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Pressurization of Axially Prestretched Tube: Consequences for Arterial Mechanics

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Abstract. It is well known that in their *in situ* position, arteries are axially prestretched. They retract upon excision as a consequence. Studies of the prestretch have shown that it depends on location in arterial tree. Axial prestretch increases with increasing distance from the heart. It has also been shown that the prestretch changes with age. Aging damages internal architecture of arteries and consequently longitudinal pretension is gradually loosen. Results obtained in laboratory experiments document that there is specific value of the prestretch under which axial deformation of the pressurized artery is negligible. Under such conditions, mechanical work performed by the pressure on axial displacements is negligible which is advantageous from biomechanical point of view. In the present study, by adopting the Gent model of the strain energy density function to characterize mechanical properties of the artery and assuming that this artery satisfies assumptions of the membrane theory, it will be shown that the value of prestretch, which minimizes axial displacement, also maximizes internal volume attained in the pressurization of the tube.

Keywords: Artery, Hyperelasticity, Inversion point, Prestretch, Pressure.

1 Introduction

Arteries *in situ* are significantly prestretched in an axial direction [1-4]. It is manifested as a retraction that is observed in autopsy or surgery when arteries are excised. The axial prestretch is expressed as the ratio of *in situ* to *ex situ* length of a cylindrical segment of an artery. Axial prestretch in arteries strongly depends on age [3-5]. It gradually decreases until totally relaxed state and even pre-compressed arteries can be observed [4-6]. The prestretch also depends on anatomical location [1, 4-7]. It increases with increasing distance from the heart and is well correlated with the cross-sectional area of the vascular wall [7].

Ex vivo mechanical tests with enzymatically treated arteries [8,9] and genetically modified animal models [10,11] suggest crucial role of elastin for bearing the prestretch. Fundamental role of elastic membranes in the axial prestretching is in accordance with distally increasing magnitude of the prestretch [7]. Caudally decreasing number of elastic lamellae, which sustain approximately constant axial force in the whole aorta [7], lead to increased force per lamella which results in increasing prestretch.

The axial prestretch of arteries has important physiological function. It was found that *in vivo* value of the prestretch enables arteries to sustain minimal changes in axial deformation and axial force during pressure cycle [12-14]. Originally, it was assumed that nearly zero axial deformation during pressure pulse transition is a consequence of significant tethering (attachment to surrounding tissue) [12]. Experimental studies, however, showed that longitudinal immobility of arteries is related to a biological tuning that couples constitutive properties, internal structure and physiological range of arterial loading.

To be more specific, it was found that a typical *in vitro* inflation behavior of an artery held at constant length is such that there is a value of the axial prestretch above which force–pressure relationship creates increasing curve and under prestretch smaller than this value the force–pressure relationship is decreasing [13-15]. Contrary, pressurization experiments with constant axial load have shown that for small values of the load the arteries elongate during pressurization, whereas with large values of the axial load they shorten [16-18]. It is generally accepted that *in vivo* value of the prestretch (and axial force) is exactly the value under which an artery neither shortens nor elongates in pressure cycle. By some authors, this value is referred to as the inversion stretch or the stretch at inversion point [16-18].

Previous studies of the axial prestretch have revealed that it is mechanically favorable for human arteries to operate in the prestretched state [12-18]. It minimizes mechanical work necessary to transmit pressure pulse because no axial displacements of the wall during pressure cycle means no work spent on these displacements. Since real arteries are rather viscoelastic than elastic, it also means, that there will be no energy dissipated in axial movement of the artery wall. In the present paper we would like to show another interesting property which can be observed when nonlinear elastic tube is inflated at constant axial load. It will be shown that at axial prestretching corresponding to the inversion point, when no axial movement is exhibited during inflation, also maximum internal volume of the pressurized tube is attained. Since large arteries serve as a conduit in blood transport, it suggests that they operate in mechanically optimal regime which allows maximum of the blood to be transferred in one pressure cycle.

2 Methods

Our study will not deal with any specific artery, instead we will focus our attention only to general properties known from arterial elasticity. By means of analytical model, it will be shown that maximal internal volume of the pressurized tube corresponds to loading at inversion axial stretch. To this end, the artery considered in our model is assumed to be made from Gent hyperelastic material [19,20]. The Gent hyperelastic

model is well-known in elastomer mechanics and soft tissue elasticity and is frequently used in biomechanical studies describing physiological principles in arterial mechanics. It was used for example by Horný et al. [21] in their study of aging impact on the pre-tension sustained by the human abdominal aorta, by Horgan and Saccamandi [22] and Sang et al. [23] to study the inflation-extension behavior of an artery and in [22] to study longitudinal oscillations of arterial strip. It is isotropic nonlinear material model exhibiting large strain stiffening which is a property common to all soft tissues.

Constitutive model. The specific form of the strain energy density function W corresponding to the Gent model is expressed in (1).

$$W = -\frac{\mu J_m}{2} \ln \left(1 - \frac{I_1 - 3}{J_m} \right) \quad (1)$$

Here μ is stress-like material parameter which at infinitesimal strains corresponds to the shear modulus and J_m is dimensionless parameter modulating nonlinear behavior of the material. I_1 is the first principal invariant of the right Cauchy-Green strain tensor $\mathbf{C}$. $\mathbf{C} = \mathbf{F}^T \mathbf{F}$ where $\mathbf{F}$ denotes tensor of the deformation gradient. The material is assumed to be incompressible and its constitutive equation is written in the form of (2).

$$\boldsymbol{\sigma} = \frac{\partial W}{\partial \mathbf{F}} \mathbf{F}^T - p \mathbf{I} \quad (2)$$

In (2), $\boldsymbol{\sigma}$ denotes Cauchy stress tensor, $\mathbf{I}$ denotes second order unit tensor and p is Lagrangean multiplier enforced by incompressibility constraint which has to be determined from force boundary condition.

Kinematics of inflation–extension response. The studied artery is considered to be long thin-walled tube with closed ends which, in the reference configuration, has a middle radius R , thickness H , and length L . Assume that during pressurization, the motion of a material particle located originally at (R, Θ, Z) , which is sufficiently distant from the ends, is described by the equations summarized in (3).

$$r = \lambda_{r\Theta} R, \quad h = \lambda_{rR} H, \quad z = \lambda_{zZ} Z, \quad \theta = \Theta \quad (3)$$

Here r and h respectively denote deformed middle radius and thickness. The equations (3) express the fact that the tube uniformly inflates and extends and that it does not twist. Stretches λ_{kK} ($k = r, \theta, z$; $K = R, \Theta, Z$) are the components of the deformation gradient $\mathbf{F}$. Consequently, $\mathbf{F} = \text{diag}[\lambda_{rR}, \lambda_{\theta\Theta}, \lambda_{zZ}]$. Incompressibility constraint then reads $\det(\mathbf{F}) = \lambda_{rR} \lambda_{\theta\Theta} \lambda_{zZ} = 1$.

Equilibrium of thin-walled tube. The equilibrium equations of a thin-walled closed tube loaded by an internal pressure P can be written in the form (4). Here σ_{rr} , $\sigma_{\theta\theta}$, and σ_{zz} respectively, denote the radial, circumferential, and axial component of the Cauchy

stress tensor. The radial component was considered to be zero due to the thin-wall assumption.

$$\sigma_{rr} = 0 \quad \sigma_{\theta\theta} = \frac{rP}{h} \quad \sigma_{zz} = \frac{rP}{2h} + \frac{F_{red}}{2\pi rh} \quad (4)$$

F_{red} in (4c) denotes additional axial force which ensures longitudinal prestretch of the tube. This loading remains constant during the pressurization of the tube. However, the axial deformation may change according to the mechanical state which results from a combination of equilibrium and constitutive equations and loading by internal pressure.

Geometry and material parameters. Constitutive parameters characterizing artery material were adopted from literature. In [21] Horný et al. conducted uniaxial tensile tests with cylindrical segments of the human abdominal aorta and approximated mechanical response by the model (1). For the purposes of our calculations, parameters corresponding to 48 years old female individual were chosen. Specific values used in the study are in Table 1.

Table 1. Material parameters and dimensions.

age	gender	J_m	μ	R	H
48	female	0.22	23	5.81	1.62
years		-	kPa	mm	mm

Simulation of inflation-extension behavior. Quasistatic inflation-extension behavior of the artery was simulated numerically in Maple 2015. The system of equations describing response to internal pressure and axial loading is obtained by substitution of (1) to (2) and subsequent substitution (2) into (4). Finally deformed radius and thickness in (4) are substituted from (3) and $\lambda_{rR} = (\lambda_{\theta\theta}\lambda_{zz})^{-1}$ is used to eliminate the radial stretch. One equation, (4a), is used to determine p coming from the incompressibility constraint. The result is the system of two nonlinear equations which contain material parameters, reference dimensions and P , F_{red} , $\lambda_{\theta\theta}$, and λ_{zz} . The final form of the governing equations is expressed in (5) and (6).

$$\left[\lambda_{\theta\theta} \frac{\partial W}{\partial \lambda_{\theta\theta}} \right]_{\lambda_{rR} = \lambda_{\theta\theta}^{-1} \lambda_{zz}^{-1}} - \left[\lambda_{rR} \frac{\partial W}{\partial \lambda_{rR}} \right]_{\lambda_{rR} = \lambda_{\theta\theta}^{-1} \lambda_{zz}^{-1}} = P \lambda_{\theta\theta}^2 \lambda_{zz} \frac{R}{H} \quad (5)$$

$$\left[\lambda_{zz} \frac{\partial W}{\partial \lambda_{zz}} \right]_{\lambda_{rR} = \lambda_{\theta\theta}^{-1} \lambda_{zz}^{-1}} - \left[\lambda_{rR} \frac{\partial W}{\partial \lambda_{rR}} \right]_{\lambda_{rR} = \lambda_{\theta\theta}^{-1} \lambda_{zz}^{-1}} = \frac{P}{2} \lambda_{\theta\theta}^2 \lambda_{zz} \frac{R}{H} + \frac{F_{red}}{2\pi RH} \lambda_{zz} \quad (6)$$

One may select two variables as independent and remaining quantities are computed from equations modeling the problem. Our solution is obtained in two steps. First, axial prestretch of the tube is induced by assigning $\lambda_{zz} = \lambda_{zz}^{ini}$ at $P = 0$. In this step, the system

(5-6) is used to compute F_{red} necessary to preelongate the tube. Simultaneously the initial circumferential stretch $\lambda_{\theta\theta}^{ini}$ is obtained. In the second step, (5-6) is used to compute $\lambda_{\theta\theta}$ and λ_{zz} at a given P and F_{red} . The sequence $0.5(i - 1)$ kPa for $i = 1, \dots, 41$ was used to simulate the pressurization of the tube. The axial prestretch λ_{zz}^{ini} considered in our study was $1 + 0.02(i - 1)$ for $i = 1, \dots, 14$ that includes typical values of the prestretch in the human abdominal aorta expected at 48 years old individual [3,4,21].

3 Results and Discussion

The results obtained in our simulation are summarized in Figure 1 – 3. The inflation-extension response expressed by means of P – $\lambda_{\theta\theta}$ and P – λ_{zz} relationships at constant axial load corresponding to $\lambda_{zz}^{ini} = 1, 1.02, 1.04, \dots, 1.26$ is in Figure 1. Colors of the curves uniquely correspond to selected values of λ_{zz}^{ini} and specific value of the axial prestretch is evident from the right panel of the figure (P – λ_{zz}). The figure clearly shows that there is a certain value of the prestretch under which the tube changes its response from inflation–extension to inflation–shortening. This is exactly inversion axial stretch and this mechanical response is highlighted by dotted curve and denoted by λ_{zz}^{inv} . Numerical computation showed that, in specific case, it is approximately equal to 1.036.

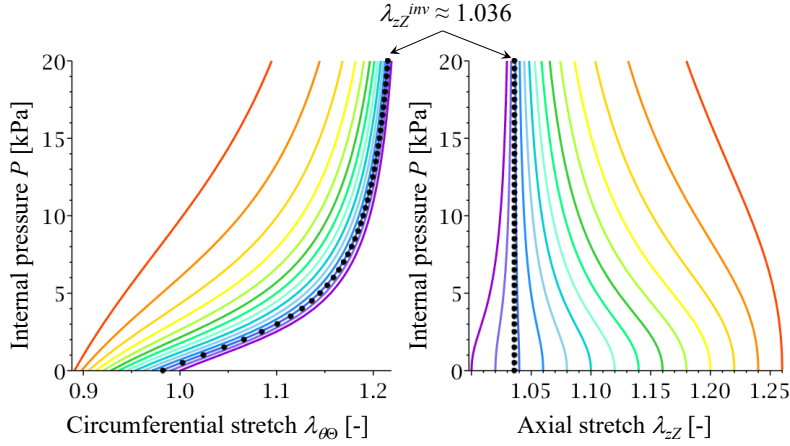


Fig. 1. Inflation-extension response obtained from model. $\lambda_{zz}^{ini} = 1, 1.02, 1.04, \dots, 1.26$ was used in calculation. Inversion axial prestretch, under which tube does not move axially, is highlighted by dotted line.

Figure 2 documents a relationship between inflating pressure and relative internal volume v , $v = \pi r^2 l / (\pi R^2 L) = \lambda_{\theta\theta}^2 \lambda_{zz}$. It is clear that maximal volume of the tube is attained when the tube is inflated at inversion axial stretch (dotted curve). If axes of the figure were changed (pressure as vertical axis and volume as horizontal axis), it would be evident that at the same time the inflation at inversion stretch also leads to minimal work performed during pressurization of the tube because it corresponds to minimal

area under P - v curve. To be exact, we should talk about work per unit volume instead of work because relative volume instead of volume as such is variable on the axis.

Finally, Figure 3 shows traces of the kinematics of tube inflation in a phase space of the deformation. Left panel is for λ_{zz} - $\lambda_{\theta\theta}$ dependence and right panel shows dependence of relative volume v on λ_{zz} . Applied axial prestretch is indicated by color of the curve. Both λ_{zz} - $\lambda_{\theta\theta}$ and v - λ_{zz} also contains contours of constant pressure at $P = 0, 1, 2.5, 5, 10$, and 20 kPa. They are depicted with thin dotted curves. One observes that λ_{zz}^{inv} intersects contours of the constant pressure at their maximal volume which again documents that the inversion prestretch leads to optimal inflation conditions.

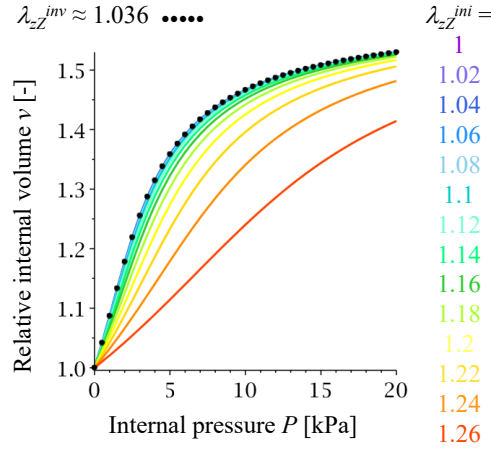


Fig. 2. Relative volume–pressure relationship at various λ_{zz}^{ini} . Dotted curve corresponds to inversion prestretch under which maximal volume is attained. Additionally to maximal volume, the figure clearly shows that minimal work is also spent when tube is inflated under inversion prestretch λ_{zz}^{inv} .

Although previous studies focused on the axial prestretch of arteries have pointed out that the prestretch is mechanically favorable, it was not, to the best of our knowledge, explicitly stated that the inversion stretch leads to maximal volume of the inflated tube. It suggests that tuning of the blood transport is highly sophisticated and optimized from the biomechanical point of view.

Our present result is however only one piece of a jigsaw puzzle of mechanical principles acting in our cardiovascular system. One should not overestimate it because there are other questions which have to be considered and were not addressed in our study. Particularly, the present result was obtained for one specific selection of material parameters. In order to reach physiological relevance it has to be generalized and proved for other, more accurate models of arterial wall which will consider anisotropic behavior of the wall. We also have to note that the thin-wall assumption should be accepted with caution because it is known that arteries exhibit residual strains which cannot be modeled under this assumption. The results are also limited by the fact that our study was based on quasistatic formulation but true blood pressure propagation is

a wave problem. However, our present contribution is only a preliminary study and in future we plan to refine it in a more accurate computational model.

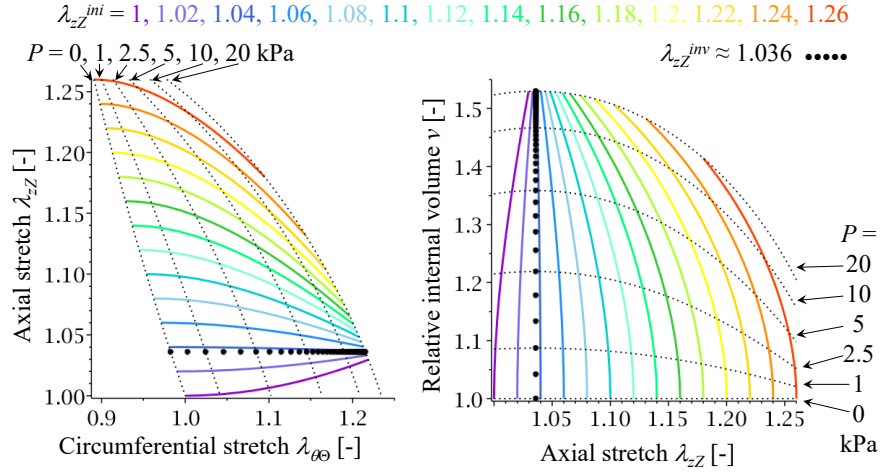


Fig. 3. Kinematics of inflation–extension response depicted in phase space. The left panel shows that $\partial\lambda_{zz}/\partial\lambda_{\theta\theta} = 0$ at inversion stretch and right panel documents that contours of constant pressure have maximum v at λ_{zz}^{inv} .

Finally, it is concluded that results obtained from the simplified model suggest that axial prestretch in human arteries ensures not only the constancy of axial force and axial deformation during the pressure cycle but also the maximum volume attained in the inflation of an artery.

Conflict of Interest

Authors declare that they have no conflict of interest.

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