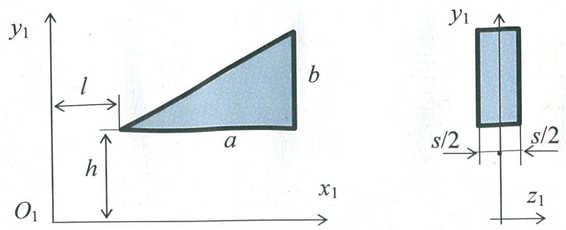
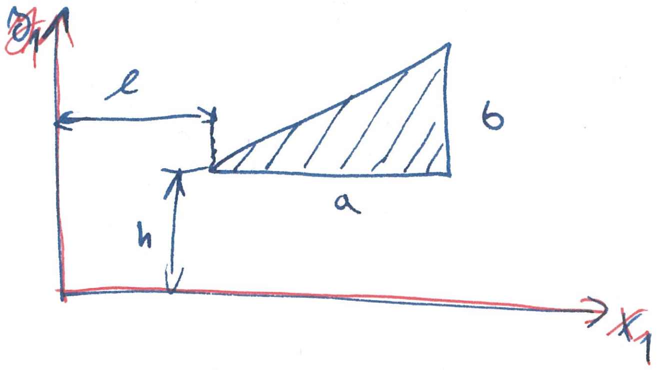
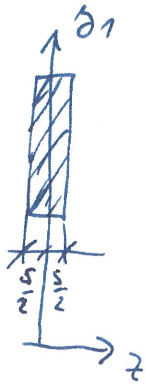
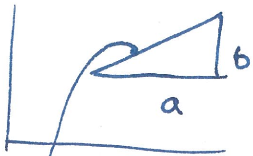


Určete deviační momenty $D_{x_1y_1}$, $D_{y_1z_1}$, $D_{z_1x_1}$ homogenní trojúhelníkové desky tloušťky s.

Dáno: a, b, s, h, l, ρ [kgm^{-3}].







$y_{hm} = h + \frac{b}{a}(x-l)$

$$D_{x_1y_1} = \int xy\,dm = \rho s \int xy\,dx\,dy = \left| \text{meze: } \begin{matrix} x \in \langle l, l+a \rangle \\ y \in \langle h, y_{hm} \rangle \end{matrix} \right| =$$

$$= \rho s \int_l^{l+a} \left[y^2 \tfrac{1}{2}\right]_h^{y_{hm}} x\,dx = \frac{1}{2}\rho s \int \left[\left(h + \frac{b}{a}(x-l)\right)^2 - h^2\right] x\,dx =$$

$$= \frac{1}{2}\rho s \int_l^{l+a} \left[\left(\frac{b}{a}\right)^2 (x^2 - 2lx + l^2) + 2\frac{hb}{a}(x-l)\right] x\,dx =$$

$$= \frac{1}{2}\rho s \int_l^{l+a} \left[\left(\frac{b}{a}\right)^2 x^3 + 2\left(\frac{bh}{a} - \left(\frac{b}{a}\right)^2 l\right)x^2 + \left(\left(\frac{bl}{a}\right)^2 - \frac{2bhl}{a}\right)x\right] dx =$$

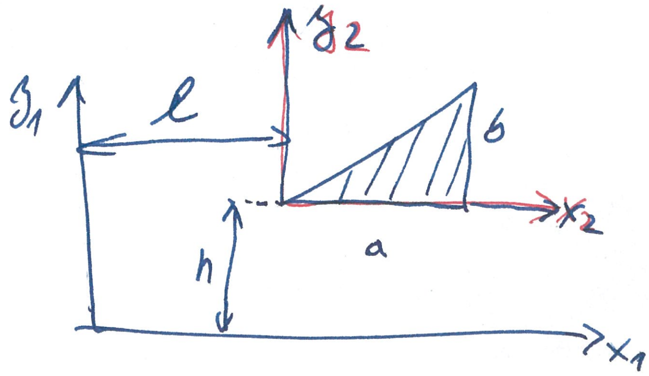
$$= \frac{1}{2}\rho s \frac{b}{a}\left[\frac{b}{a}\frac{x^4}{4} + 2\left(h - \frac{b}{a}l\right)\frac{x^3}{3} + \left(\frac{b}{a}l^2 - 2hl\right)\frac{x^2}{2}\right]_l^{l+a} = \text{DLOUHÁ ÚPRAVA} =$$

$$= \frac{1}{24}\rho s\, ab\left[3ab + 8ah + 4bl + 12hl\right]$$

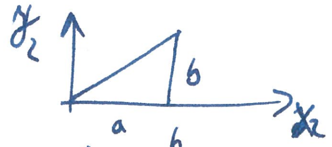
$$m = \frac{1}{2}\rho s\, ab$$

$$D_{x_1y_1} = m\left[\frac{1}{4}ab + \frac{2}{3}ah + \frac{1}{3}bl + hl\right]$$

→ JEDNODUŠŠÍ POSTUP :



ZAVEDENÝ SOUŘADNICOVÝ SYSTÉM PEVNĚ NA TĚLESE



$y_2 = \frac{b}{a} x_2$

→ SPOČTEME $D_{x_2 y_2}$:

$$D_{x_2 y_2} = \int x_2 y_2 \, dm = \rho s \int x_2 y_2 \, dx_2 \, dy_2 = \left| \text{meze: } \begin{array}{l} x_2 \in \langle 0, a \rangle \\ y_2 \in \langle 0, y_{hm} \rangle \end{array} \right| =$$

$$= \rho s \int \left[\frac{1}{2} y_2^2 \right]_0^{\frac{b}{a} x_2} x_2 \, dx_2 = \frac{1}{2} \rho s \frac{b^2}{a^2} \int x_2^3 \, dx_2 = \frac{1}{2} \rho s \frac{b^2}{a^2} \left[\frac{x^4}{4} \right]_0^a =$$

$$= \frac{1}{8} \rho s b^2 a^2 = \left| m = \frac{1}{2} \rho s a b \right| = \frac{1}{4} a b \, m$$

PRO $x_1 y_1$:

$$D_{x_1 y_1} = \int (x_2 + l)(y_2 + h) \, dm = \rho s \int_0^a \int_0^{\frac{b}{a} x_2} \left[x_2 y_2 + x_2 h + y_2 l + l h \right] dx_2 \, dy_2 =$$

$$= D_{x_2 y_2} + \rho s \left[\int x_2 h \, dx_2 dy_2 + \int y_2 l \, dx_2 dy_2 + \int l h \, dx_2 dy_2 \right] =$$

$$= D_{x_2 y_2} + \rho s \left[\int h x_2 \frac{b}{a} x_2 \, dx_2 + l \int \frac{1}{2} x_2^2 \frac{b^2}{a^2} \, dx_2 + \int l h \frac{b}{a} x_2 \, dx_2 \right] =$$

$$= D_{x_2 y_2} + \rho s \frac{b}{a} \left[h \frac{1}{3} a^3 + l \frac{b}{a} \frac{1}{6} a^3 + l h \frac{1}{2} a^2 \right] =$$

$$= D_{x_2 y_2} + m \left[\frac{2}{3} a h + \frac{1}{3} l b + l h \right]$$

$D_{x_2 z_2} = 0 = D_{y_2 z_2}$ ... Z DŮVODU SYMETRIE