) (23)</td>
</tr>
<tr>
<td>(b) Un baffled staggered tubes, 10 rows, Approximate equation for turbulent flow³</td>
<td>McAdams</td>
<td>260</td>
<td>(10-3)</td>
<td></td>
</tr>
<tr>
<td>(c) Un baffled in-line tubes, 10 rows, Approximate equation for turbulent flow³</td>
<td>McAdams</td>
<td>260</td>
<td>(10-3)</td>
<td></td>
</tr>
<tr>
<td>Gmax ( = \text{min} ) area of tube. Coefficients for tube banks depend greatly on geometrical details. These values approximate only.</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>