Effect of Axial Prestretch on Internal Volume of Nonlinear Anisotropic Tube in its Inflation

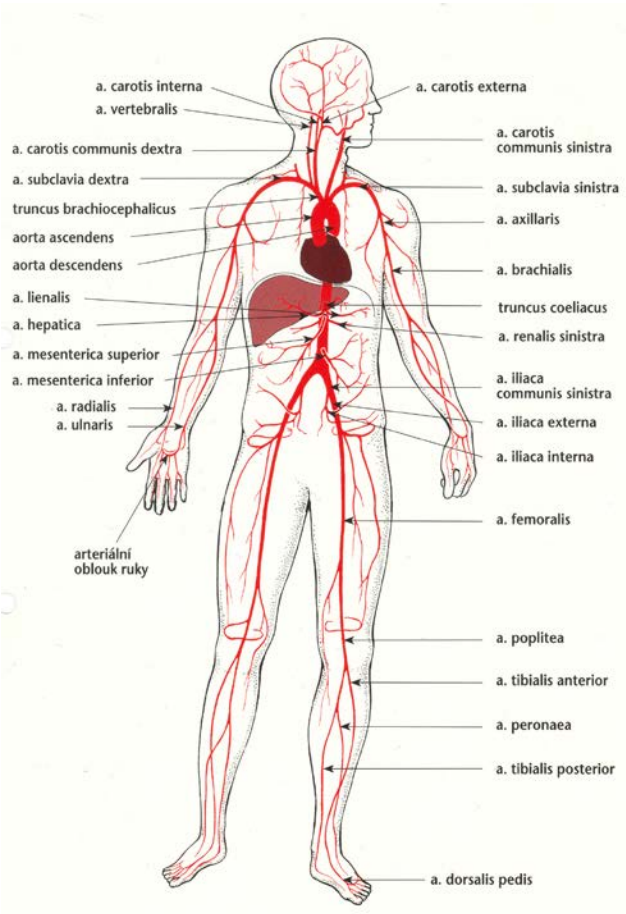
Consequences for Arterial Mechanics

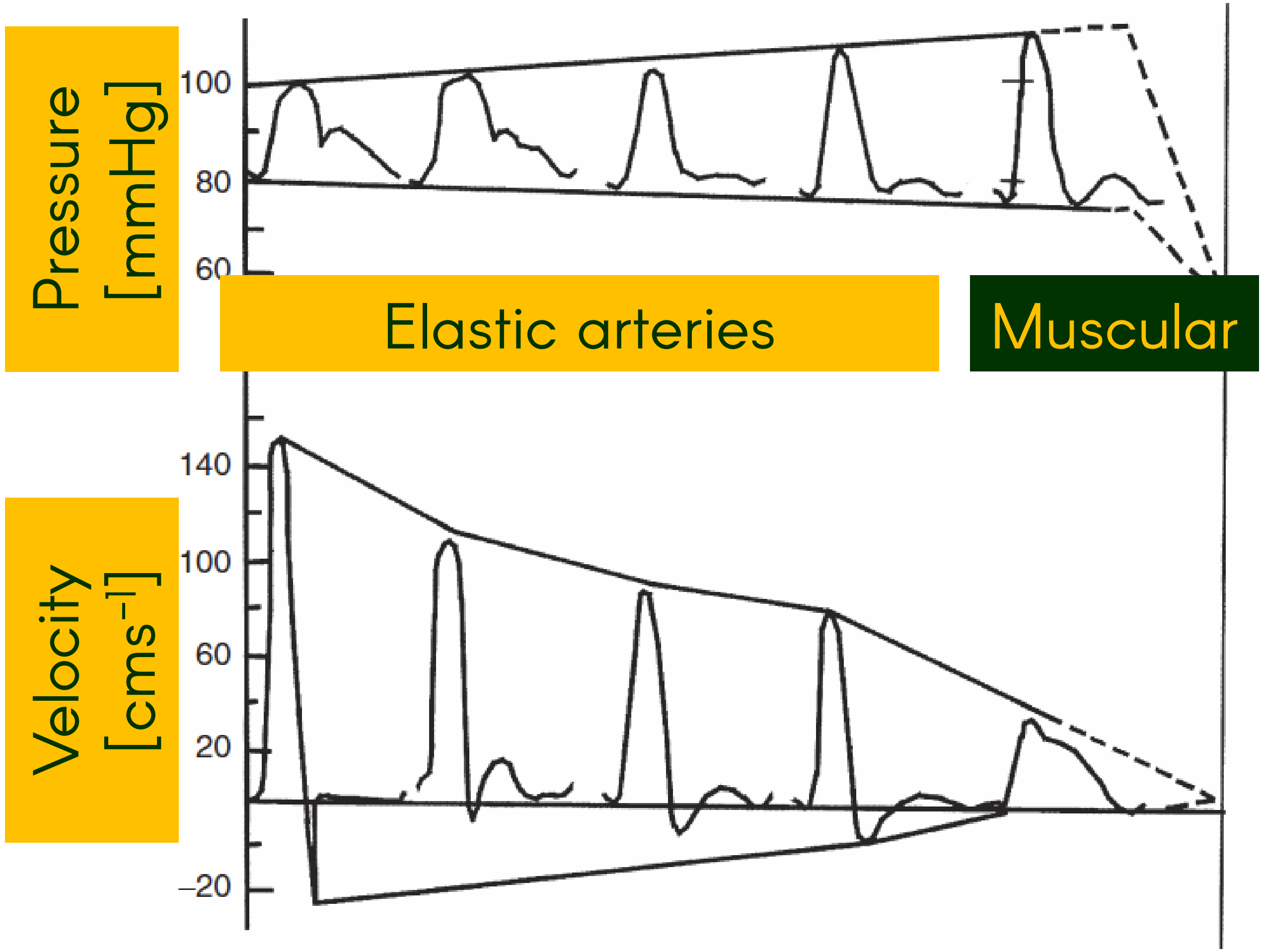
Lukáš Horný and Zdeněk Petřivý
Czech Technical University in Prague, Faculty of Mechanical Engineering

http://users.fs.cvut.cz/~hornyluk/files/COMPLAS-2019.pdf

Arteries

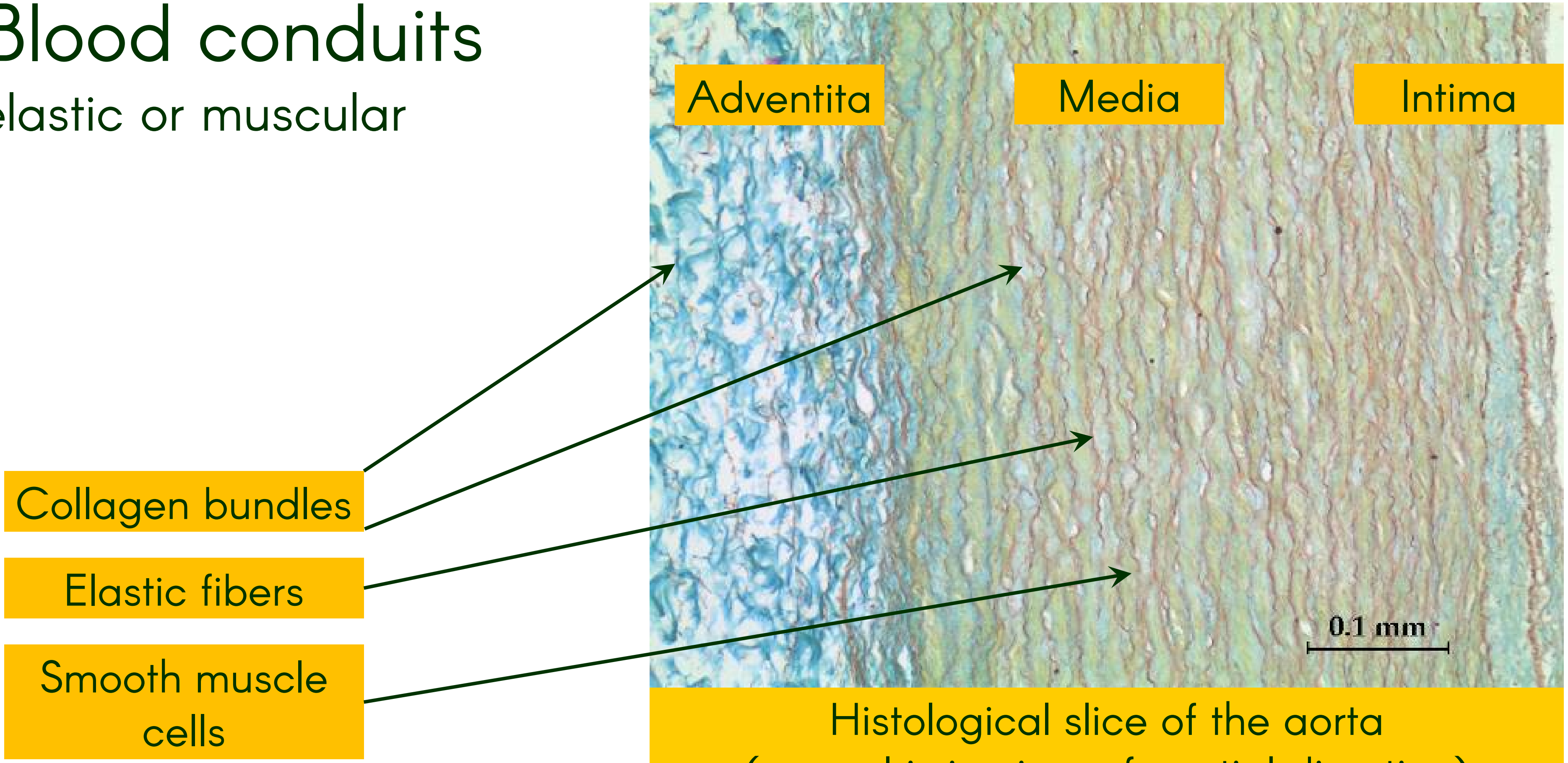
- Blood conduits
  elastic or muscular





Arteries

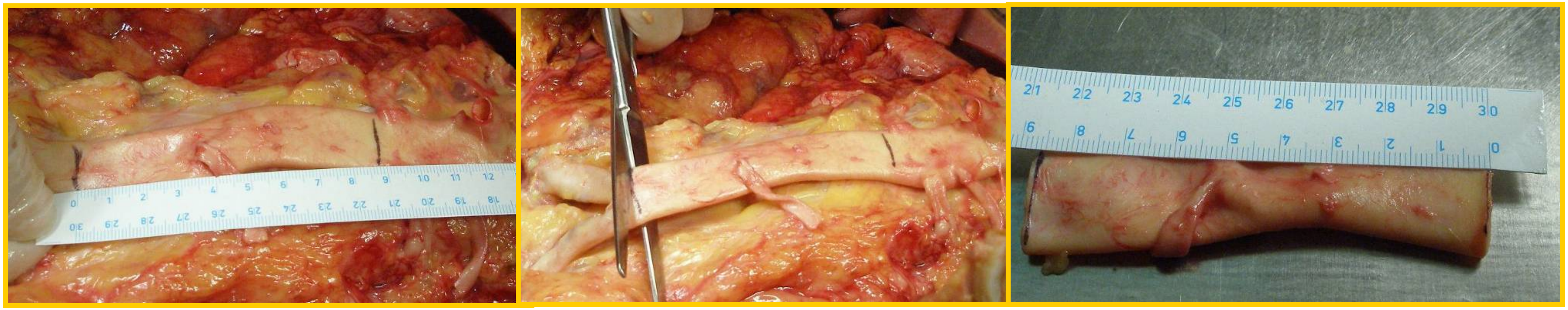
- Blood conduits
  elastic or muscular

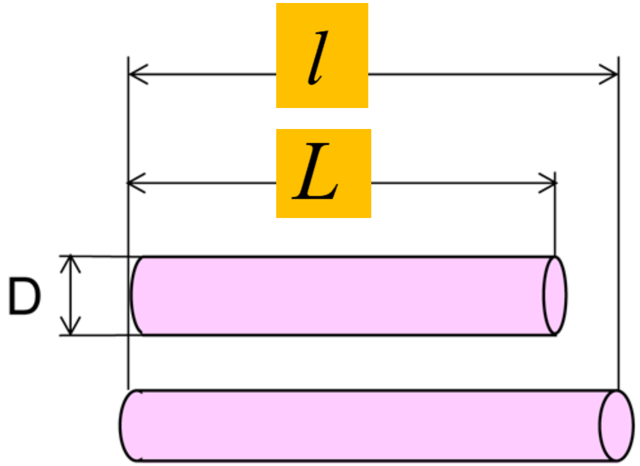


Histological slice of the aorta
(normal is in circumferential direction)

Axial prestretch

- Ratio of in situ to ex situ length

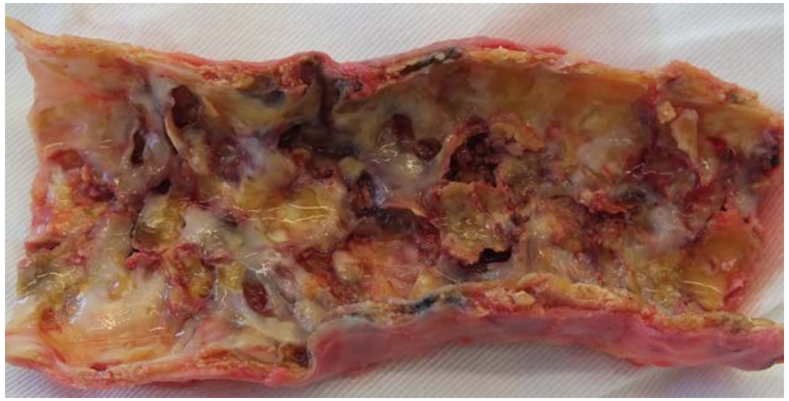


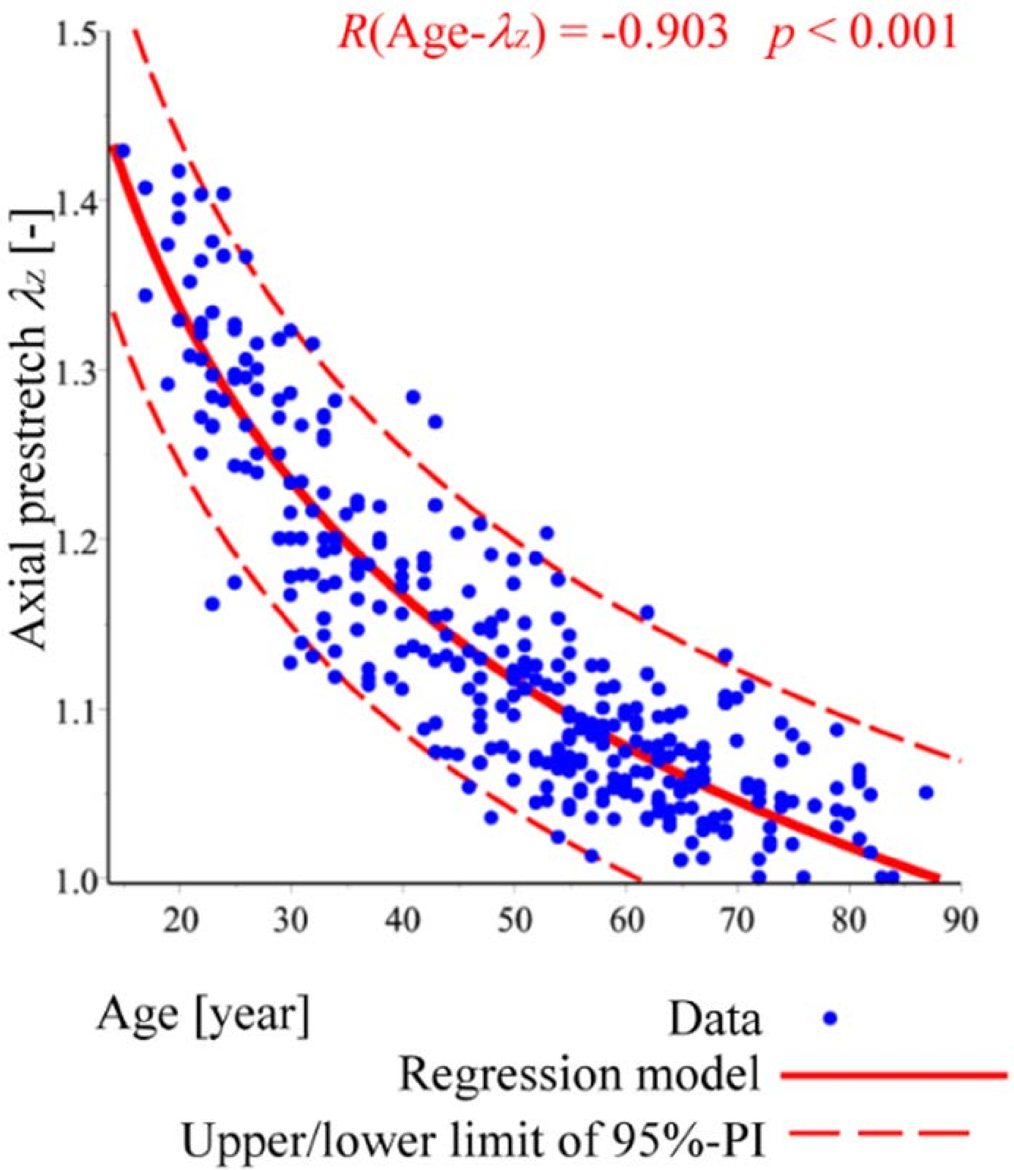


$$\lambda_{zZ}^{ini} = \frac{l}{L}$$

Axial prestretch

- Age-related changes





We should ask: What are the mechanical consequences of the prestretch?

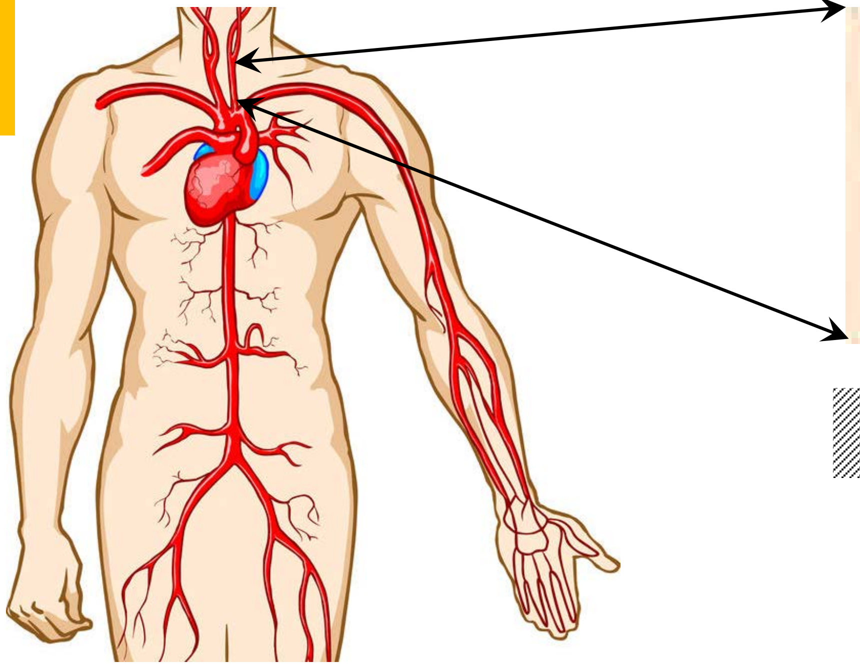
Analytical model

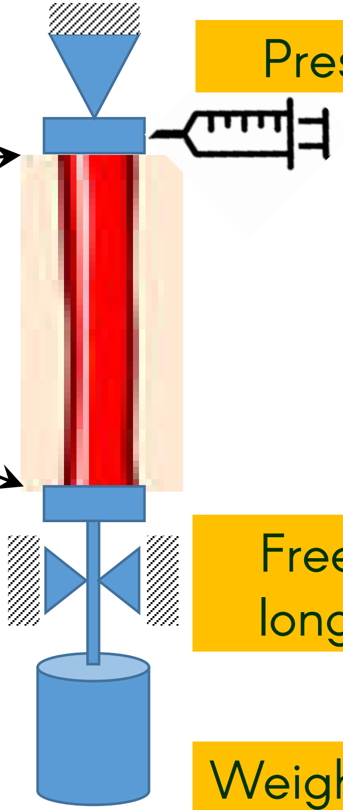
- To simulate an effect of the axial prestretch on pressure-radius relationship assume that we have
    - hyperelastic and incompressible
    - thin-walled tube
    - preloaded by axial weight
    - which mechanically responses to the loading in the way that deformed geometry still retains character of uniform circular cylinder (only radius and length are changed)

Analytical model

- To simulate an effect of the axial prestretch on pressure-radius relationship assume that we have

Human left common carotid artery





Analytical model

- Constitutive description by exponential strain energy

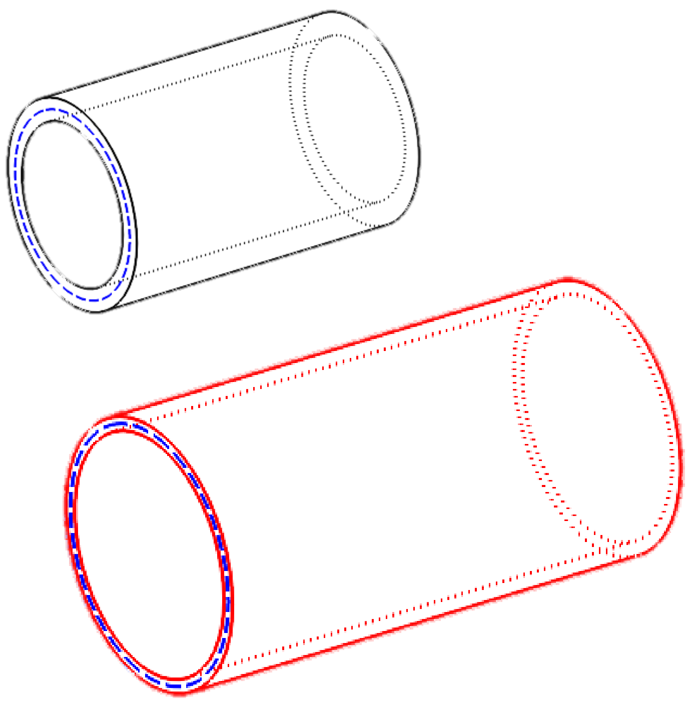
$$W = \frac{c_0}{2}(I_1 - 3) + \frac{c_1^{circ}}{4c_2^{circ}}\left(e^{c_2^{circ}\left(\lambda_{\theta\Theta}^2 - 1\right)^2} - 1\right) + \frac{c_1^{ax}}{4c_2^{ax}}\left(e^{c_2^{ax}\left(\lambda_{zZ}^2 - 1\right)^2} - 1\right) + \frac{c_1^{helix}}{2c_2^{helix}}\left(e^{c_2^{helix}\left(\lambda_{\theta\Theta}^2 \cos^2\beta + \lambda_{zZ}^2 \sin^2\beta - 1\right)^2} - 1\right)$$

$c_0 = 5.94$ kPa $c_1{}^{ax} = 2.53$ kPa $c_2{}^{ax} = 2.44$ $c_1{}^{circ} = 7.41$ kPa
$c_2{}^{circ} = 0.93$ $c_1{}^{helix} = 4.96$ kPa $c_2{}^{circ} = 4.38$ $\beta = 48.13°$
57 years old female

Kamenskiy A.V., Dzenis Y.A., Kazmi S.A.J., Pemberton M.A., Pipinos I.I., Phillips N.Y., Herber K., (…), MacTaggart J.N. (2014) Biaxial mechanical properties of the human thoracic and abdominal aorta, common carotid, subclavian, renal and common iliac arteries. Biomech Model Mechanobiol 13(6):1341-1359. https://link.springer.com/article/10.1007%2Fs10237-014-0576-6

Analytical model

- Kinematics



$$h = \lambda_{rR} H$$
$$r = \lambda_{\theta\Theta} R$$
$$z = \lambda_{zZ} Z$$

$$\mathbf{F} = \begin{pmatrix} \lambda_{rR} & 0 & 0 \\ 0 & \lambda_{\theta\Theta} & 0 \\ 0 & 0 & \lambda_{zZ} \end{pmatrix} = \begin{pmatrix} \frac{h}{H} & 0 & 0 \\ 0 & \frac{r}{R} & 0 \\ 0 & 0 & \frac{z}{Z} \end{pmatrix}$$

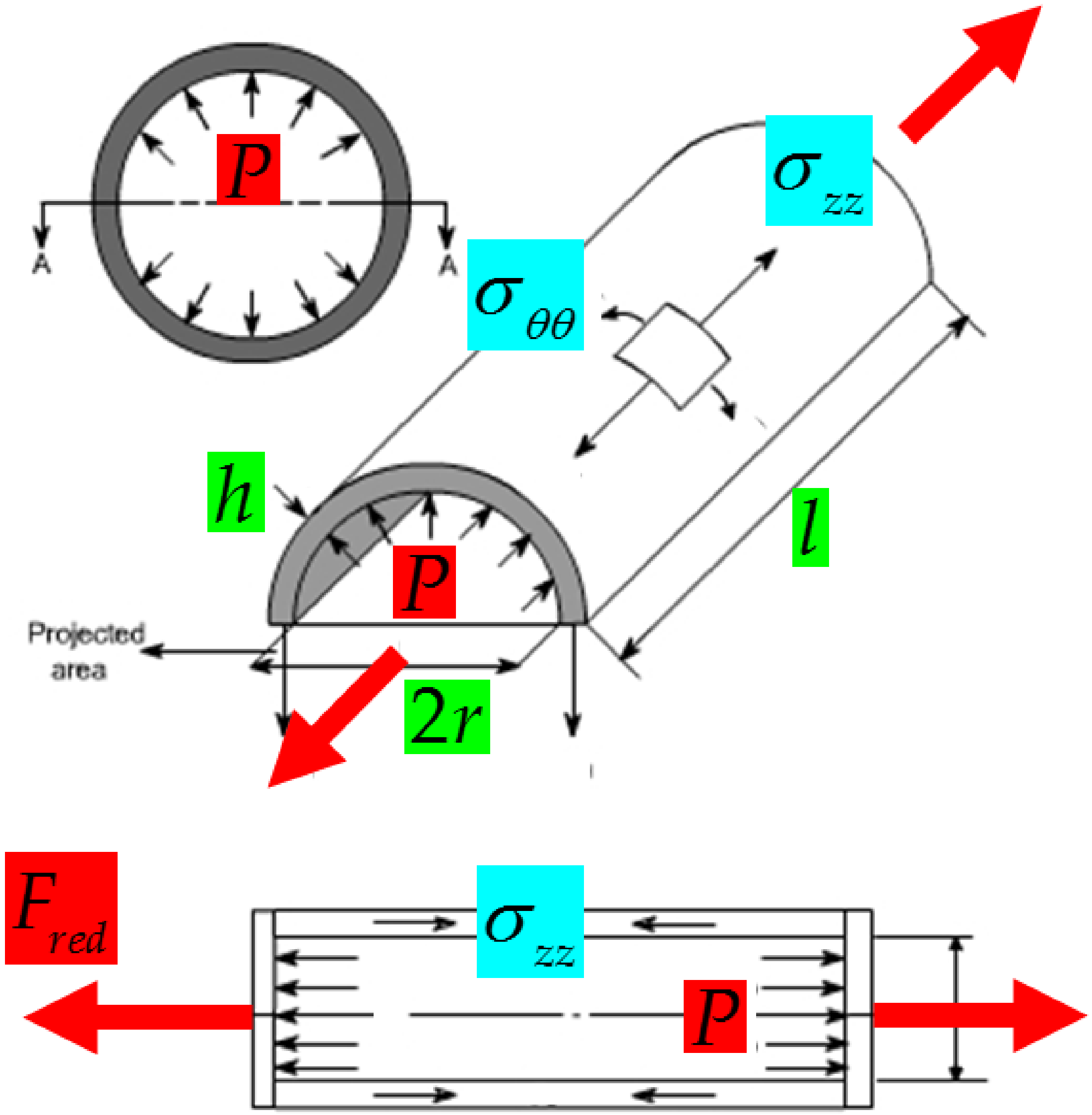
Analytical model

- Equilibrium equations

$$\sigma_{\theta\theta} = \frac{rP}{h}$$

$$\sigma_{zz} = \frac{F_{red}}{2\pi rh} + \frac{rP}{2h}$$

$$\sigma_{rr} = 0$$



Analytical model

$$W = \frac{c_0}{2}(I_1 - 3) + \frac{c_1^{circ}}{4c_2^{circ}}\left(e^{c_2^{circ}\left(\lambda_{\theta\Theta}^2 - 1\right)^2} - 1\right) + \frac{c_1^{ax}}{4c_2^{ax}}\left(e^{c_2^{ax}\left(\lambda_{zZ}^2 - 1\right)^2} - 1\right) + \frac{c_1^{helix}}{2c_2^{helix}}\left(e^{c_2^{helix}\left(\lambda_{\theta\Theta}^2 \cos^2\beta + \lambda_{zZ}^2 \sin^2\beta - 1\right)^2} - 1\right)$$

Strain energy density

$$\sigma_{rr} = \lambda_{rR}\frac{\partial W}{\partial \lambda_{rR}} - p \qquad \sigma_{\theta\theta} = \lambda_{\theta\Theta}\frac{\partial W}{\partial \lambda_{\theta\Theta}} - p \qquad \sigma_{zz} = \lambda_{zZ}\frac{\partial W}{\partial \lambda_{zZ}} - p$$

Constitutive equations

$$\sigma_{rr} = 0 \qquad \sigma_{\theta\theta} = \frac{rP}{h} \qquad \sigma_{zz} = \frac{rP}{2h} + \frac{F_{red}}{2\pi rh}$$

Equilibrium equations

$$r = \lambda_{\theta\Theta}R \qquad h = \lambda_{rR}H \qquad z = \lambda_{zZ}Z$$

Kinematics

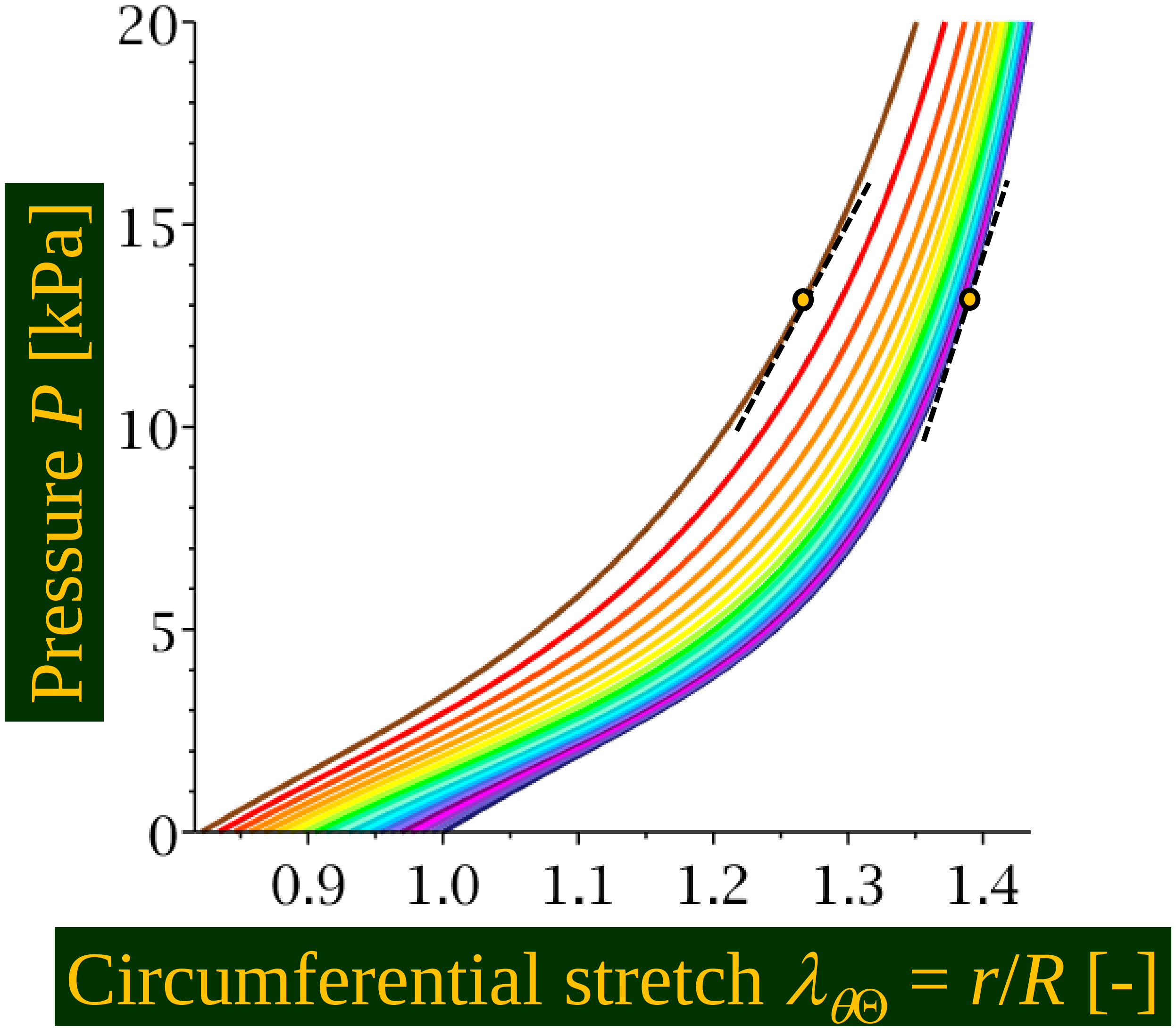
$$\lambda_{rR}\lambda_{\theta\Theta}\lambda_{zZ} = 1$$

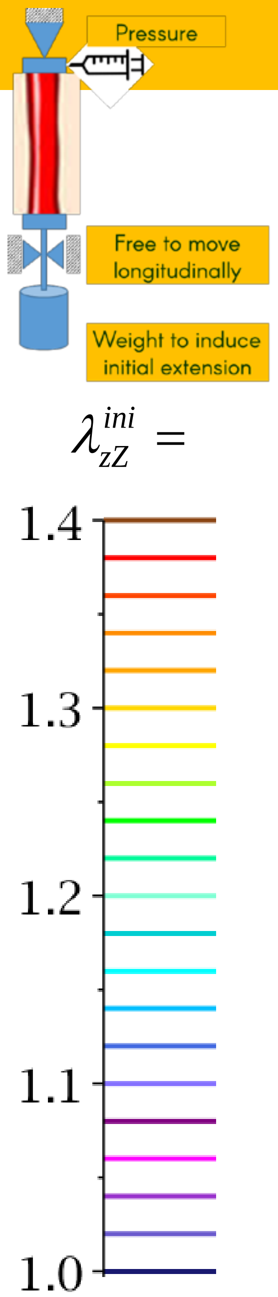
Constraint

Predicted mechanical response

Model predictions

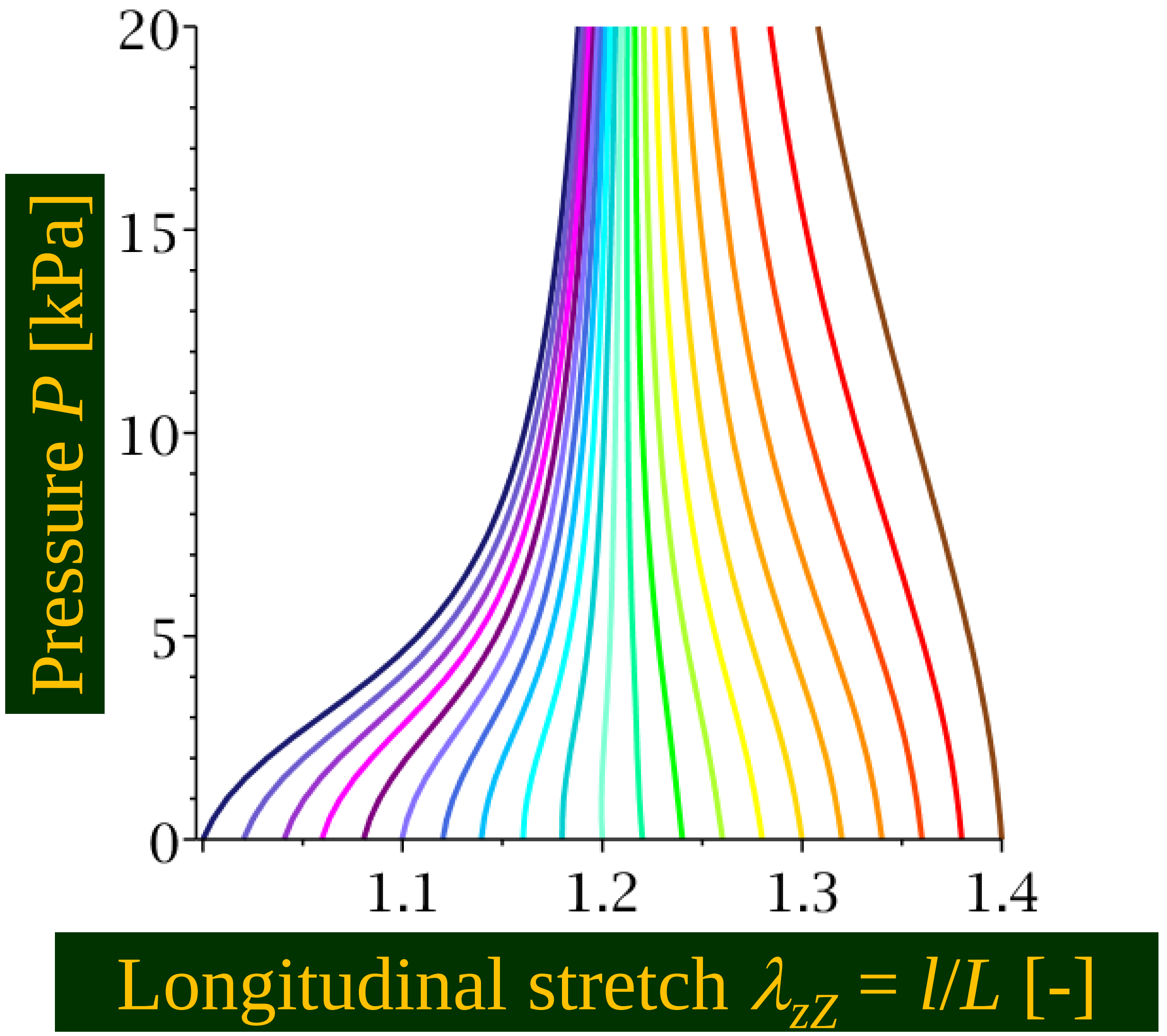
- Inflation

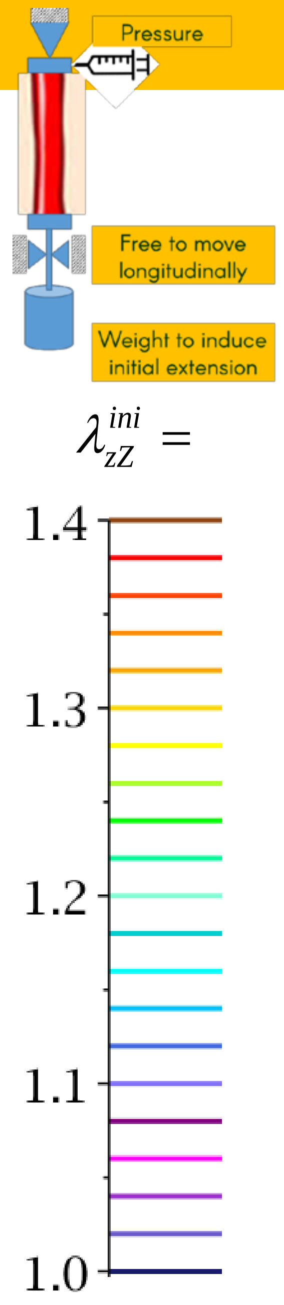




Model predictions

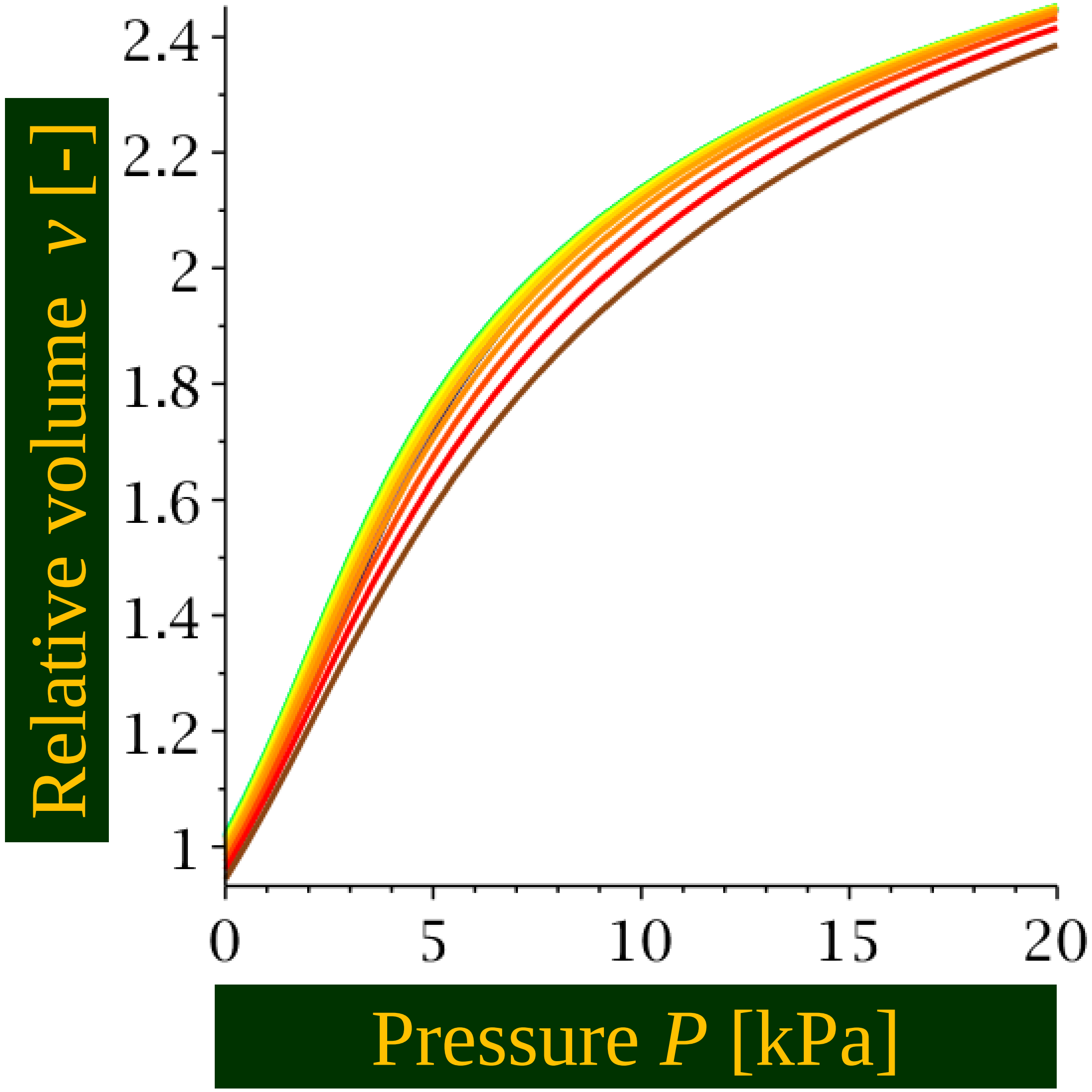
- Extension

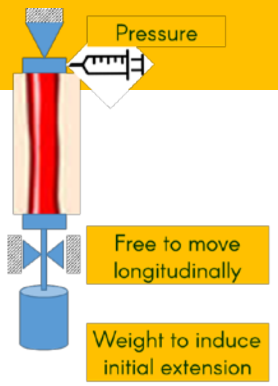


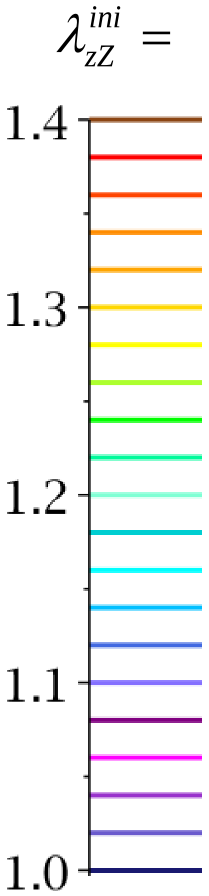


Model predictions

- Volume inside the tube

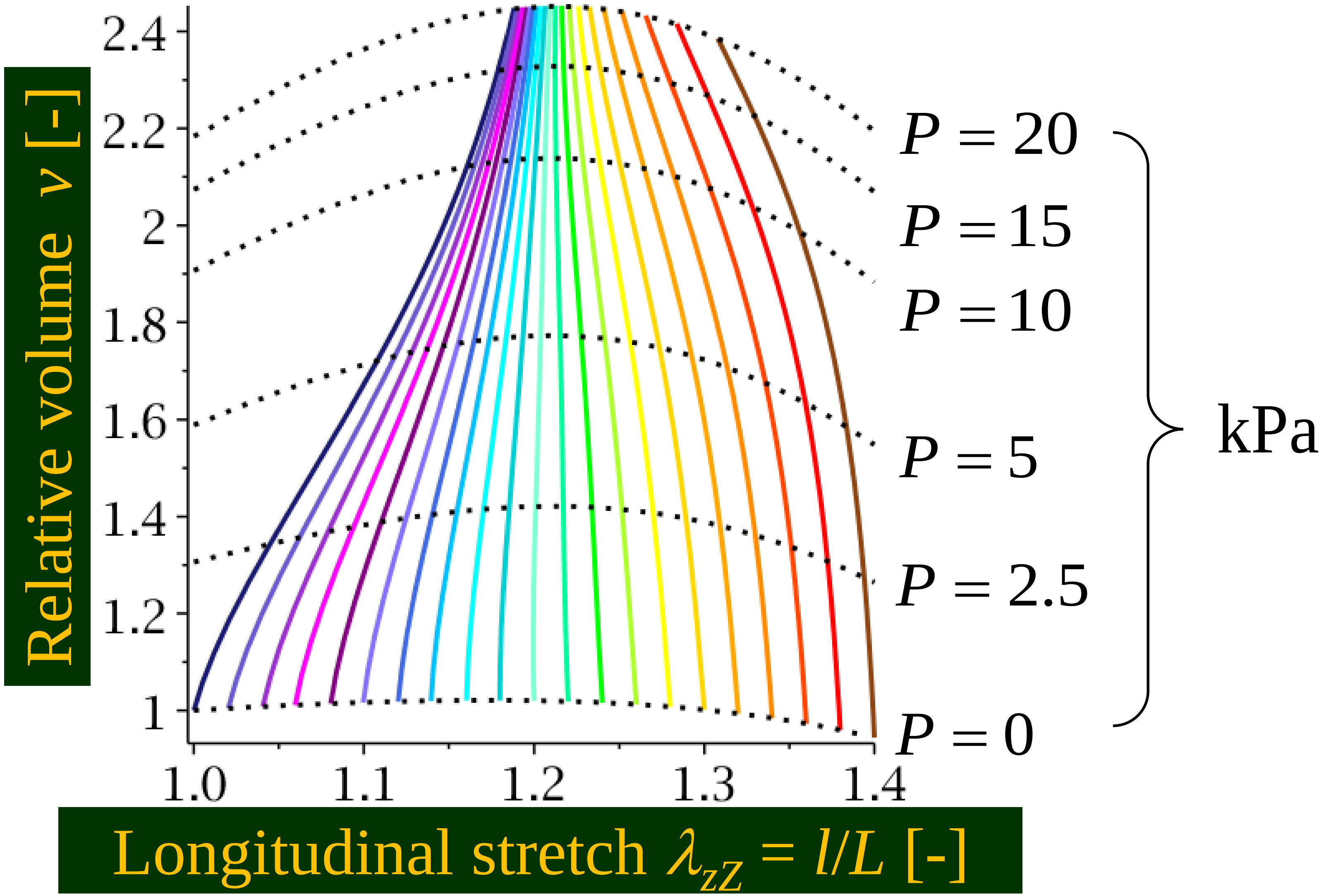






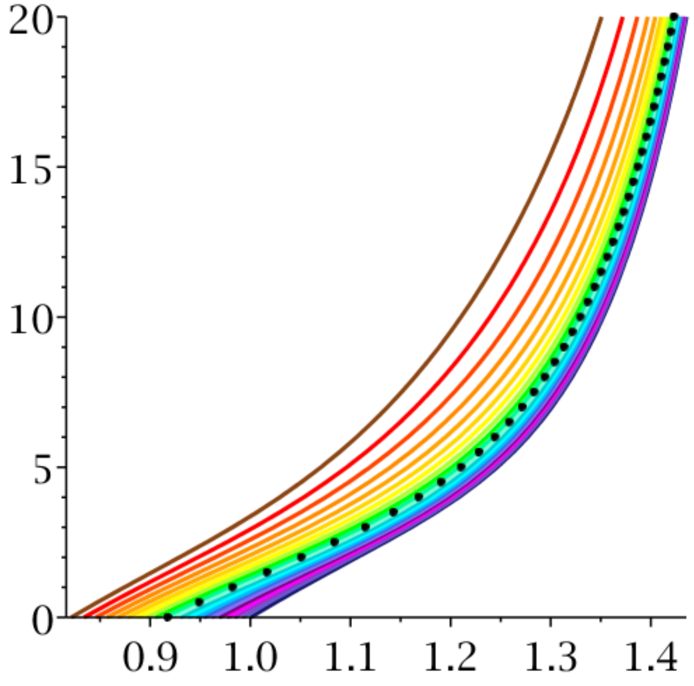
Model predictions

- Volume inside tube

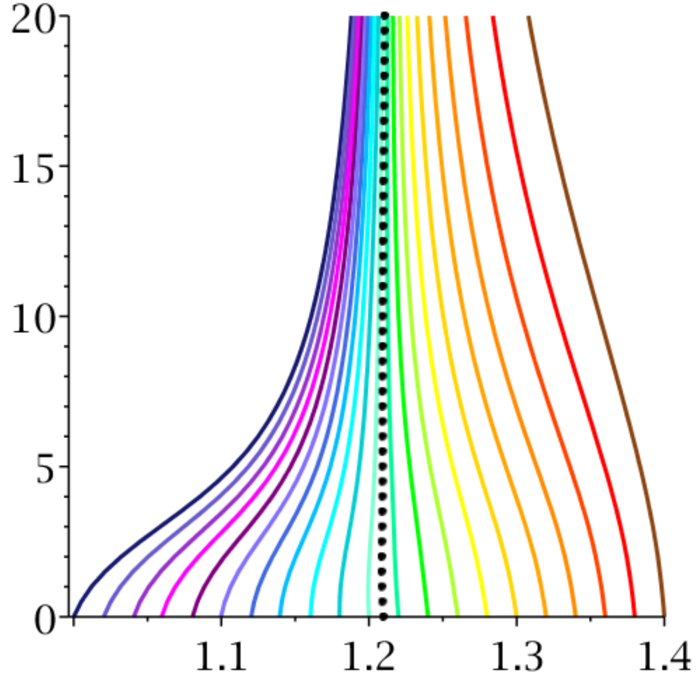


Derived hypothesis

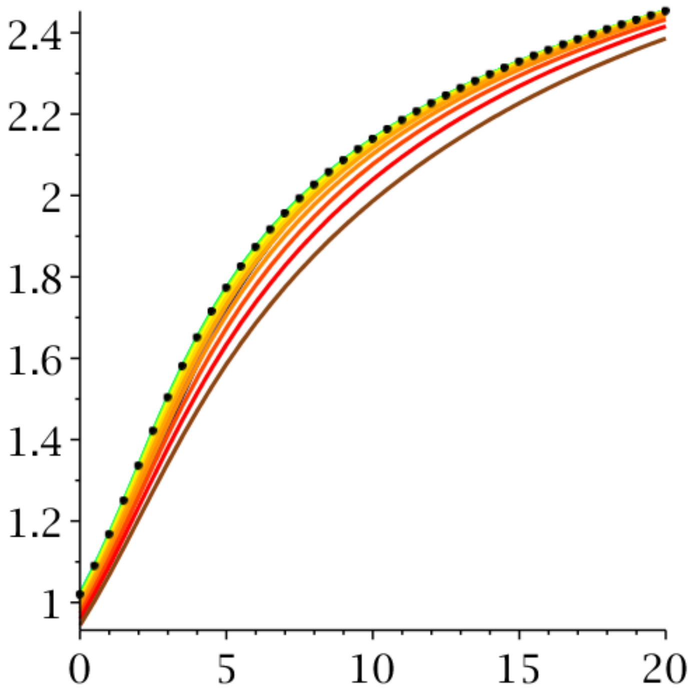
- Axial prestretch which maximizes volume



P [kPa] – $\lambda_{\theta\Theta}$ [-]



P [kPa] – λ_{zZ} [-]



v [-] – P [kPa]

Maximum volume $\lambda_{zZ}^{ini} \doteq 1.21$

Conclusion and questions

- From above mentioned we hypothesize that there is a value of axial prestretch which maximizes volume
- Does this value correspond to the prestretch measured in autopsy?
- Thus, is arterial mechanics an example of optimal design?

Thank you for your attention.

http://users.fs.cvut.cz/~hornyluk/files/COMPLAS-2019.pdf