Momentum transfer

Continuity equation

$$\frac{\mathrm{D}\varrho}{\mathrm{D}t} + \varrho \nabla \bullet \vec{u} = 0$$

$$\left\{ \frac{\partial \varrho}{\partial t} + (\vec{u} \bullet \nabla)\, \varrho \right\} + \varrho \nabla \bullet \vec{u} = 0$$

Incompressible fluids

$$\nabla \bullet \vec{u} = 0$$

Cartesian coordinates

$$\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} + \frac{\partial u_z}{\partial z} = 0$$

Cylindrical coordinates

$$\frac{1}{r}\frac{\partial}{\partial r}(r u_r) + \frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{\partial u_z}{\partial z} = 0$$

Cauchy's momentum equation

$$\varrho \frac{\mathrm{D}\vec{u}}{\mathrm{D}t} = -\nabla p + \nabla \bullet \vec{\vec{\tau}} + \varrho\, \vec{f}$$

$$\varrho \left\{ \frac{\partial \vec{u}}{\partial t} + (\vec{u} \bullet \nabla)\, \vec{u} \right\} = -\nabla p + \nabla \bullet \vec{\vec{\tau}} + \varrho\, \vec{f}$$

Cartesian coordinates

$$\varrho \left\{ \frac{\partial u_x}{\partial t} + u_x \frac{\partial u_x}{\partial x} + u_y \frac{\partial u_x}{\partial y} + u_z \frac{\partial u_x}{\partial z} \right\} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + \varrho\, f_x$$

$$\varrho \left\{ \frac{\partial u_y}{\partial t} + u_x \frac{\partial u_y}{\partial x} + u_y \frac{\partial u_y}{\partial y} + u_z \frac{\partial u_y}{\partial z} \right\} = -\frac{\partial p}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + \varrho\, f_y$$

$$\varrho \left\{ \frac{\partial u_z}{\partial t} + u_x \frac{\partial u_z}{\partial x} + u_y \frac{\partial u_z}{\partial y} + u_z \frac{\partial u_z}{\partial z} \right\} = -\frac{\partial p}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z} + \varrho\, f_z$$

Cylindrical coordinates

$$\varrho\left\{\frac{\partial u_r}{\partial t}+u_r\,\frac{\partial u_r}{\partial r}+\frac{u_\varphi}{r}\,\frac{\partial u_r}{\partial \varphi}-\frac{u_\varphi^2}{r}+u_z\,\frac{\partial u_r}{\partial z}\right\}=-\frac{\partial p}{\partial r}+$$
$$+\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rr}\right)+\frac{1}{r}\frac{\partial \tau_{\varphi r}}{\partial \varphi}-\frac{\tau_{\varphi\varphi}}{r}+\frac{\partial \tau_{zr}}{\partial z}+\varrho\, f_r$$

$$\varrho\left\{\frac{\partial u_\varphi}{\partial t}+u_r\,\frac{\partial u_\varphi}{\partial r}+\frac{u_\varphi}{r}\,\frac{\partial u_\varphi}{\partial \varphi}+\frac{u_r u_\varphi}{r}+u_z\,\frac{\partial u_\varphi}{\partial z}\right\}=-\frac{1}{r}\frac{\partial p}{\partial \varphi}+$$
$$+\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\tau_{r\varphi}\right)+\frac{1}{r}\frac{\partial \tau_{\varphi\varphi}}{\partial \varphi}+\frac{\partial \tau_{z\varphi}}{\partial z}+\varrho\, f_\varphi$$

$$\varrho\left\{\frac{\partial u_z}{\partial t}+u_r\,\frac{\partial u_z}{\partial r}+\frac{u_\varphi}{r}\,\frac{\partial u_z}{\partial \varphi}+u_z\,\frac{\partial u_z}{\partial z}\right\}=-\frac{\partial p}{\partial z}+$$
$$+\frac{1}{r}\frac{\partial}{\partial r}\left(r\tau_{rz}\right)+\frac{1}{r}\frac{\partial \tau_{\varphi z}}{\partial \varphi}+\frac{\partial \tau_{zz}}{\partial z}+\varrho\, f_z$$

Newtonian fluids

$$\vec{\vec{\tau}}=-\frac{2}{3}\mu\,\vec{\vec{\delta}}\,\mathrm{tr}\vec{\vec{\Delta}}+2\mu\vec{\vec{\Delta}}=2\mu\left[-\frac{1}{3}\vec{\vec{\delta}}\,(\nabla\bullet\vec{u})+\vec{\vec{\Delta}}\right]$$

Newtonian liquids

$$\vec{\vec{\tau}}=2\mu\vec{\vec{\Delta}}$$

Deformation rate tensor

$$\vec{\vec{\Delta}}=\frac{1}{2}\left[\nabla\vec{u}+(\nabla\vec{u})^{\mathrm{T}}\right]$$

Cartesian coordinates

$$\Delta_{xx}=\frac{\partial u_x}{\partial x}$$
$$\Delta_{yy}=\frac{\partial u_y}{\partial y}$$
$$\Delta_{zz}=\frac{\partial u_z}{\partial z}$$
$$\Delta_{xy}=\Delta_{yx}=\frac{1}{2}\left[\frac{\partial u_x}{\partial y}+\frac{\partial u_y}{\partial x}\right]$$
$$\Delta_{xz}=\Delta_{zx}=\frac{1}{2}\left[\frac{\partial u_x}{\partial z}+\frac{\partial u_z}{\partial x}\right]$$
$$\Delta_{yz}=\Delta_{zy}=\frac{1}{2}\left[\frac{\partial u_y}{\partial z}+\frac{\partial u_z}{\partial y}\right]$$

Cylindrical coordinates

$$\Delta_{rr}=\frac{\partial u_r}{\partial r}$$
$$\Delta_{\varphi\varphi}=\frac{1}{r}\frac{\partial u_\varphi}{\partial \varphi}+\frac{u_r}{r}$$
$$\Delta_{zz}=\frac{\partial u_z}{\partial z}$$
$$\Delta_{r\varphi}=\Delta_{\varphi r}=\frac{1}{2}\left[r\frac{\partial}{\partial r}\left(\frac{u_\varphi}{r}\right)+\frac{1}{r}\frac{\partial u_r}{\partial \varphi}\right]$$
$$\Delta_{rz}=\Delta_{zr}=\frac{1}{2}\left[\frac{\partial u_r}{\partial z}+\frac{\partial u_z}{\partial r}\right]$$
$$\Delta_{\varphi z}=\Delta_{z\varphi}=\frac{1}{2}\left[\frac{\partial u_\varphi}{\partial z}+\frac{1}{r}\frac{\partial u_z}{\partial \varphi}\right]$$

Navier–Stokes equation

$$\varrho \frac{\mathrm{D}\vec{u}}{\mathrm{D}t} = -\nabla p + \frac{1}{3}\mu\nabla\left(\nabla \bullet \vec{u}\right) + \mu\,\nabla^2\vec{u} + \varrho\,\vec{f}$$

Incompressible fluids

$$\varrho \frac{\mathrm{D}\vec{u}}{\mathrm{D}t} = -\nabla p + \mu\,\nabla^2\vec{u} + \varrho\,\vec{f}$$

$$\varrho\left\{\frac{\partial\vec{u}}{\partial t} + \left(\vec{u}\bullet\nabla\right)\vec{u}\right\} = -\nabla p + \mu\,\nabla^2\vec{u} + \varrho\,\vec{f}$$

Cartesian coordinates

$$\varrho\left\{\frac{\partial u_x}{\partial t} + u_x\frac{\partial u_x}{\partial x} + u_y\frac{\partial u_x}{\partial y} + u_z\frac{\partial u_x}{\partial z}\right\} = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_x}{\partial z^2}\right) + \varrho\,f_x$$

$$\varrho\left\{\frac{\partial u_y}{\partial t} + u_x\frac{\partial u_y}{\partial x} + u_y\frac{\partial u_y}{\partial y} + u_z\frac{\partial u_y}{\partial z}\right\} = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 u_y}{\partial x^2} + \frac{\partial^2 u_y}{\partial y^2} + \frac{\partial^2 u_y}{\partial z^2}\right) + \varrho\,f_y$$

$$\varrho\left\{\frac{\partial u_z}{\partial t} + u_x\frac{\partial u_z}{\partial x} + u_y\frac{\partial u_z}{\partial y} + u_z\frac{\partial u_z}{\partial z}\right\} = -\frac{\partial p}{\partial z} + \mu\left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_z}{\partial y^2} + \frac{\partial^2 u_z}{\partial z^2}\right) + \varrho\,f_z$$

Cylindrical coordinates

$$\varrho\left\{\frac{\partial u_r}{\partial t} + u_r\frac{\partial u_r}{\partial r} + \frac{u_\varphi}{r}\frac{\partial u_r}{\partial \varphi} - \frac{u_\varphi^2}{r} + u_z\frac{\partial u_r}{\partial z}\right\} = -\frac{\partial p}{\partial r} + \\ + \mu\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r u_r\right)\right] + \frac{1}{r^2}\frac{\partial^2 u_r}{\partial \varphi^2} - \frac{2}{r^2}\frac{\partial u_\varphi}{\partial \varphi} + \frac{\partial^2 u_r}{\partial z^2}\right\} + \varrho\,f_r$$

$$\varrho\left\{\frac{\partial u_\varphi}{\partial t} + u_r\frac{\partial u_\varphi}{\partial r} + \frac{u_\varphi}{r}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_r u_\varphi}{r} + u_z\frac{\partial u_\varphi}{\partial z}\right\} = -\frac{1}{r}\frac{\partial p}{\partial \varphi} + \\ + \mu\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r u_\varphi\right)\right] + \frac{1}{r^2}\frac{\partial^2 u_\varphi}{\partial \varphi^2} + \frac{2}{r^2}\frac{\partial u_r}{\partial \varphi} + \frac{\partial^2 u_\varphi}{\partial z^2}\right\} + \varrho\,f_\varphi$$

$$\varrho\left\{\frac{\partial u_z}{\partial t} + u_r\frac{\partial u_z}{\partial r} + \frac{u_\varphi}{r}\frac{\partial u_z}{\partial \varphi} + u_z\frac{\partial u_z}{\partial z}\right\} = -\frac{\partial p}{\partial z} + \\ + \mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left[r\frac{\partial u_z}{\partial r}\right] + \frac{1}{r^2}\frac{\partial^2 u_z}{\partial \varphi^2} + \frac{\partial^2 u_z}{\partial z^2}\right\} + \varrho\,f_z$$

Invariants of the deformation rate tensor

First invariant of the deformation rate tensor

$$\mathrm{I}_\Delta = \mathrm{tr}\,\vec{\vec{\Delta}} = \Delta_{ii}$$

Second invariant of the deformation rate tensor

$$\mathrm{II}_\Delta = \vec{\vec{\Delta}} : \vec{\vec{\Delta}} = \Delta_{ij}\,\Delta_{ji}$$

Alternative definition

$$\mathrm{II}'_\Delta = \frac{1}{2}\left[\left(\mathrm{tr}\,\vec{\vec{\Delta}}\right)^2 - \vec{\vec{\Delta}} : \vec{\vec{\Delta}}\right]$$

Third invariant of the deformation rate tensor

$$\mathrm{III}_\Delta = \det\vec{\vec{\Delta}} = \varepsilon_{ijk}\Delta_{1i}\,\Delta_{2j}\,\Delta_{3k}$$

Levi-Civita tensor of 3rd order

`https://www.wolframalpha.com/input/?i=LeviCivitaTensor[3]`

$$\varepsilon_{ijk} = \frac{1}{2}\,(i-j)\,(j-k)\,(k-i)$$

Non-Newtonian fluids

General viscous liquid

$$\vec{\vec{\tau}} = 2\eta\vec{\vec{\Delta}}$$

Power-law fluid

$$\eta = K\left(\sqrt{2\,\mathrm{II}_\Delta}\right)^{n-1}$$

Bingham plastic fluid

$$\eta = \mu_p + \frac{\tau_0}{\sqrt{2\,\mathrm{II}_\Delta}}\,;\quad \frac{1}{2}\vec{\vec{\tau}} : \vec{\vec{\tau}} > \tau_0^2$$

Heat transfer

$$\varrho c_{\mathrm{p}} \frac{\mathrm{D}T}{\mathrm{D}t} = -\nabla \bullet \vec{q} + \vec{\vec{\tau}} \, \mathbf{:} \, \vec{\vec{\Delta}} + \dot{Q}^{(\mathrm{g})}$$

Fourier-Kirchhoff equation $\lambda = \mathrm{const.}$

$$\varrho c_{\mathrm{p}} \frac{\mathrm{D}T}{\mathrm{D}t} = \lambda \nabla^2 T + \vec{\vec{\tau}} \, \mathbf{:} \, \vec{\vec{\Delta}} + \dot{Q}^{(\mathrm{g})}$$

$$\varrho c_{\mathrm{p}} \left\{ \frac{\partial T}{\partial t} + (\vec{u} \bullet \nabla)\, T \right\} = \lambda \nabla^2 T + \vec{\vec{\tau}} \, \mathbf{:} \, \vec{\vec{\Delta}} + \dot{Q}^{(\mathrm{g})}$$

Cartesian coordinates

$$\varrho c_p \left\{ \frac{\partial T}{\partial t} + u_x \frac{\partial T}{\partial x} + u_y \frac{\partial T}{\partial y} + u_z \frac{\partial T}{\partial z} \right\} = \lambda \left\{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right\} + \tau_{ij} \Delta_{ji} + \dot{Q}^{(\mathrm{g})}$$

Cylindrical coordinates

$$\varrho c_p \left\{ \frac{\partial T}{\partial t} + u_r \frac{\partial T}{\partial r} + \frac{u_\varphi}{r} \frac{\partial T}{\partial \varphi} + u_z \frac{\partial T}{\partial z} \right\} = \lambda \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 T}{\partial \varphi^2} + \frac{\partial^2 T}{\partial z^2} \right\} + \tau_{ij} \Delta_{ji} + \dot{Q}^{(\mathrm{g})}$$

Fourier's law

$$\vec{q} = -\lambda \nabla T$$

Cartesian coordinates

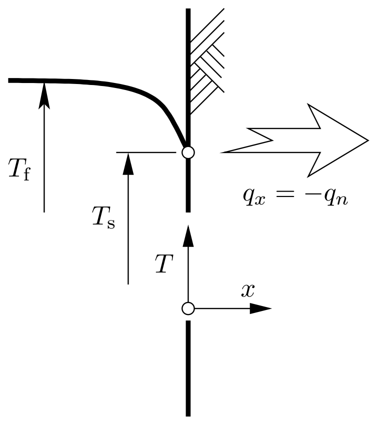
$$q_x = -\lambda \frac{\partial T}{\partial x}$$
$$q_y = -\lambda \frac{\partial T}{\partial y}$$
$$q_z = -\lambda \frac{\partial T}{\partial z}$$

Cylindrical coordinates

$$q_r = -\lambda \frac{\partial T}{\partial r}$$
$$q_\varphi = -\lambda \frac{1}{r} \frac{\partial T}{\partial \varphi}$$
$$q_z = -\lambda \frac{\partial T}{\partial z}$$

Newton's hypothesis

$$q_n = \vec{q} \bullet \vec{n} = \alpha \left(T_{\mathrm{s}} - T_{\mathrm{f}} \right)$$



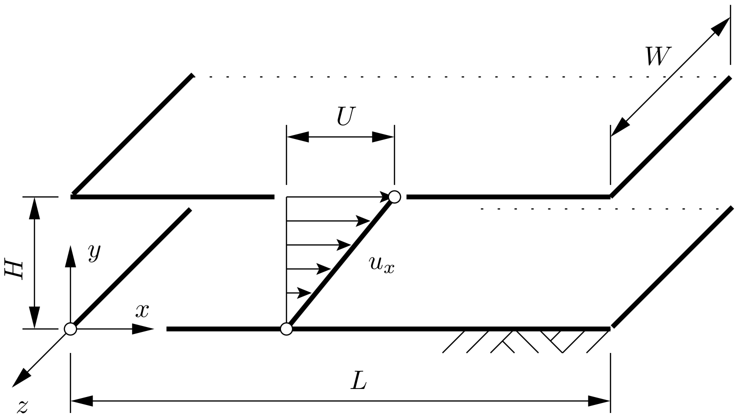
Stephan-Boltzmann law

$$E_{E,0} = \sigma^{(\mathrm{S})}\, T^4$$

$$\sigma^{(S)} = 5.6705 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$$

Momentum transport/Exact solutions

Flow between two plates; $u_x|_{y=0} = 0$**,** $u_x|_{y=H} = U$



$$u_x = U\frac{y}{H}$$

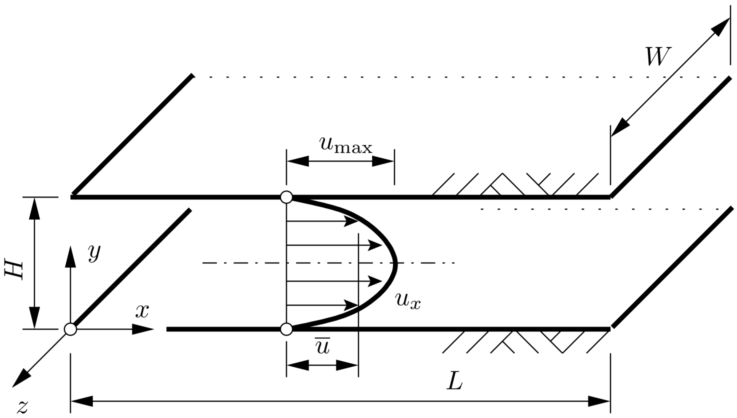
$$\tau_{yx} = \mu\frac{U}{H}$$

$$\dot{V} = \frac{1}{2}UWH$$

$$0 = \frac{\mathrm{d}\tau_{yx}}{\mathrm{d}y}$$

$$\tau_{yx} = \mu\frac{\mathrm{d}u_x}{\mathrm{d}y}$$

Pressure-driven flow between two plates; $u_x|_{y=0} = 0$**,** $u_x|_{y=H} = 0$



$$u_x = \frac{\Delta p H^2}{2\mu L}\left[\left(\frac{y}{H}\right)^2 - \frac{y}{H}\right]$$

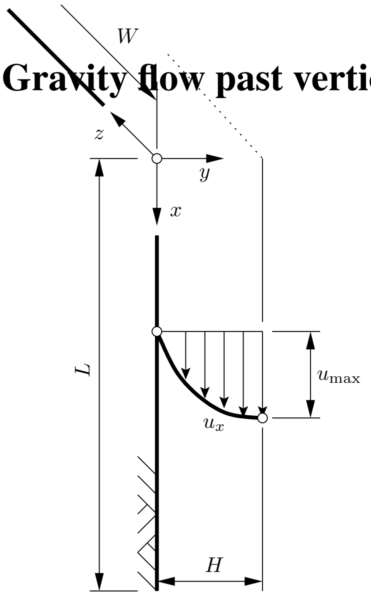
$$\tau_{yx} = \frac{\Delta p H}{2L}\left[2\frac{y}{H} - 1\right]$$

$$\dot{V} = -\frac{\Delta p W H^3}{12\mu L}$$

$$0 = -\frac{\mathrm{d}p}{\mathrm{d}x} + \frac{\mathrm{d}\tau_{yx}}{\mathrm{d}y}$$

$$\tau_{yx} = \mu\frac{\mathrm{d}u_x}{\mathrm{d}y}$$

Gravity flow past vertical wall; $u_x|_{y=0} = 0$, $\mathrm{d}u_x/\mathrm{d}y|_{y=H} = 0$



$$u_x = \frac{\varrho g H^2}{\mu}\left[\frac{y}{H} - \frac{1}{2}\left(\frac{y}{H}\right)^2\right]$$

$$\tau_{yx} = \varrho g H\left[1 - \frac{y}{H}\right]$$

$$\dot{V} = \frac{\varrho g W H^3}{3\mu}$$

$$0 = \frac{\mathrm{d}\tau_{yx}}{\mathrm{d}y} + \varrho g$$

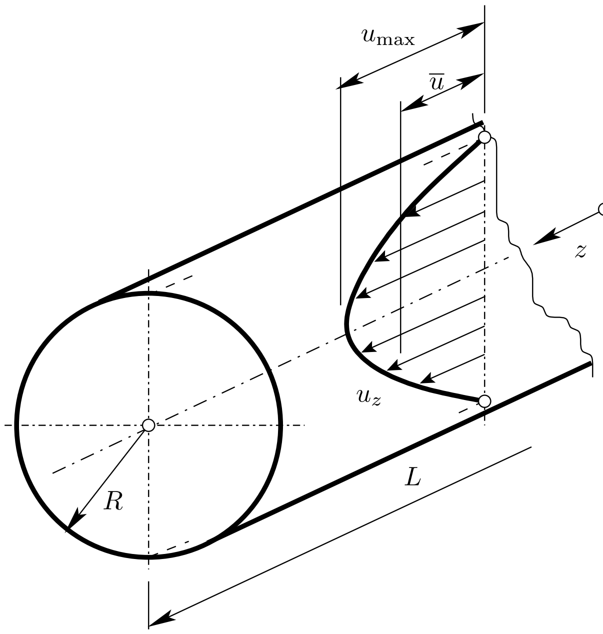
$$\tau_{yx} = \mu\frac{\mathrm{d}u_x}{\mathrm{d}y}$$

Flow regime

$$\mathrm{Re} = \frac{4(\dot{V}/W)\varrho}{\mu} = \frac{4\bar{u}H\varrho}{\mu}$$

$\mathrm{Re} < 25$	laminar flow regime
$25 < \mathrm{Re} < 1000$	pseudolaminar flow regime (laminar in the core, waves and vortices at the surface
$1000 < \mathrm{Re} < 1500$	transient flow regime
$1500 < \mathrm{Re}$	turbulent flow regime

Pressure-driven axial flow in pipe; $u_z|_{r=R} = 0$, $\mathrm{d}u_z/\mathrm{d}r|_{r=0} = 0$



$$u_z = -\frac{\Delta p R^2}{4\mu L}\left[1 - \left(\frac{r}{R}\right)^2\right] = 2\overline{u}\left[1 - \left(\frac{r}{R}\right)^2\right]$$

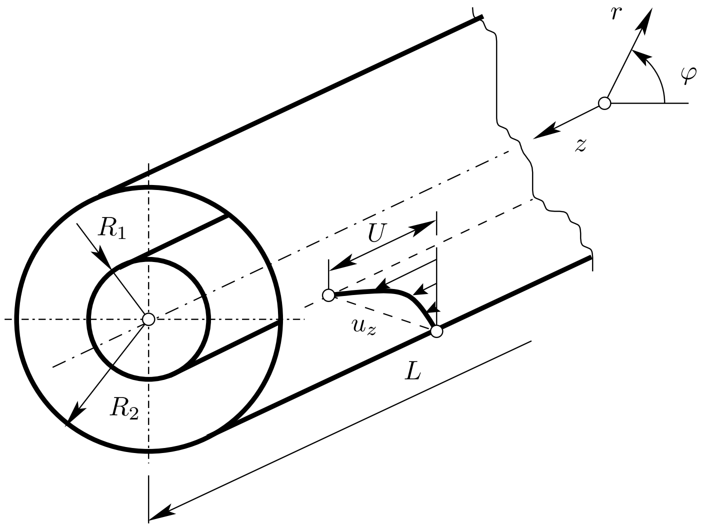
$$\tau_{rz} = \frac{\Delta p r}{2L}$$

$$\dot{V} = -\frac{\pi \Delta p R^4}{8\mu L}$$

$$0 = -\frac{\Delta p}{L} + \frac{1}{r}\frac{\mathrm{d}\left(r\tau_{rz}\right)}{\mathrm{d}r}$$

$$\tau_{rz} = \mu\frac{\mathrm{d}u_z}{\mathrm{d}r}$$

Axial flow in cylindrical gap



$$u_z = \frac{U}{\ln\kappa}\ln\frac{r}{R_2}; \quad \kappa = \frac{R_1}{R_2}$$

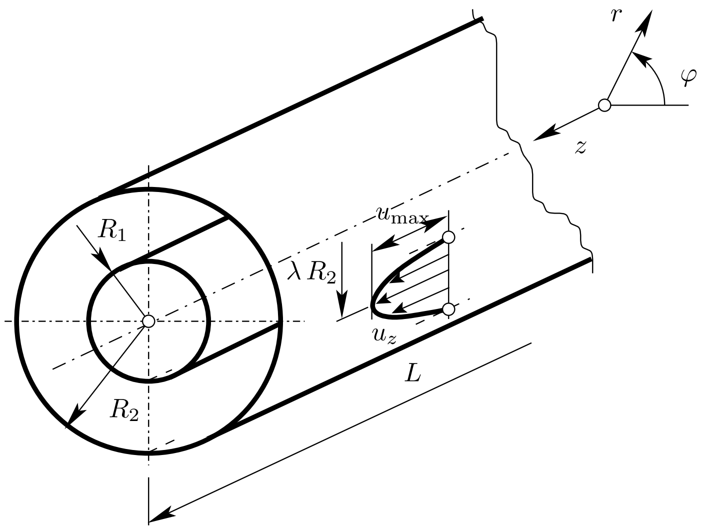
$$\tau_{rz} = \frac{\mu\, U}{\ln\kappa}\,\frac{1}{r}$$

$$\dot{V} = -\frac{\pi\, R_2^2\, U}{2\ln\kappa}\left[1-\kappa^2+2\kappa^2\ln\kappa\right]$$

$$\dot{V} = \frac{\pi\, R_2^2\, U}{2}\left[\frac{1-\kappa^2}{\ln(1/\kappa)}-2\kappa^2\right]$$

$$0 = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\tau_{rz}\right)$$
$$\tau_{rz} = \mu\frac{\mathrm{d}u_z}{\mathrm{d}r}$$

Pressure-driven axial flow in cylindrical gap; $u_z|_{r=R_1} = 0$, $u_z|_{r=R_2} = 0$

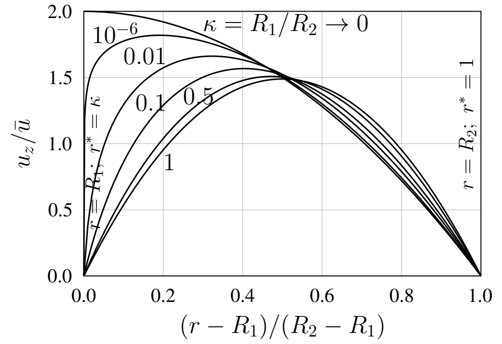


$$u_z = -\frac{\Delta p R_2^2}{4\mu L}\left[1-\left(\frac{r}{R_2}\right)^2+\frac{1-\kappa^2}{\ln(1/\kappa)}\ln\frac{r}{R_2}\right]$$
$$\kappa = \frac{R_1}{R_2}$$

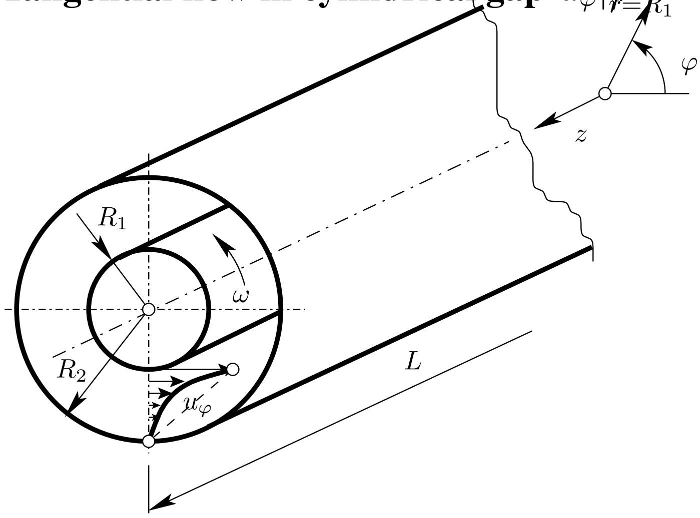
$$\tau_{rz} = \frac{\Delta p R_2}{2L}\left[\frac{r}{R_2}-\lambda^2\frac{R_2}{r}\right];\ \lambda^2 = \frac{1}{2}\frac{1-\kappa^2}{\ln(1/\kappa)}$$

$$\dot{V} = -\frac{\pi\Delta p R_2^4}{8\mu L}\left[1-\kappa^4-\frac{\left(1-\kappa^2\right)^2}{\ln(1/\kappa)}\right]$$

$$0 = -\frac{\Delta p}{L}+\frac{1}{r}\frac{\mathrm{d}\left(r\tau_{rz}\right)}{\mathrm{d}r}$$
$$\tau_{rz} = \mu\frac{\mathrm{d}u_z}{\mathrm{d}r}$$



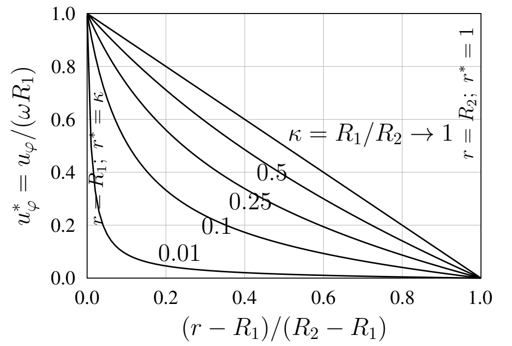
Tangential flow in cylindrical gap $u_\varphi|_{r=R_1} = \omega\, R_1$, $u_\varphi|_{r=R_2} = 0$



$$u_\varphi = \omega R_1 \frac{\kappa}{1-\kappa^2}\left[\frac{R_2}{r} - \frac{r}{R_2}\right];\ \kappa = \frac{R_1}{R_2}$$

$$\tau_{r\varphi} = -2\mu\omega \frac{\kappa^2}{1-\kappa^2}\left(\frac{R_2}{r}\right)^2$$

$$M_k = \frac{4\pi\mu\omega R_1^2 L}{1-\kappa^2}$$

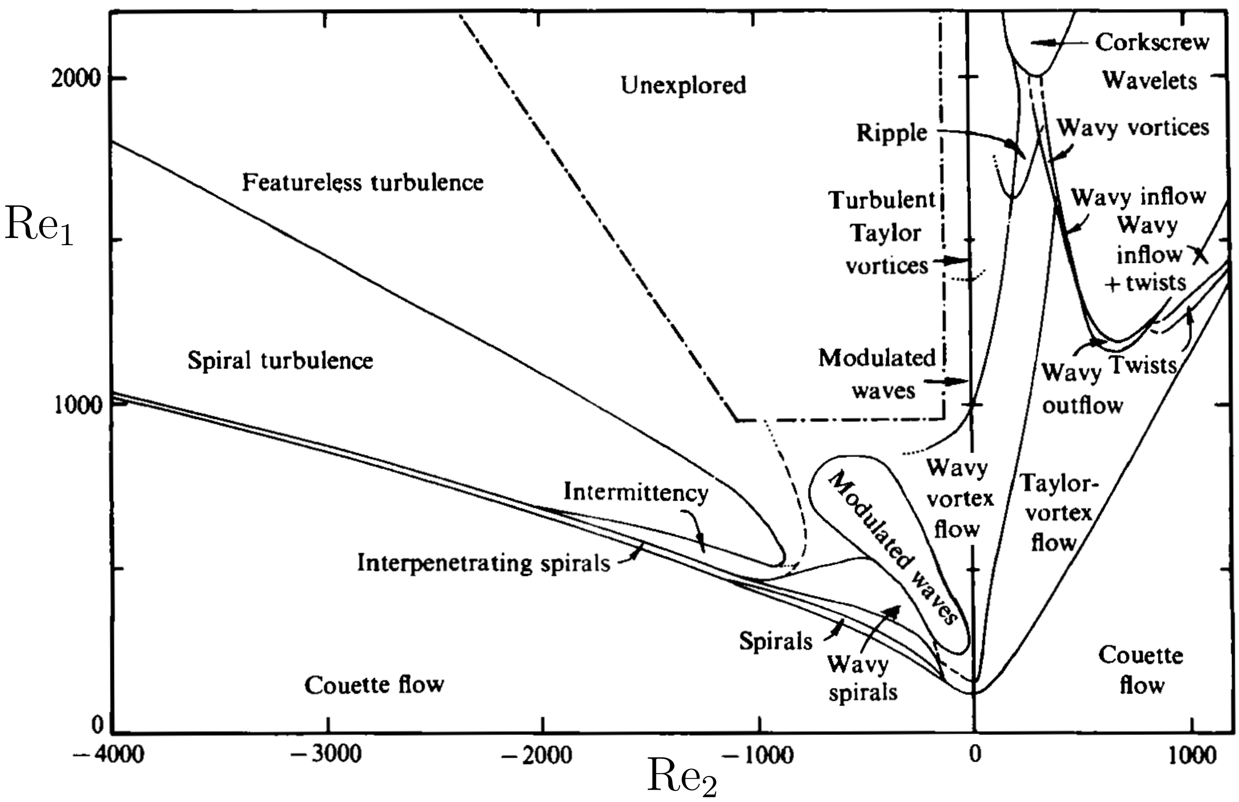


$$0 = \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\tau_{r\varphi}\right)$$

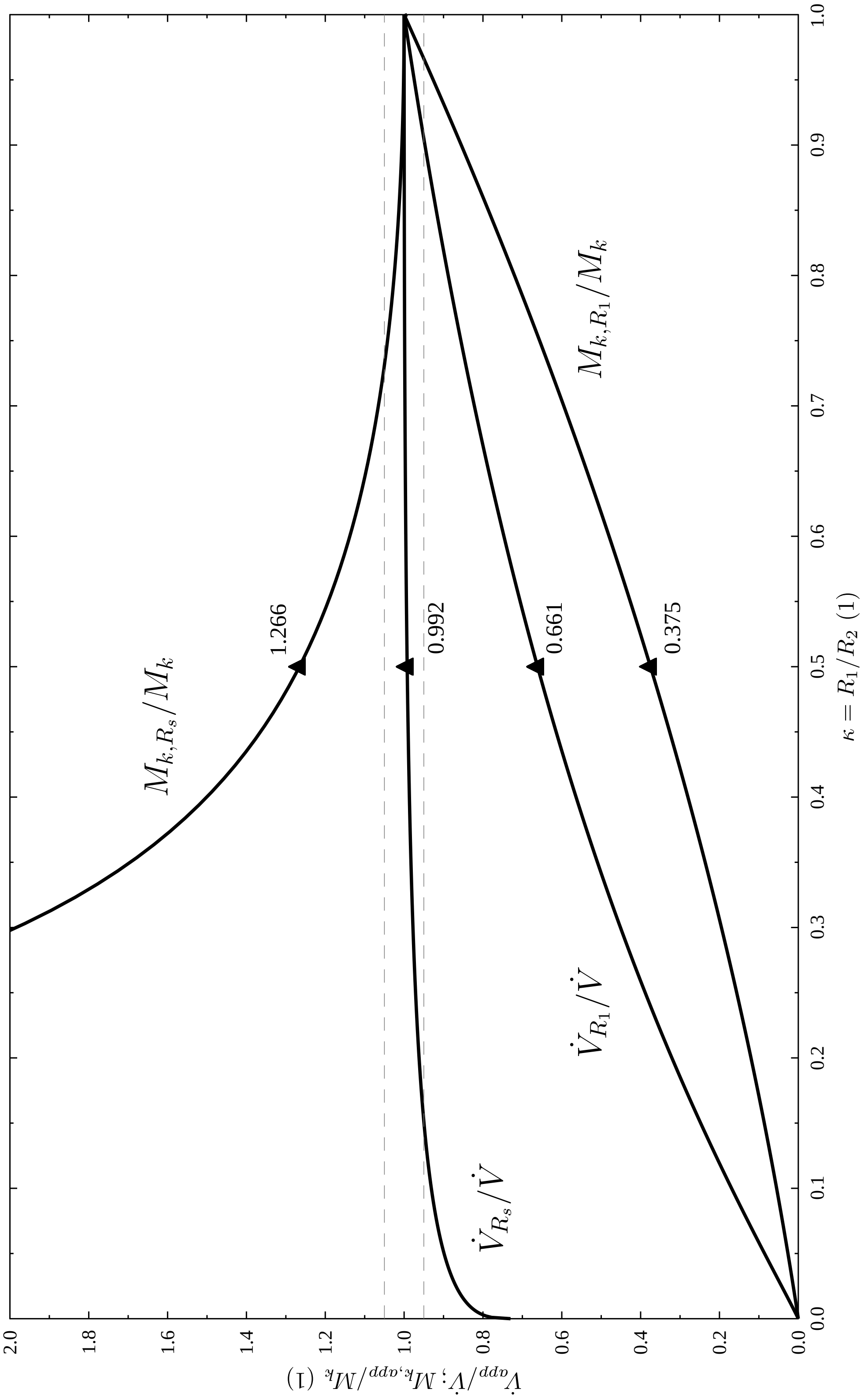
$$\tau_{r\varphi} = \mu\, r \frac{\mathrm{d}}{\mathrm{d}r}\left(\frac{u_\varphi}{r}\right)$$

Flow regime

$$\mathrm{Re}_1 = \frac{\omega_1 R_1 (R_2 - R_1)\,\varrho}{\mu}, \qquad \mathrm{Re}_2 = \frac{\omega_2 R_2 (R_2 - R_1)\,\varrho}{\mu}$$



Graphical illustration of flow regimes in a gap between two independently rotating cylinders. The dashed lines depict the transition betwee regimes which are difficult to observe. The dotted lines are expected transition boundaries. (Andredeck, C. D., Liu, S. S., Swinney, H. L., Flow regimes in a circular Couette system with independently rotating cylinders, J. Fluid. Mech., 164 (1986), pp. 155–183.)



$$\frac{M_{k,R_1}}{M_k} = \frac{\mu\frac{\omega R_1}{R_2-R_1}2\pi R_1^2 L}{\frac{4\pi\mu\omega R_1^2 L}{1-\kappa^2}} = \frac{\frac{R_1}{R_2-R_1}}{\frac{2}{1-\kappa^2}} = \frac{1}{2}\kappa\,(1+\kappa)$$

$$\frac{M_{k,R_s}}{M_k} = \frac{\mu\frac{\omega R_s}{R_2-R_1}2\pi R_s^2 L}{\frac{4\pi\mu\omega R_1^2 L}{1-\kappa^2}} = \frac{\frac{R_s^3}{R_2-R_1}}{\frac{2R_1^2}{1-\kappa^2}} = \frac{(1+\kappa)^4}{16\,\kappa^2}$$

Graphical illustration of approximate and accurate solution ratio for the case of pressure-driven axial flow in cylindrical gap ($\dot{V}$) and tangential flow in the cylindrical gap with rotating inner cylinder (M_k) with respect to ratio of radiuses. The approximate solution (transformed to a plane) is performed for two cases, one is for the transformation with respect to the inner radius, R_1, and the second one is for the mean radius $R_s = (R_1 + R_2)/2$ (see indices).

Pressure-driven axial flow of power-law liquid in pipe

$$0 = -\frac{\Delta p}{L} + \frac{1}{r}\frac{\mathrm{d}\left(r\tau_{rz}\right)}{\mathrm{d}r}$$

$$\tau_{rz} = K\left|\frac{\mathrm{d}u_z}{\mathrm{d}r}\right|^{n-1}\frac{\mathrm{d}u_z}{\mathrm{d}r}$$

$$u_z|_{r=R} = 0;\ \mathrm{d}u_z/\mathrm{d}r|_{r=0} = 0\ \left(\tau_{rz}|_{r=0} = 0\right)$$

Integrating the momentum equation and applying the second boundary condition will give

$$\tau_{rz} = \frac{\Delta p\, r}{2L}\,.$$

We add the constitutive equation.

$$\tau_{rz} = \frac{\Delta p\, r}{2L} = K\left|\frac{\mathrm{d}u_z}{\mathrm{d}r}\right|^{n-1}\frac{\mathrm{d}u_z}{\mathrm{d}r}$$

Assuming positive value of Δp, the value of $\mathrm{d}u_z/\mathrm{d}r$ will be positive as well. After removed the absolute value, we can integrate (using the second boundary condition).

$$\frac{\Delta p\, r}{2L} = K\left(\frac{\mathrm{d}u_z}{\mathrm{d}r}\right)^n;\ \Delta p > 0;\ 0 \le r \le R$$

$$\left(\frac{\Delta p r}{2KL}\right)^{1/n} = \frac{\mathrm{d}u_z}{\mathrm{d}r}$$

$$\int\limits_R^r \left(\frac{\Delta p r}{2KL}\right)^{1/n}\mathrm{d}r = \int\limits_0^{u_z}\mathrm{d}u_z$$

$$u_z = \frac{n}{n+1}\left(\frac{\Delta p}{2KL}\right)^{1/n} R^{\frac{n+1}{n}}\left[\left(\frac{r}{R}\right)^{\frac{n+1}{n}} - 1\right]$$

The volumetric flow rate can be determined by integration

$$\dot{V} = 2\pi\int\limits_0^R r u_z \mathrm{d}r = \left|\begin{array}{c} r^* = r/R \\ \mathrm{d}r^* = \mathrm{d}r/R\end{array}\right| = 2\pi R^2\int\limits_0^1 r^* u_z \mathrm{d}r^* =$$

$$= 2\pi R^2\frac{n}{n+1}\left(\frac{\Delta p}{2KL}\right)^{1/n} R^{\frac{n+1}{n}}\int\limits_0^1 r^*\left[r^{*\,\frac{n+1}{n}} - 1\right]\mathrm{d}r^* =$$

$$= -\pi R^2\frac{n}{3n+1}\left(\frac{\Delta p}{2KL}\right)^{1/n} R^{\frac{n+1}{n}}$$

Mean (average) volumetric velocity.

$$\bar{u} = \frac{\dot{V}}{\pi R^2} = -\frac{n}{3n+1}\left(\frac{\Delta p}{2KL}\right)^{1/n} R^{\frac{n+1}{n}}$$

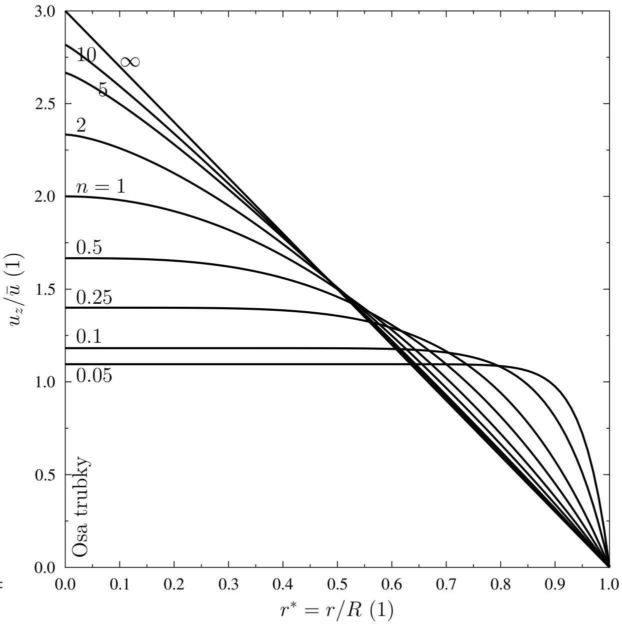
Dimensionless velocity related to the average volumetric velocity for constant volumetric flow rate.

$$u_z^* = \frac{u_z}{\bar{u}} = \frac{3n+1}{n+1}\left[1 - \left(\frac{r}{R}\right)^{\frac{n+1}{n}}\right]$$

From the relation describing the radial profile of axial flow velocity in the channel, we can see that for positive value of pressure difference $\Delta p > 0$, the velocity profile is negative therefore the liquid flow is in the opposite direction to axis z. This is the consequence of larger pressure at the end than at the beginning of the channel. The pressure difference have to be negative in the case of flow direction same as direction of z-axis, that is $\Delta p < 0$. In such case, the velocity profile will be postive. The problem of calculating the the velocity from the derived relation is that general root of negative value is not defined. Nevertheless, we can easily transform the relation to a form which enables the calculation even for negative pressure differences.

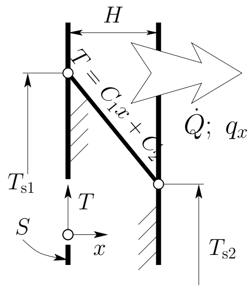
$$u_z = \frac{n}{n+1}\left|\frac{\Delta p}{2KL}\right|^{1/n} R^{\frac{n+1}{n}}\left[\left(\frac{r}{R}\right)^{\frac{n+1}{n}} - 1\right]\mathrm{sig}\left(\Delta p\right)$$

Function $\mathrm{sig}\left(\Delta p\right)$ just express the sign of the pressure difference. This trick can be used in other cases, especially when we do not want to define a general root of real number as a new mathematical operation.



Heat transfer

Heat conduction in plane wall



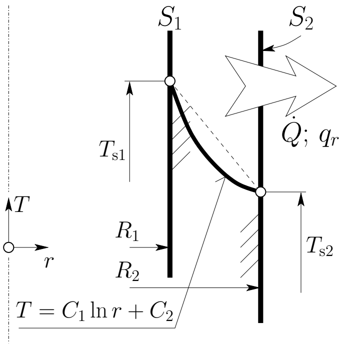
$$T = -\left(T_{s1} - T_{s2}\right)\frac{x}{H} + T_{s1}$$

$$q_x = \frac{\lambda}{H}\left(T_{s1} - T_{s2}\right)$$

$$\dot{Q} = \frac{\lambda}{H} S \left(T_{s1} - T_{s2}\right)$$

$$0 = \frac{\mathrm{d}^2 T}{\mathrm{d}x^2}$$

Heat conduction in cylindrical wall



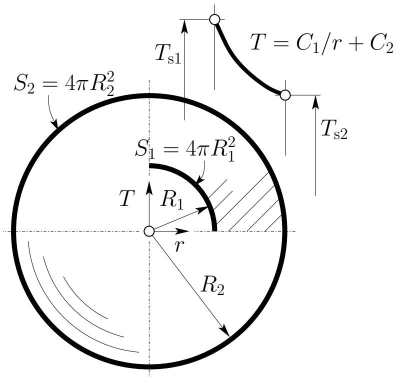
$$\frac{T - T_{s2}}{T_{s1} - T_{s2}} = \frac{\ln r/R_2}{\ln R_1/R_2}$$

$$q_r = \frac{\lambda}{\ln R_2/R_1}\left(T_{s1} - T_{s2}\right)\frac{1}{r}$$

$$\dot{Q} = \frac{2\pi L \lambda}{\ln R_2/R_1}\left(T_{s1} - T_{s2}\right)$$

$$0 = \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}T}{\mathrm{d}r}\right)$$

Heat conduction in spherical wall



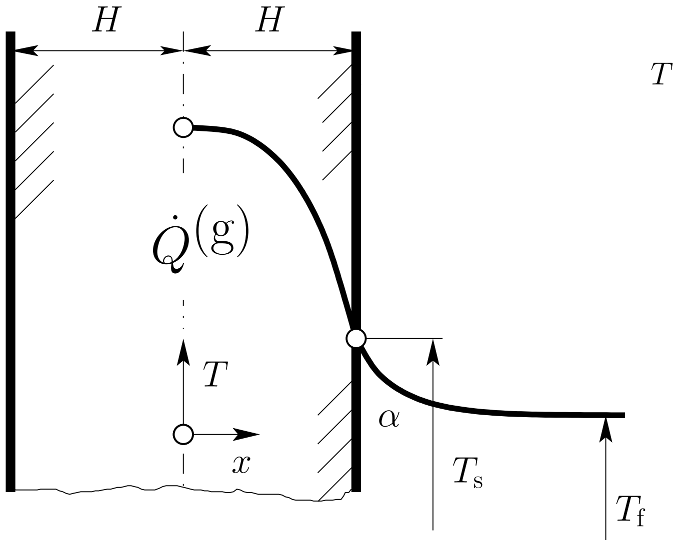
$$\frac{T - T_{s2}}{T_{s1} - T_{s2}} = \frac{1/R_2}{\frac{1}{R_1} - \frac{1}{R_2}}\left(1 - \frac{R_2}{r}\right)$$

$$q_r = \frac{\lambda}{\frac{1}{R_1} - \frac{1}{R_2}}\left(T_{s1} - T_{s2}\right)\frac{1}{r^2}$$

$$\dot{Q} = \frac{4\pi\lambda}{\frac{1}{R_1} - \frac{1}{R_2}}\left(T_{s1} - T_{s2}\right)$$

$$0 = \frac{1}{r^2}\frac{\mathrm{d}}{\mathrm{d}r}\left(r^2\frac{\mathrm{d}T}{\mathrm{d}r}\right)$$

Heat conduction in plane wall with volume heat source



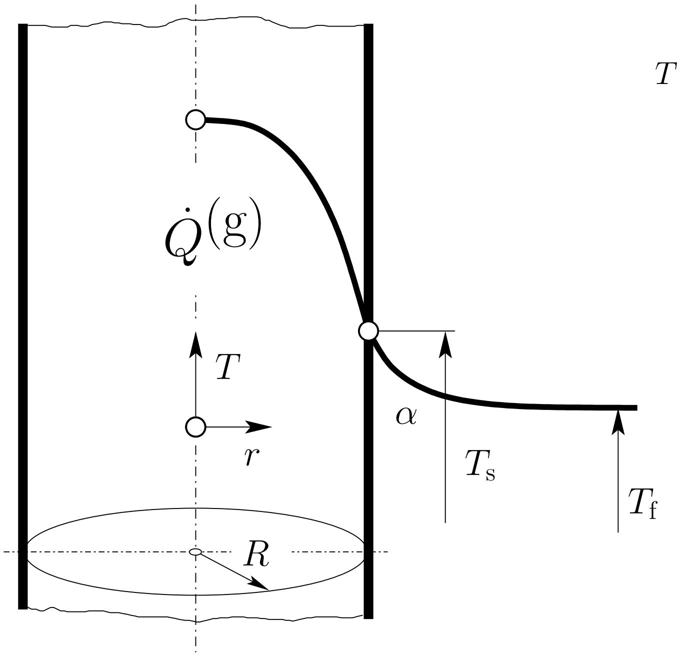
$$T - T_f = \frac{\dot{Q}^{(g)}}{2\lambda} H^2 \left[1 + \frac{4}{\mathrm{Bi}} - \left(\frac{x}{H} \right)^2 \right]$$

$$\mathrm{Bi} = \frac{2\alpha H}{\lambda}$$

$$\left[\dot{Q}^{(g)} \right] = \mathrm{W\,m^{-3}}$$

$$0 = \lambda \frac{d^2 T}{dx^2} + \dot{Q}^{(g)}$$

Heat conduction in cylindrical wall with volume heat source



$$T - T_f = \frac{\dot{Q}^{(g)}}{4\lambda} R^2 \left[1 + \frac{4}{\mathrm{Bi}} - \left(\frac{r}{R} \right)^2 \right]$$

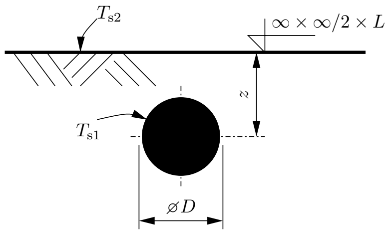
$$\mathrm{Bi} = \frac{2\alpha R}{\lambda}$$

$$\left[\dot{Q}^{(g)} \right] = \mathrm{W\,m^{-3}}$$

$$0 = \lambda \frac{1}{r} \frac{d}{dr} \left(r \frac{dT}{dr} \right) + \dot{Q}^{(g)}$$

Two-dimensional heat conduction and shape factor

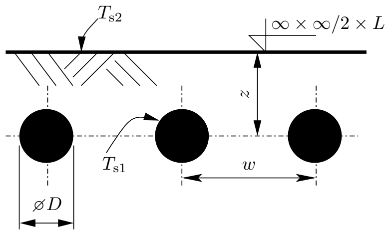
$$\dot{Q} = \lambda \, S_\mathrm{T} \; (T_\mathrm{s1} - T_\mathrm{s2})$$



$$S_\mathrm{T} = \frac{2\,\pi\,L}{\cosh^{-1}\dfrac{2\,z}{D}}$$

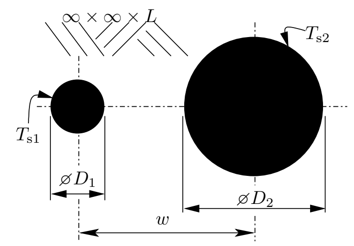
$$L \gg D$$

See [Incropera, F. P., DeWitt, D. P., Bergman, T. L., Lavine, A. S.: Fundamentals of Heat and Mass Transfer, 6th Edition, John Wiley & Sons, Inc. (2007)] except …



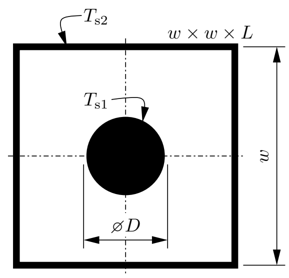
$$S_\mathrm{T} = \frac{2\,\pi\,L}{\ln\left[\dfrac{2\,w}{\pi\,D}\sinh\left(2\pi\dfrac{z}{w}\right)\right]}$$

[Šesták, J., Rieger, F.: Přenos hybnosti, tepla a hmoty, Vydavatelství ČVUT, Praha (1998)]



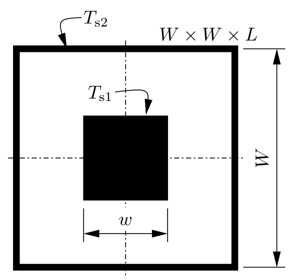
$$S_\mathrm{T} = \frac{2\,\pi\,L}{\cosh^{-1}\left(\dfrac{4\,w^2 - D_1^2 - D_2^2}{2\,D_1 D_2}\right)}$$

$$L \gg D_1,\; D_2$$
$$L \gg w$$



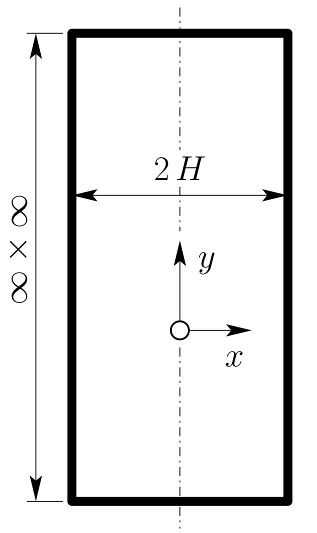
$$S_\mathrm{T} = \frac{2\,\pi\,L}{\ln\dfrac{1.08\,w}{D}}$$

$$L \gg w$$
$$w > D$$



$$S_\mathrm{T} = \frac{2\,\pi\,L}{0.785\ln\dfrac{W}{w}}; \quad W/w < 1.4; \quad L \gg w$$

$$S_\mathrm{T} = \frac{2\,\pi\,L}{0.930\ln\dfrac{W}{w} - 0.05}; \quad W/w > 1.4; \quad L \gg w$$



$$\frac{\partial T}{\partial t} = a\frac{\partial^2 T}{\partial x^2}$$

$$T^* = \frac{T - T_\mathrm{f}}{T_0 - T_\mathrm{f}}$$

$$x^* = \frac{x}{H}$$

$$t^* = \frac{at}{H^2}$$

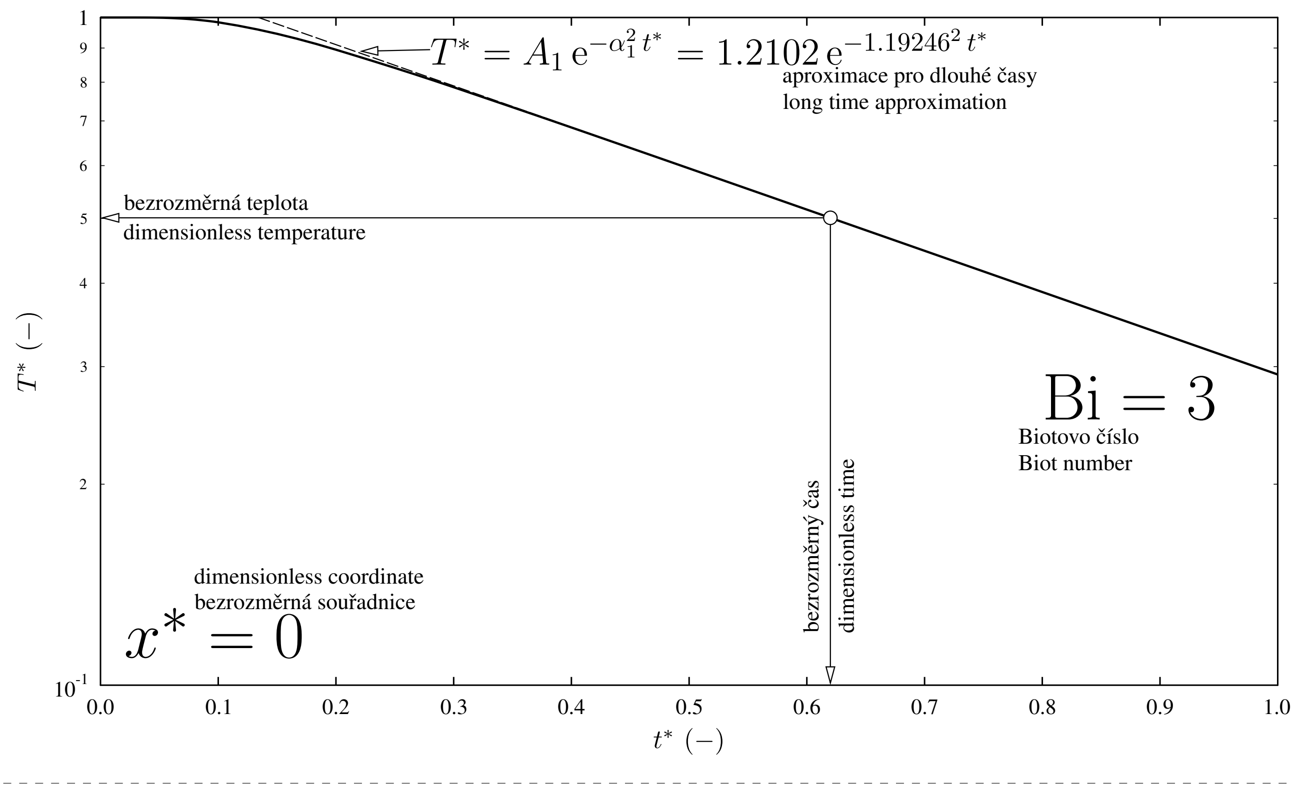
$$\mathrm{Bi} = \frac{\alpha H}{\lambda}$$

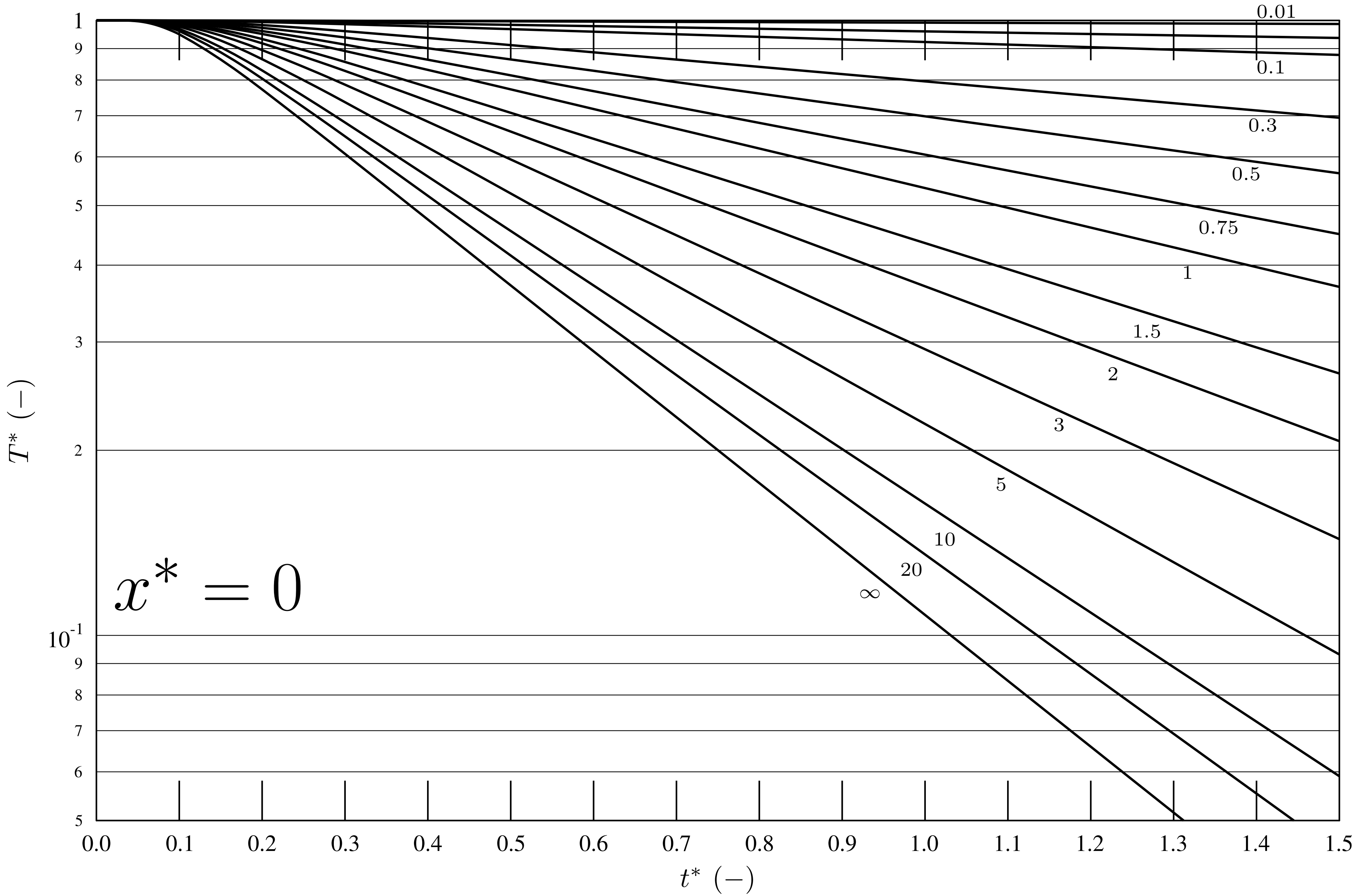
$$T^* = \sum_{j=1}^{\infty} \underbrace{\frac{2\sin\alpha_j}{\alpha_j + \sin\alpha_j \cos\alpha_j}}_{A_n} \cos\left(\alpha_j x^*\right) \exp\left(-\alpha_j^2 t^*\right)$$

Eigenvalue α_j is a solution of equation

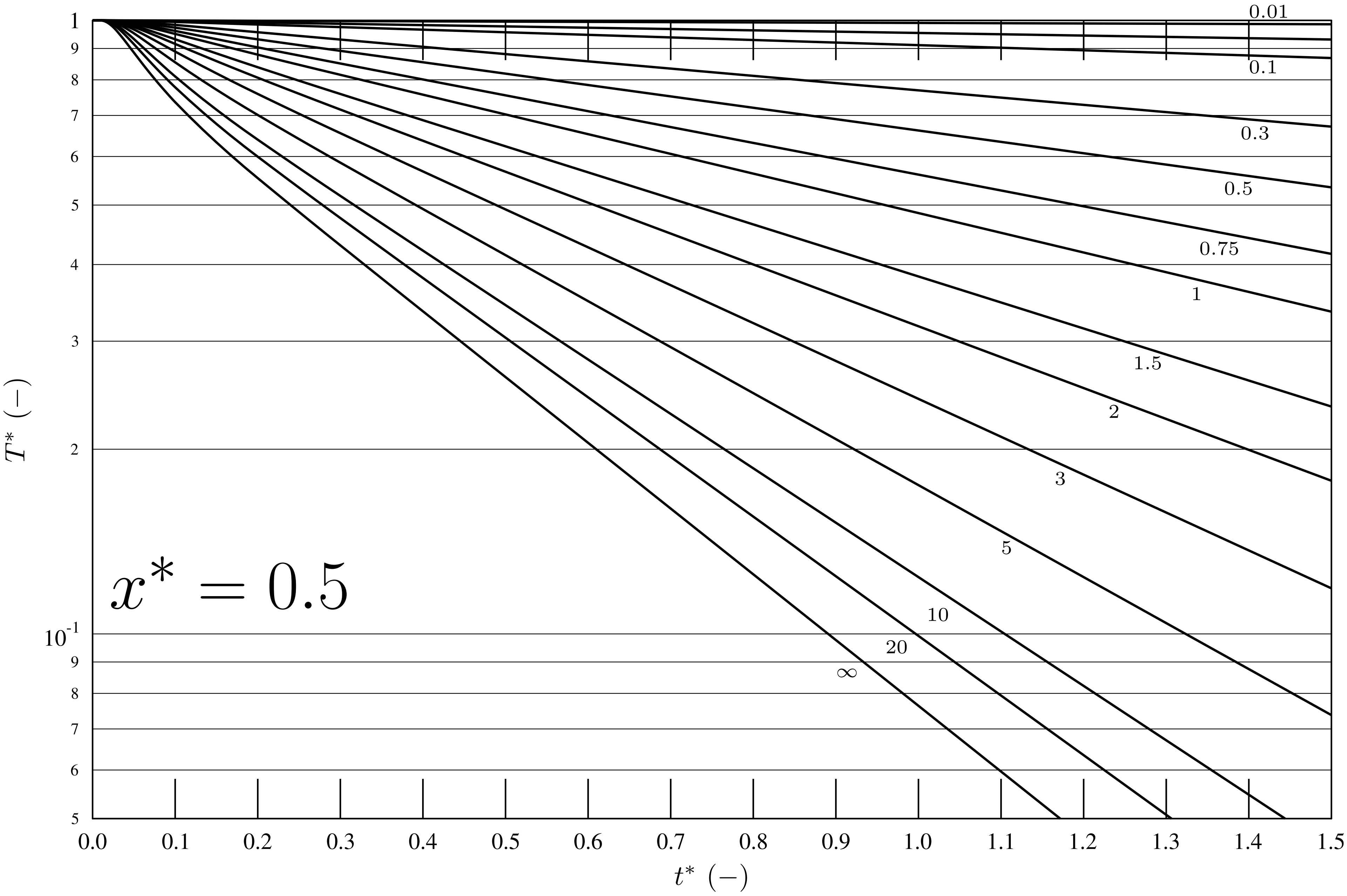
$$0 = \alpha_j - \mathrm{Bi}\,\mathrm{cotan}\,\alpha_j$$

The basic principle of reading the dimensionless temperature with respect to dimensionless time for infinite plate, infinite cylinder, and sphere, for large times.

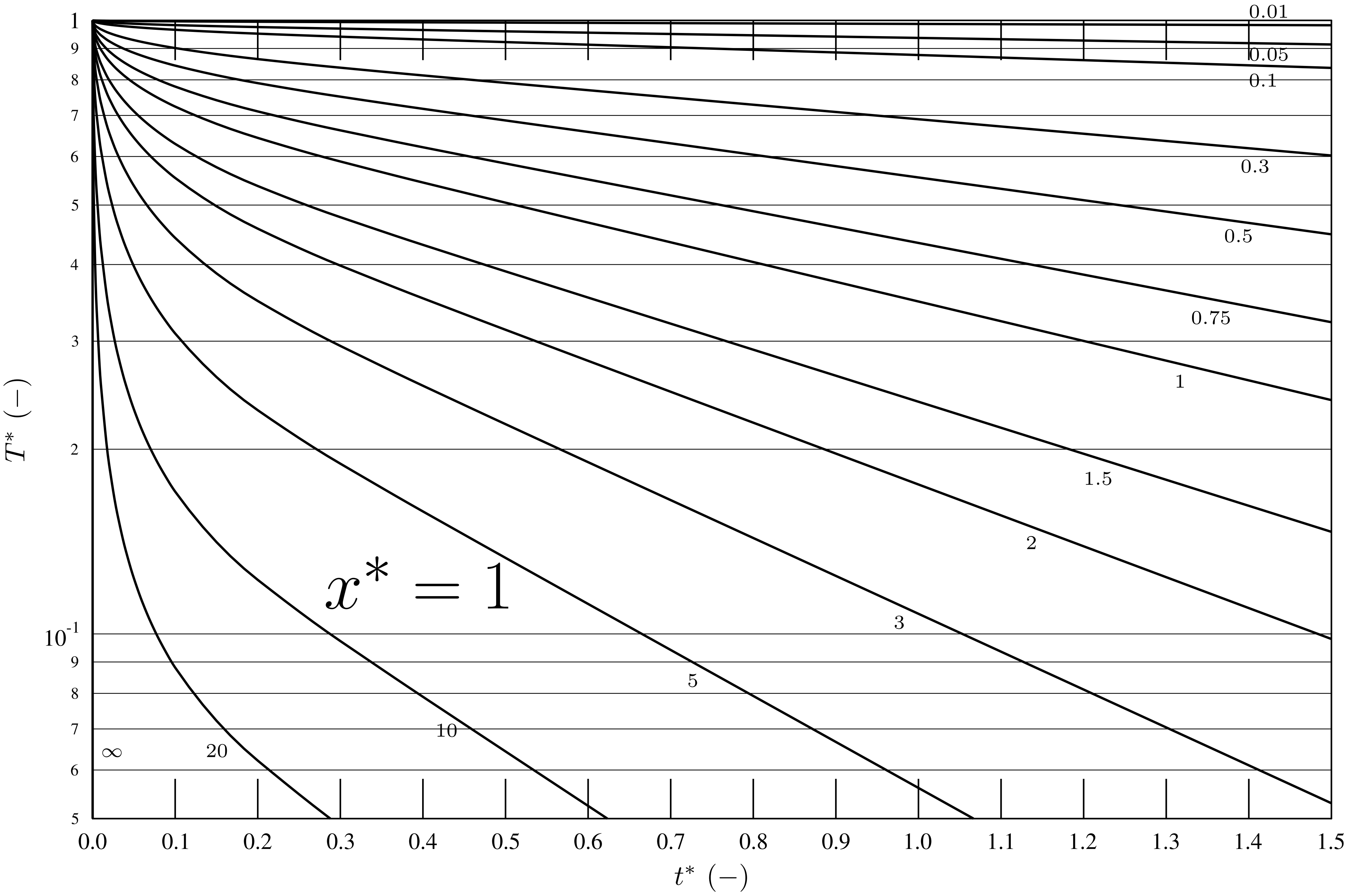




$x^* = 0$
T^* (−)
t^* (−)
1
9
8
7
6
5
4
3
2
10^{-1}
9
8
7
6
5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
0.01
0.1
0.3
0.5
0.75
1
1.5
2
3
5
10
20
∞

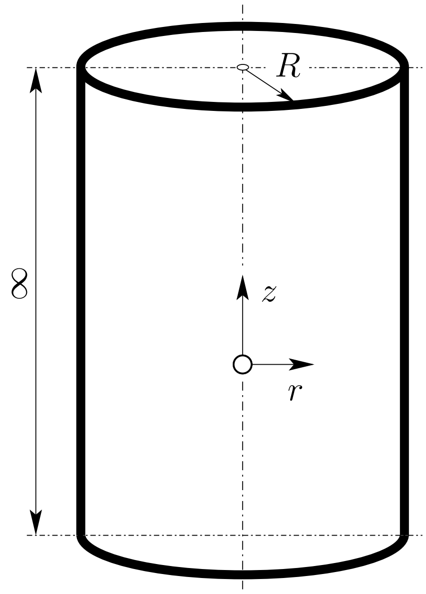


$x^* = 0.5$
0.01
0.1
0.3
0.5
0.75
1
1.5
2
3
5
10
20
∞
T^* (−)
1
9
8
7
6
5
4
3
2
10^{-1}
9
8
7
6
5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
t^* (−)



$x^* = 1$
T^* (−)
t^* (−)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1
9
8
7
6
5
4
3
2
10^{-1}
9
8
7
6
5
0.01
0.05
0.1
0.3
0.5
0.75
1
1.5
2
3
5
10
20
∞

Transient heat conduction in infinite cylinder



$$T^* = \frac{T - T_\mathrm{f}}{T_0 - T_\mathrm{f}}$$

$$r^* = \frac{r}{R}$$

$$t^* = \frac{at}{R^2}$$

$$\mathrm{Bi} = \frac{\alpha R}{\lambda}$$

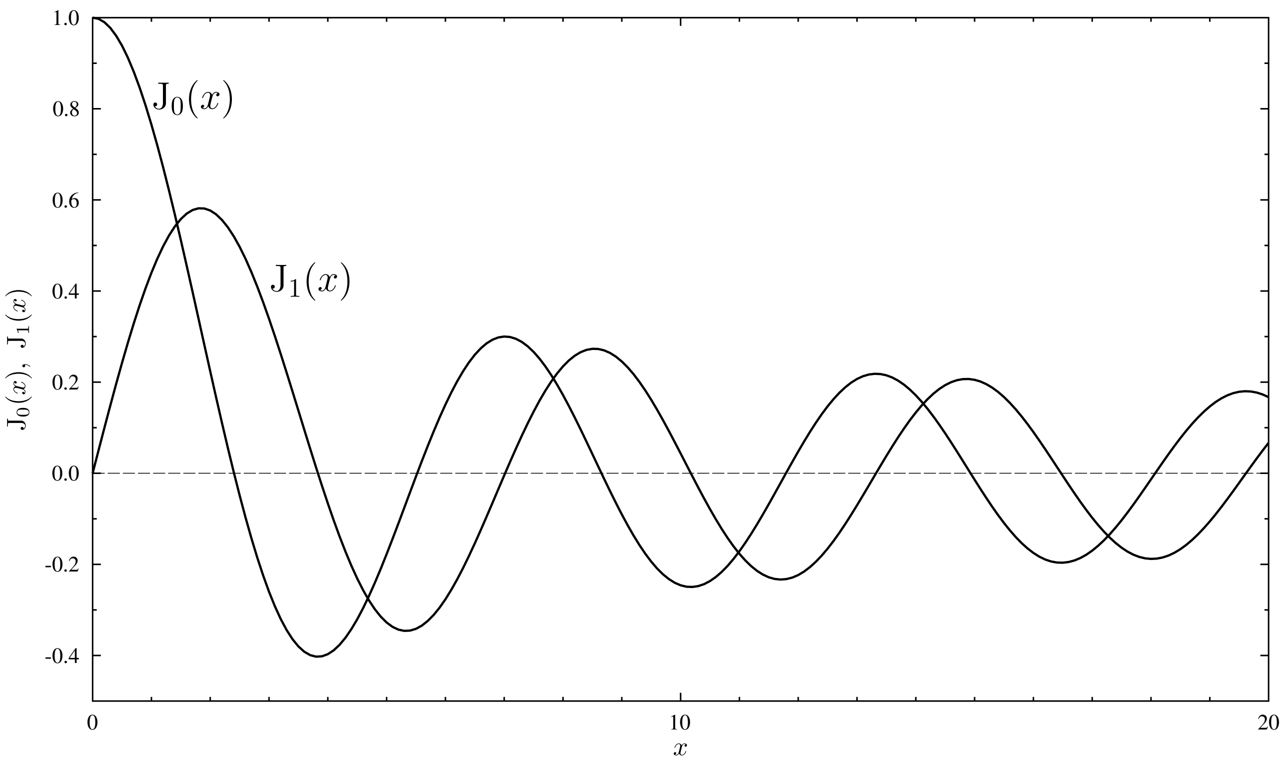
$$T^* = \sum_{j=1}^{\infty} \underbrace{\frac{2}{\alpha_j \mathrm{J}_1(\alpha_j)\left[(\alpha_j/\mathrm{Bi})^2 + 1\right]}}_{A_n} \mathrm{J}_0(\alpha_j r^*) \exp\left(-\alpha_j^2 t^*\right)$$

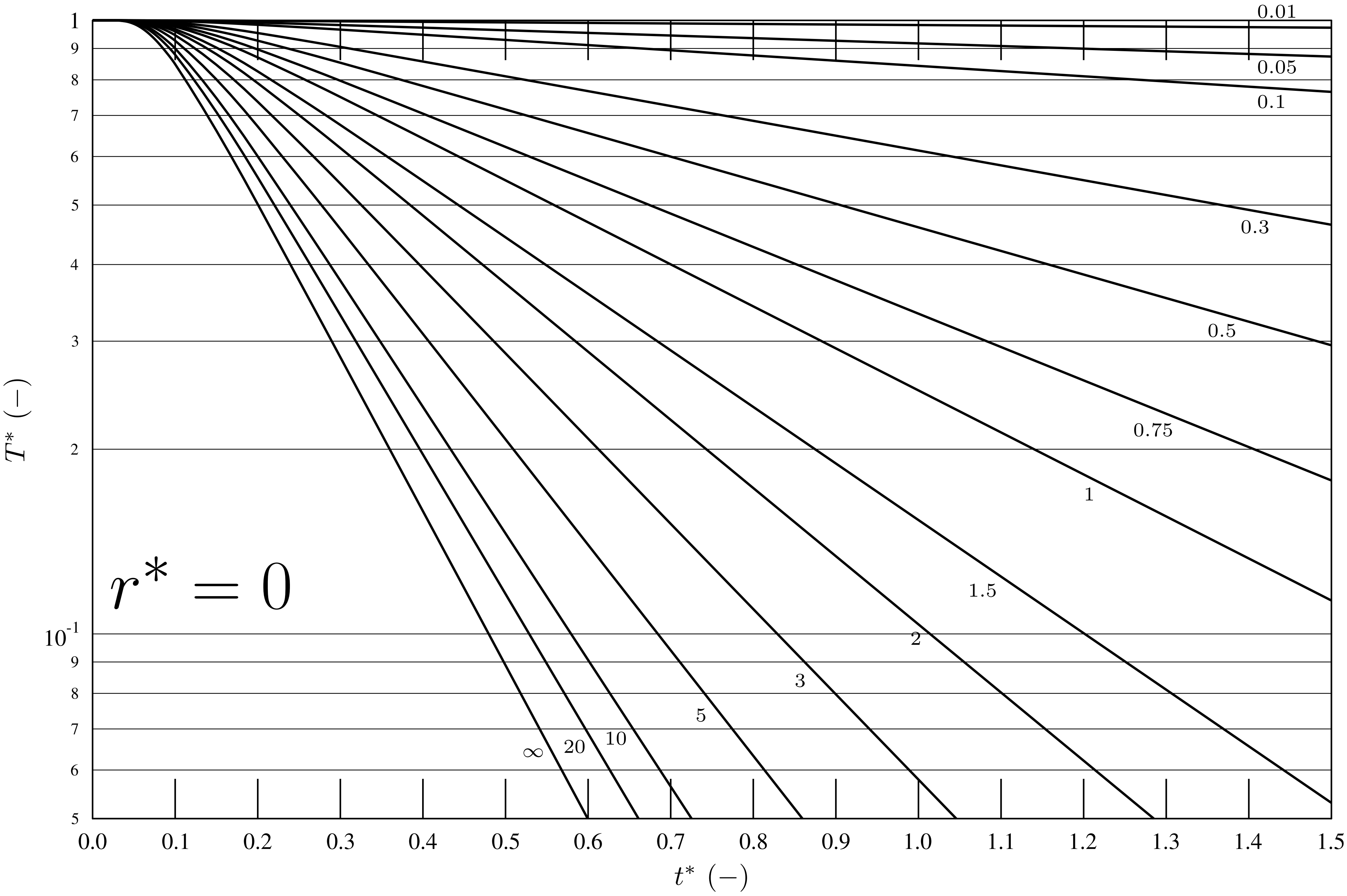
$$\frac{\partial T}{\partial t} = a \frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r \frac{\mathrm{d}T}{\mathrm{d}r}\right)$$

Eigenvalue α_j is a solution of equation

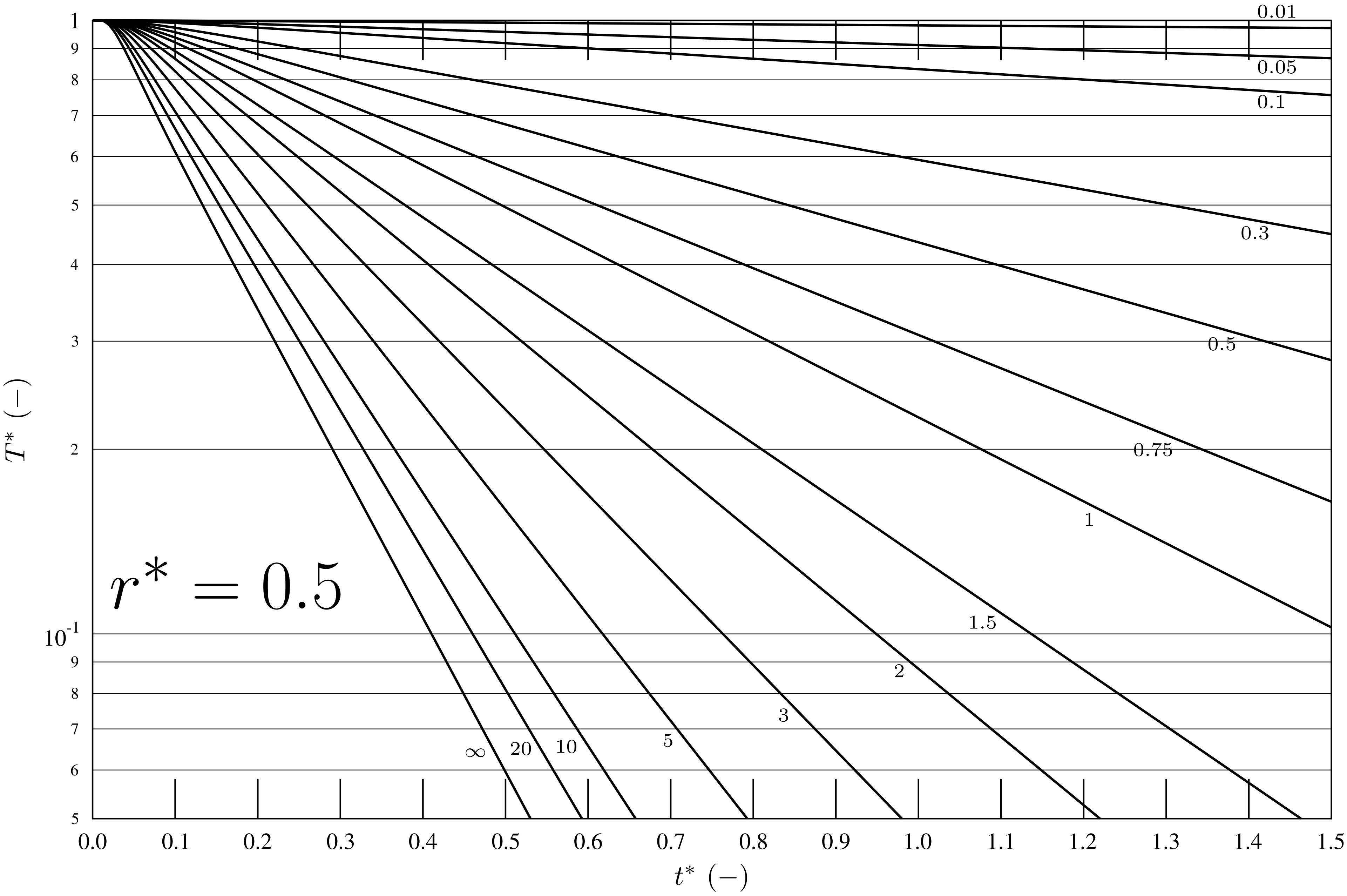
$$0 = \alpha\, \mathrm{J}_1(\alpha_j) - \mathrm{Bi}\, \mathrm{J}_0(\alpha_j)$$

First kind Bessel functions of zeroth and first order

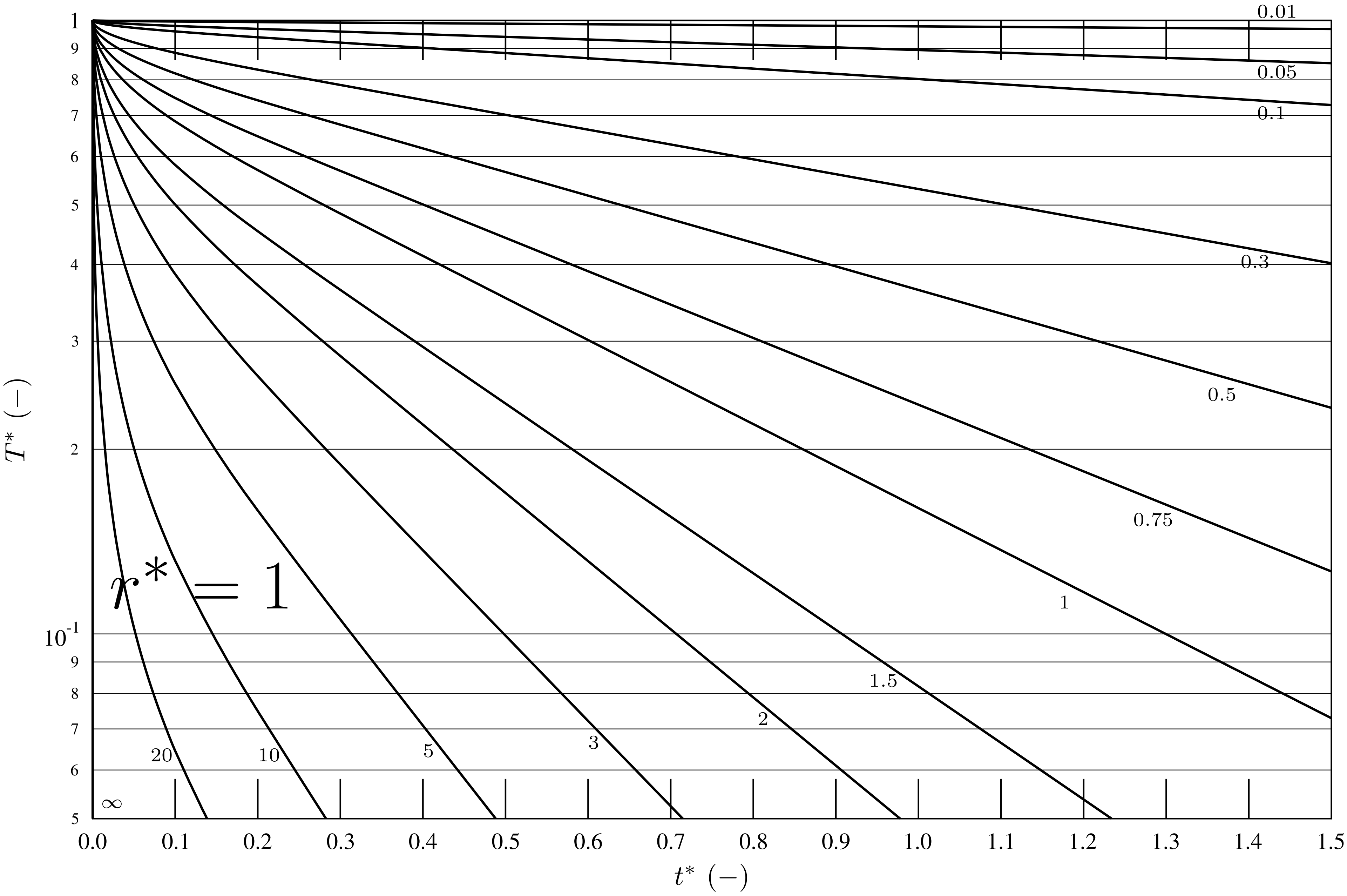




$r^* = 0$
0.01
0.05
0.1
0.3
0.5
0.75
1
1.5
2
3
5
10
20
∞
T^* (−)
1
9
8
7
6
5
4
3
2
10^{-1}
9
8
7
6
5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
t^* (−)

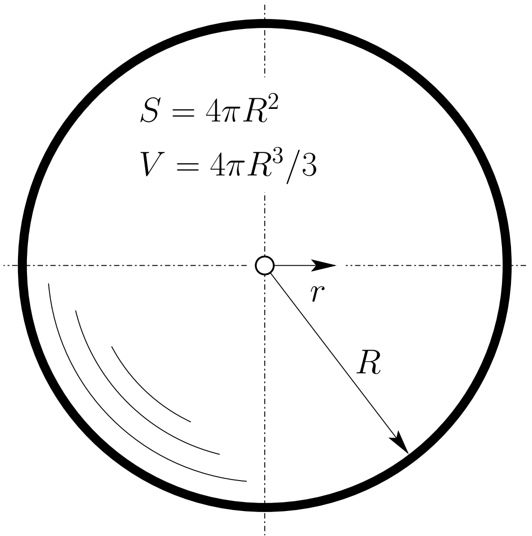


$r^* = 0.5$
T^* (−)
t^* (−)
1
9
8
7
6
5
4
3
2
10^{-1}
9
8
7
6
5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
0.01
0.05
0.1
0.3
0.5
0.75
1
1.5
2
3
5
10
20
∞



$r^* = 1$
T^* (−)
t^* (−)
1
9
8
7
6
5
4
3
2
10^{-1}
9
8
7
6
5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
0.01
0.05
0.1
0.3
0.5
0.75
1
1.5
2
3
5
10
20
∞

Transient heat conduction in sphere



$$T^* = \frac{T - T_\mathrm{f}}{T_0 - T_\mathrm{f}}$$

$$r^* = \frac{r}{R}$$

$$t^* = \frac{at}{R^2}$$

$$\mathrm{Bi} = \frac{\alpha R}{\lambda}$$

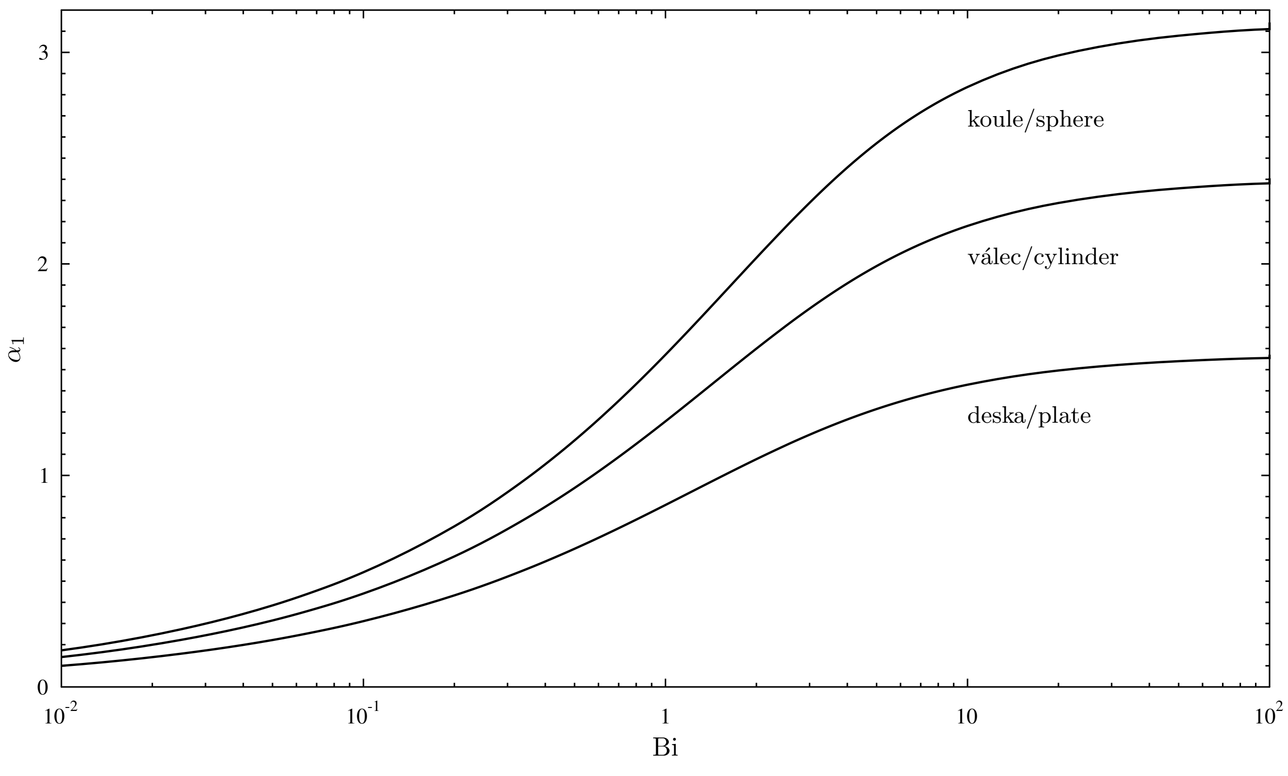
$$\frac{\partial T}{\partial t} = a \frac{1}{r^2} \frac{\mathrm{d}}{\mathrm{d}r} \left(r^2 \frac{\mathrm{d}T}{\mathrm{d}r} \right)$$

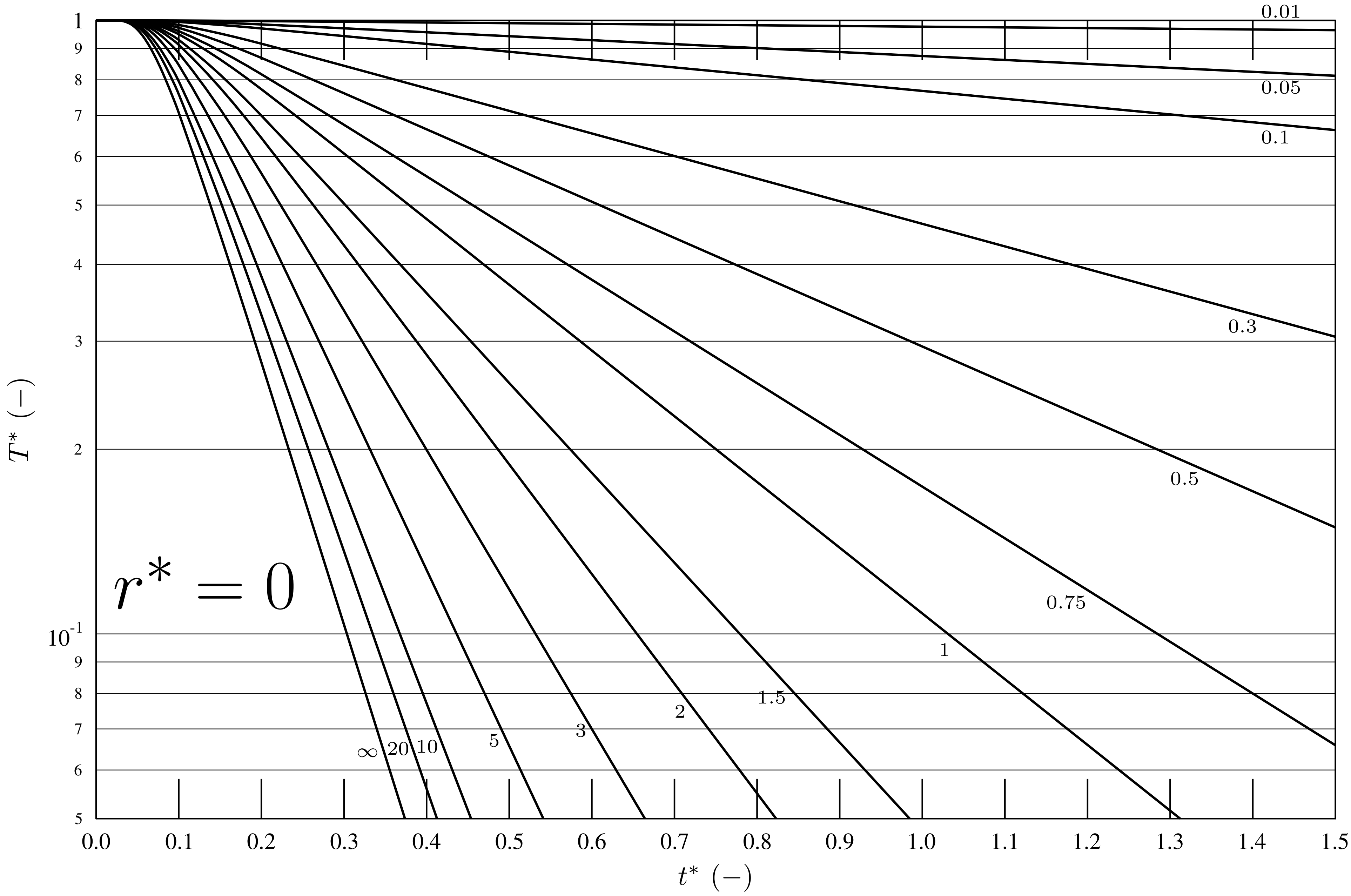
$$T^* = \sum_{j=1}^{\infty} \underbrace{\frac{2\left(\sin \alpha_j - \alpha_j \cos \alpha_j\right)}{\alpha_j - \sin \alpha_j \cos \alpha_j}}_{A_n} \frac{\sin\left(\alpha_j r^*\right)}{\alpha_j r^*} \exp\left(-\alpha_j^2 t^*\right)$$

Eigenvalue α_j is a solution of equation

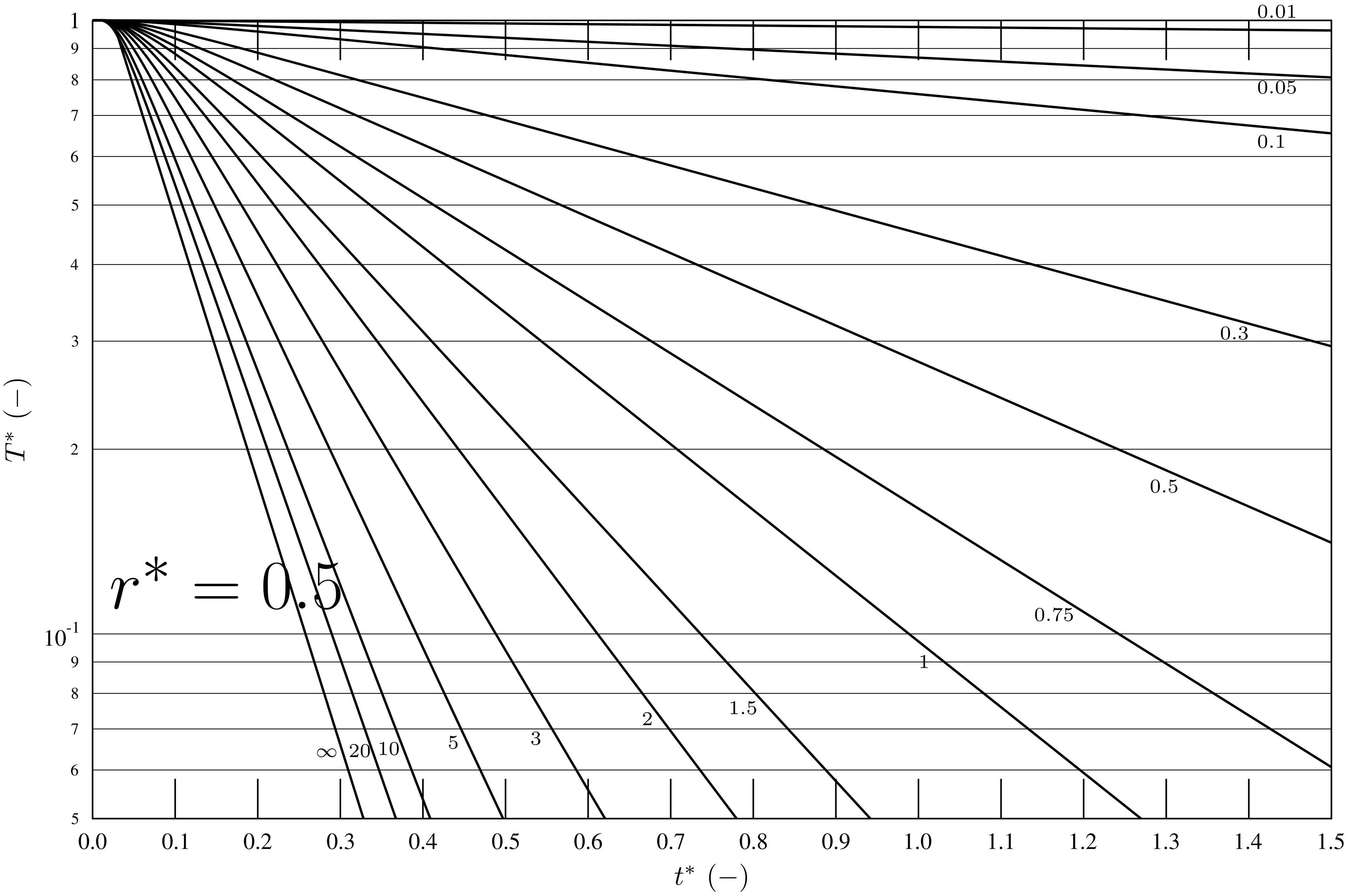
$$0 = 1 - \mathrm{Bi} - \alpha_j \,\mathrm{cotan}\, \alpha_j$$

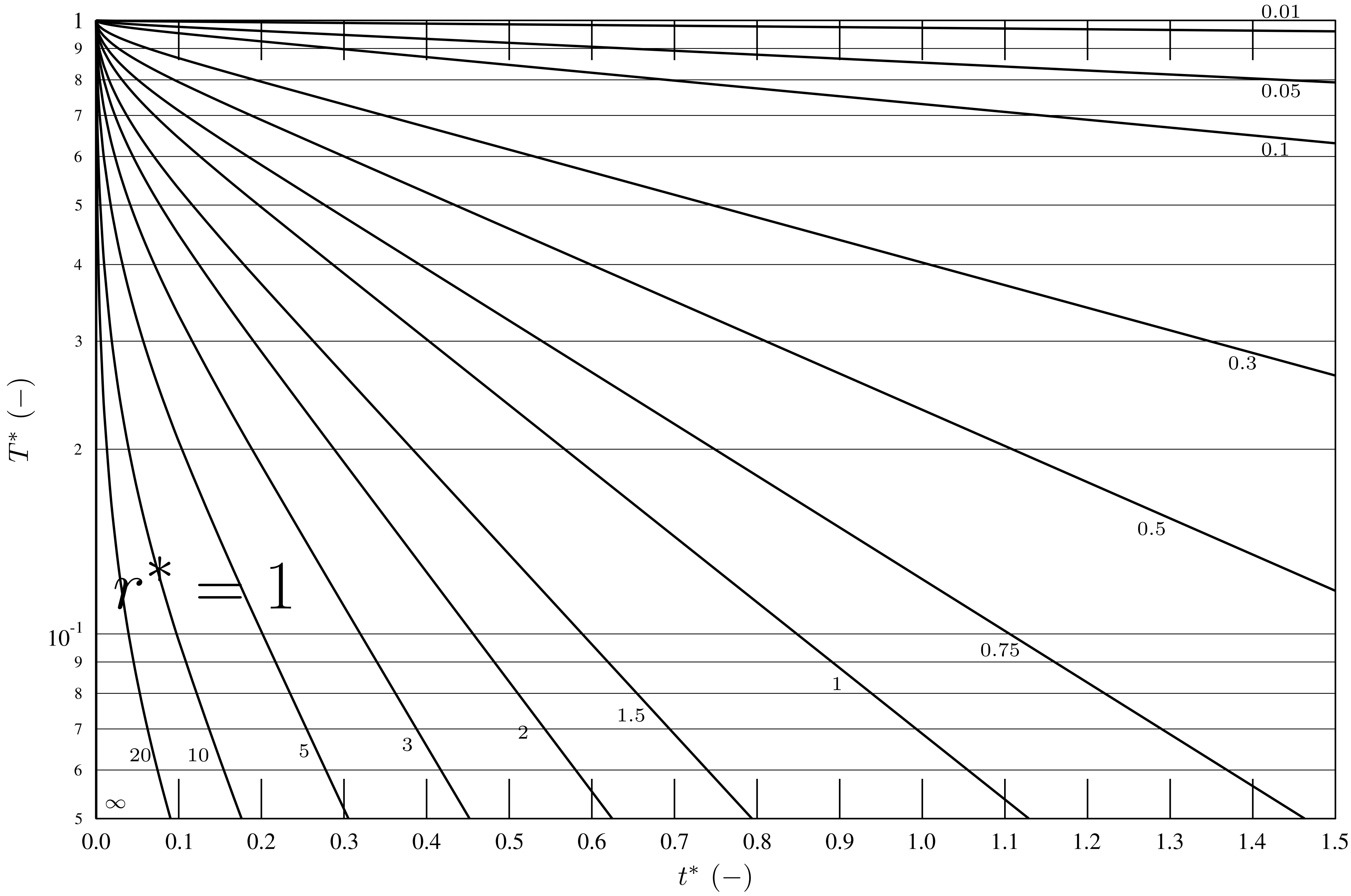
The first eigenvalue of the characteristic equation for infinite plate, infinite cylinder, and sphere (see Table Approximation for large times)





r* = 0
T* (−)
t* (−)
0.01
0.05
0.1
0.3
0.5
0.75
1
1.5
2
3
5
10
20
∞


r* = 0.5
T* (−)
t* (−)
0.01
0.05
0.1
0.3
0.5
0.75
1
1.5
2
3
5
10
20
∞
1
9
8
7
6
5
4
3
2
10-1
9
8
7
6
5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5



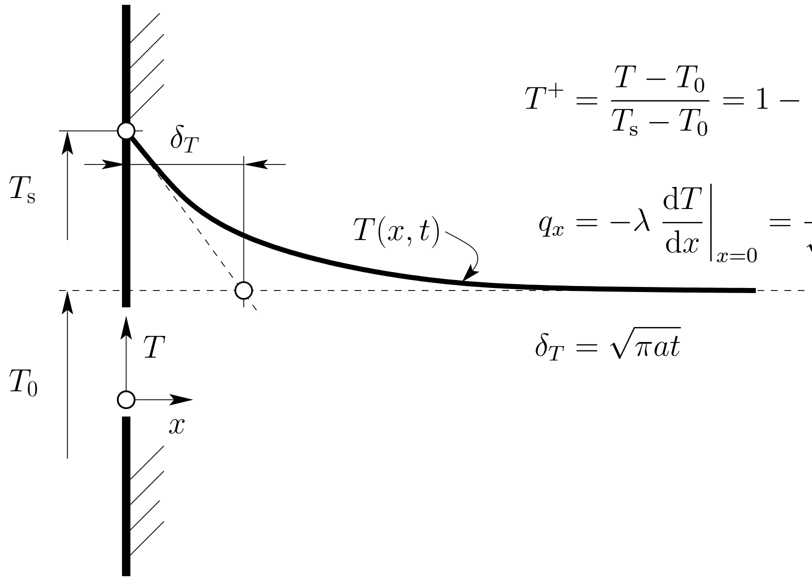
$r^* = 1$
T^* (−)
t^* (−)
0.01
0.05
0.1
0.3
0.5
0.75
1
1.5
2
3
5
10
20
∞
1
9
8
7
6
5
4
3
2
10^{-1}
9
8
7
6
5
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5

Approximation for large times

	plate		cylinder		sphere	
Bi	α_1	A_1	α_1	A_1	α_1	A_1
0.010	0.09983	1.0017	0.14124	1.0025	0.17303	1.0030
0.025	0.15746	1.0041	0.22291	1.0062	0.27318	1.0075
0.050	0.22176	1.0082	0.31426	1.0124	0.38537	1.0150
0.075	0.27048	1.0122	0.38370	1.0185	0.47080	1.0224
0.100	0.31105	1.0161	0.44168	1.0246	0.54228	1.0298
0.200	0.43284	1.0311	0.61697	1.0483	0.75931	1.0592
0.300	0.52179	1.0450	0.74646	1.0712	0.92079	1.0880
0.400	0.59324	1.0580	0.85158	1.0931	1.05279	1.1164
0.500	0.65327	1.0701	0.94077	1.1143	1.16556	1.1441
0.600	0.70507	1.0814	1.01844	1.1345	1.26440	1.1713
0.700	0.75056	1.0918	1.08725	1.1539	1.35252	1.1978
0.750	0.77136	1.0968	1.11891	1.1633	1.39325	1.2108
0.800	0.79103	1.1016	1.14897	1.1724	1.43203	1.2236
0.900	0.82740	1.1107	1.20484	1.1902	1.50442	1.2488
1.000	0.86033	1.1191	1.25578	1.2071	1.57080	1.2732
1.100	0.89035	1.1270	1.30251	1.2232	1.63199	1.2970
1.200	0.91785	1.1344	1.34558	1.2387	1.68868	1.3201
1.300	0.94316	1.1412	1.38543	1.2533	1.74140	1.3424
1.400	0.96655	1.1477	1.42246	1.2673	1.79058	1.3640
1.500	0.98824	1.1537	1.45695	1.2807	1.83660	1.3850
1.600	1.00842	1.1593	1.48917	1.2934	1.87976	1.4052
1.700	1.02725	1.1645	1.51936	1.3055	1.92035	1.4247
1.800	1.04486	1.1695	1.54769	1.3170	1.95857	1.4436
1.900	1.06136	1.1741	1.57434	1.3279	1.99465	1.4618
2.000	1.07687	1.1785	1.59945	1.3384	2.02876	1.4793
2.500	1.14223	1.1966	1.70602	1.3836	2.17463	1.5578
3.000	1.19246	1.2102	1.78866	1.4191	2.28893	1.6227
3.500	1.23227	1.2206	1.85449	1.4473	2.38064	1.6761
4.000	1.26459	1.2287	1.90808	1.4698	2.45564	1.7202
4.500	1.29134	1.2351	1.95248	1.4880	2.51795	1.7567
5.000	1.31384	1.2402	1.98981	1.5029	2.57043	1.7870
5.500	1.33302	1.2444	2.02162	1.5151	2.61515	1.8124
6.000	1.34955	1.2479	2.04901	1.5253	2.65366	1.8338
6.500	1.36396	1.2508	2.07283	1.5339	2.68713	1.8519
7.000	1.37662	1.2532	2.09373	1.5411	2.71646	1.8673
7.500	1.38782	1.2552	2.11220	1.5473	2.74235	1.8806
8.000	1.39782	1.2570	2.12864	1.5526	2.76536	1.8920
8.500	1.40678	1.2585	2.14336	1.5572	2.78593	1.9020
9.000	1.41487	1.2598	2.15661	1.5611	2.80443	1.9106
9.500	1.42220	1.2610	2.16860	1.5646	2.82113	1.9182
10.000	1.42887	1.2620	2.17950	1.5677	2.83630	1.9249
20.000	1.49613	1.2699	2.28805	1.5919	2.98572	1.9781
50.000	1.54001	1.2727	2.35724	1.6002	3.07884	1.9962
100.000	1.55525	1.2731	2.38090	1.6015	3.11019	1.9990
∞	1.57080	1.2732	2.40483	1.6020	3.14159	2.0000

```
https://www.wolframalpha.com/input/?i=solve+0=x-1*cot(x)
https://www.wolframalpha.com/input/?i=solve0=x*J1(x)-1*J0(x)
https://www.wolframalpha.com/input/?i=solve0=1-1-x*cot(x)
```

Transient heat conduction in semi-infinite body



$$T^{+} = \frac{T - T_0}{T_s - T_0} = 1 - \frac{2}{\sqrt{\pi}} \int_0^{\frac{x}{2\sqrt{at}}} e^{-\eta^2} d\eta = 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{at}}\right)$$

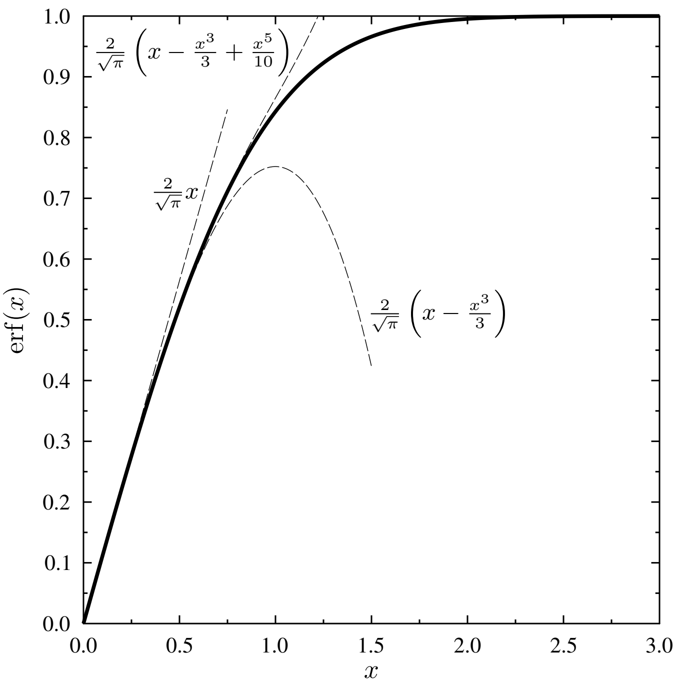
$$q_x = -\lambda \left.\frac{dT}{dx}\right|_{x=0} = \frac{\lambda}{\sqrt{\pi a t}} (T_s - T_0) = \frac{\lambda}{\delta_T} (T_s - T_0)$$

$$\delta_T = \sqrt{\pi a t}$$

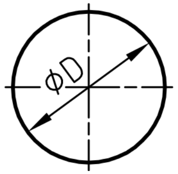
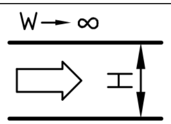
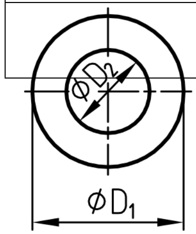
$$\frac{\partial T}{\partial t} = a \frac{\partial^2 T}{\partial x^2}$$

Error function (Gauss error function)

```
https://www.wolframalpha.com/input/?i=plot+erf(x)+from+0+to+2
https://www.wolframalpha.com/input/?i=erf(1)
https://www.wolframalpha.com/input/?i=solve+erf(x)=0.5
```



erf(x)	x
0.999	2.3268
0.995	1.9849
0.99	1.8214
0.95	1.3859
0.9	1.1631
0.8	0.9062
0.7	0.7329
0.6	0.5951
0.5	0.4769
0.4	0.3708
0.3	0.2725
0.2	0.1791
0.1	0.0889

Geometry	Correlation	Char. len.	Char. temperature	Limitations	Comment
forced convection in pipe					
	$\mathrm{Nu} = 0.027\,\mathrm{Re}^{0.8}\mathrm{Pr}^{1/3}$ 0.023 (more conservative) Colburn	D	$\frac{1}{2}(T_{in} + T_{out})$	$\mathrm{Re} \geq 10^4 (3 \times 10^3)$ $0.5 < \mathrm{Pr} < 17000$	$10 < L/D < 60$ $\left[1 + (D/L)^{2/3}\right]$ Sieder–Tate $(\bar{\mu}/\mu_w)^{0.14}$
	$\mathrm{Nu} = 0.015\,\mathrm{Re}^{0.83}\mathrm{Pr}^{0.42}$ Whitaker	D	$\frac{1}{2}(T_{in} + T_{out})$	$2300 < \mathrm{Re} < 10^5$ $0.48 < \mathrm{Pr} < 592$	Sieder–Tate $(\bar{\mu}/\mu_w)^{0.14}$
	$\mathrm{Nu} = 1.86\,\mathrm{Gz}^{1/3}$ 1.615 (Leveque for short pipes and constant wall temperature)	D	$\frac{1}{2}(T_{in} + T_{out})$	$\mathrm{Re} < 2100$ $0.5 < \mathrm{Pr} < 17000$ $\mathrm{Nu} > 3.72$	Sieder–Tate $(\bar{\mu}/\mu_w)^{0.14}$
	$\mathrm{Nu} = 3.66 + \dfrac{0.0668\,\mathrm{Gz}}{1 + 0.04\,\mathrm{Gz}^{2/3}}$ Hausen	D	$\frac{1}{2}(T_{in} + T_{out})$	$\mathrm{Re} < 2300$	Sieder–Tate $(\bar{\mu}/\mu_w)^{0.14}$
	$\mathrm{Nu} = 7.55 + \dfrac{0.024\,\mathrm{Gz}^{1.14}}{1 + 0.0358\,\mathrm{Gz}^{2/3}}$	$2H$	$\frac{1}{2}(T_{in} + T_{out})$	$\mathrm{Re} < 2200$ $0.1 < \mathrm{Pr} < 1000$	
	$\mathrm{Nu} = 3.66 + 1.2\kappa^{0.8} + \dfrac{0.19\left[1 + 0.14\kappa^{0.5}\right]\mathrm{Gz}^{0.8}}{1 + 0.117\,\mathrm{Gz}^{0.467}}$	$D_1 - D_2$	$\frac{1}{2}(T_{in} + T_{out})$	laminar flow	

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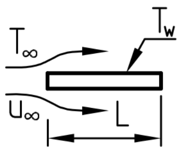
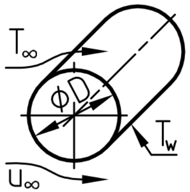
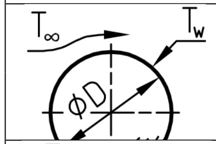
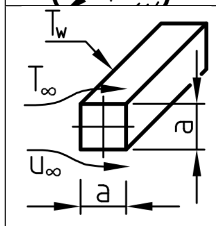
Geometry	**Correlation**	**Char. len.**	**Char. temperature**	**Limitations**	**Comment**
	where $\kappa = D_2/D_1$; $\mathrm{Gz} = \mathrm{Re}\,\mathrm{Pr}\,D_e/L$ Gnielinski			in cylindrical gap	
External flows					
	$\mathrm{Nu} = 0.664\,\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3}$	L	$\frac{1}{2}(T_\infty + T_w)$	$\mathrm{Re} < 5 \times 10^5$ $0.6 < \mathrm{Pr} < 60$	
	$\mathrm{Nu} = (0.037\,\mathrm{Re}^{0.8} - 870)\,\mathrm{Pr}^{1/3}$	L	$\frac{1}{2}(T_\infty + T_w)$	$5 \times 10^5 < \mathrm{Re} < 10^8$ $0.6 < \mathrm{Pr} < 60$	
	$\mathrm{Nu} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282000}\right)^{5/8}\right]^{4/5}$	D	$\frac{1}{2}(T_\infty + T_w)$	$\mathrm{RePr} > 0.2$	
	$\mathrm{Nu} = 0.25 + \left(0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}\right)\mathrm{Pr}^{0.4}$	D	T_∞	$1 < \mathrm{Re} < 10^5$ $0.67 < \mathrm{Pr} < 300$	Sieder–Tate $(\mu_\infty/\mu_w)^{0.25}$
	$\mathrm{Nu} = 2 + \left(0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}\right)\mathrm{Pr}^{0.4}$	D	T_∞	$3.5 < \mathrm{Re} < 8 \times 10^4$ $0.7 < \mathrm{Pr} < 380$	Sieder–Tate $(\mu_\infty/\mu_w)^{0.25}$
	$\mathrm{Nu} = 0.102\,\mathrm{Re}^{0.675}\mathrm{Pr}^{1/3}$	a	$\frac{1}{2}(T_\infty + T_w)$	$5000 < \mathrm{Re} < 10^5$	

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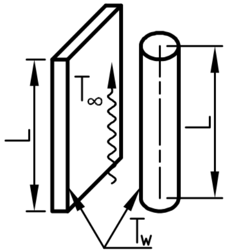
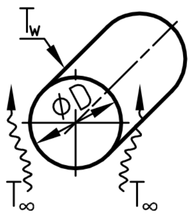
Geometry	**Correlation**	**Char. len.**	**Char. temperature**	**Limitations**	**Comment**
Free (natural) convection					
	$\mathrm{Nu} = \left\{0.825 + \frac{0.387\,\mathrm{Ra}^{1/6}}{[1 + (0.492/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$	L	$\frac{1}{2}(T_\infty + T_w)$	$0.1 \le \mathrm{Ra} \le 10^{12}$	$\mathrm{Nu}_{cyl}/\mathrm{Nu}_{plate} = = \left[1 + 1.43\left(\frac{L}{D\mathrm{Gr}^{0.25}}\right)^{0.9}\right]$
	$\mathrm{Nu} = 0.59\,\mathrm{Ra}^{1/4}$	L	$\frac{1}{2}(T_\infty + T_w)$	$10^4 \le \mathrm{Ra} \le 10^9$	
	$\mathrm{Nu} = 0.1\,\mathrm{Ra}^{1/3}$	L	$\frac{1}{2}(T_\infty + T_w)$	$10^9 \le \mathrm{Ra} \le 10^{13}$	experiment
	$\mathrm{Nu} = \left\{0.6 + \frac{0.387\,\mathrm{Ra}^{1/6}}{[1 + (0.559/\mathrm{Pr})^{9/16}]^{8/27}}\right\}^2$	D	$\frac{1}{2}(T_\infty + T_w)$	$10^5 \le \mathrm{Ra} \le 10^{12}$	
	$\mathrm{Nu} = 0.53\,\mathrm{Ra}^{1/4}$	D	$\frac{1}{2}(T_\infty + T_w)$	$10^4 \le \mathrm{Ra} \le 10^9$	
	$\mathrm{Nu} = 0.13\,\mathrm{Ra}^{1/3}$	D	$\frac{1}{2}(T_\infty + T_w)$	$10^9 \le \mathrm{Ra} \le 10^{12}$	experiment

Thermophysical properties

Air*)

T (°C)	ϱ (kg m^{-3})	c_p (J kg^{-1} K^{-1})	λ (W m^{-1} K^{-1})	ν (m^2 s^{-1})	β (K^{-1})
0	1.2760	1006	0.0241	13.5×10^{-6}	3.67×10^{-3}
20	1.1887	1006	0.0256	15.3×10^{-6}	3.43×10^{-3}
40	1.1119	1006	0.0270	17.2×10^{-6}	3.20×10^{-3}
60	1.0456	1007	0.0285	19.2×10^{-6}	3.00×10^{-3}
80	0.9867	1009	0.0299	21.2×10^{-6}	2.83×10^{-3}
100	0.9334	1011	0.0314	23.8×10^{-6}	2.68×10^{-3}
200	0.7359	1026	0.0385	35.0×10^{-6}	2.11×10^{-3}
300	0.6076	1047	0.0447	48.5×10^{-6}	1.75×10^{-3}
400	0.5173	1068	0.0502	63.4×10^{-6}	1.49×10^{-3}
500	0.4504	1093	0.0555	79.5×10^{-6}	
600	0.3988	1114	0.0607	96.8×10^{-6}	
700	0.3578	1137	0.0660	115×10^{-6}	
800	0.3244	1160	0.0706	135×10^{-6}	
900	0.2970	1182	0.0750	155×10^{-6}	
1000	0.2730	1193	0.0791	175×10^{-6}	

*) Trensport and thermophysical properties of dry air at pressure 101325 Pa. According to Šesták, J., Bukovský, J., Houška, M. Tepelné pochody. Transportní a termodynamická data, Vydavatelství ČVUT, Praha (1986). In the last column, there is the thermal expansion coefficient which is used in Grashoff number Gr, for example.

Water**)

T (°C)	ϱ (kg m^{-3})	c_p (J kg^{-1} K^{-1})	λ (W m^{-1} K^{-1})	ν (m^2 s^{-1})	β (K^{-1})
0	999.8	4218	0.569	1.751×10^{-6}	-0.07×10^{-3}
10	999.7	4192	0.587	1.304×10^{-6}	0.088×10^{-3}
20	998.2	4182	0.604	1.004×10^{-6}	0.206×10^{-3}
30	995.7	4178	0.618	0.801×10^{-6}	0.303×10^{-3}
40	992.2	4178	0.632	0.658×10^{-6}	0.385×10^{-3}
50	988.0	4181	0.643	0.553×10^{-6}	0.457×10^{-3}
60	983.2	4184	0.654	0.474×10^{-6}	0.523×10^{-3}
70	977.8	4190	0.662	0.413×10^{-6}	0.585×10^{-3}
80	971.8	4196	0.669	0.365×10^{-6}	0.643×10^{-3}
90	965.3	4205	0.676	0.326×10^{-6}	0.698×10^{-3}
100	958.4	4216	0.682	0.295×10^{-6}	0.753×10^{-3}

**) Thermophysical properties of water on the lower saturation curve. According to Šesták, J., Bukovský, J., Houška, M. Tepelné pochody. Transportní a termodynamická data, Vydavatelství ČVUT, Praha (1986). In the last column, there is the thermal expansion coefficient which is used in Grashoff number Gr, for example.

Properties of some other materials

	$\varrho_{20\ °C}$ kg m^{-3}	$c_{p,20\ °C}$ J kg^{-1} K^{-1}	$\lambda_{20\ °C}$ W m^{-1} K^{-1}
aluminium	2710	902	236
duralumin (96 % Al, 4 % Cu, Mg)	2790	881	169
copper	8930	386	398
aluminium bronze (90 % Cu, 10 % Al)	8360	420	56
bronze (89 % Cu, 11 % Sn)	8800	343	24.8
brass (70 % Cu, 30 % Zn)	8440	377	109
gold	19300	127	315
lead	11340	128	35.3
nickel	8900	444	91.4
platinum	21450	133	71.4
silver	10500	234	427
tin	7310	228	67
titanium	4500	520	22
wolfram	19350	134	179
uranium	19070	116	27.4
bricks (dry)	1760 – 1800	840	0.38 – 0.57
glass	2710	840	0.76
fire clay	2000 – 2050	960	1.22 – 1.35 (400 °C)

Šesták, J., Bukovský, J., Houška, M. Tepelné pochody. Transportní a termodynamická data, Vydavatelství ČVUT, Praha (1986).

Saturated vapor pressure

$$\left.\frac{\mathrm{d}p}{\mathrm{d}T}\right|_{\mathrm{fr}} = \frac{\Delta h^{\mathrm{fr}}}{T\Delta v^{\mathrm{fr}}} \qquad \left.\frac{\mathrm{d}\ln p}{\mathrm{d}T}\right|_{\mathrm{lg}} = \frac{\Delta \tilde{H}^{\mathrm{lg}}}{\mathrm{R}T^2} \qquad \ln p'' = A - \frac{B}{T+C} \quad (\mathrm{Pa,\ K})$$

	$\mathcal{M}$ kg mol^{-1}	$\varrho_{T_{\mathrm{ref}}}$ kg m^{-3}	T_{ref} K	A	B	C
H_2O	0.018015	998	293.15	23.1964	3816.44	−46.13
CO	0.028010	803	81	19.2614	530.22	−13.15
NH_3	0.017031	639	273.15	21.8409	2132.5	−32.98
N_2H_4	0.032045	1008	293.15	22.8827	3877.65	−45.15
CH_4	0.016043	425	111.7	20.1171	897.84	−7.16
C_8H_{18}	0.114232	692	293.15	20.5778	2896.28	−52.41
C_6H_6	0.078114	885	289.15	20.7936	2788.51	−52.36
C_2H_5OH	0.046069	789	293.15	23.8047	3803.98	−41.68
CH_3OH	0.032042	791	293.15	23.4803	3626.55	−34.29

Šesták, J., Bukovský, J., Houška, M. Tepelné pochody. Transportní a termodynamická data, Vydavatelství ČVUT, Praha (1986).