1 Vectors: $\vec{u} \cdot \vec{v}$, $\vec{u} \times \vec{v}$, linear independence

1. Find $\vec{u} \cdot \vec{v}$ and $\vec{u} \times \vec{v}$. Are the vectores perpedicular?
   (a) $\vec{u} = (10; -2; 3)$, $\vec{v} = (1; 2; -1)$
   (b) $\vec{u} = (-3; 2; 3)$, $\vec{v} = (1; 6; -3)$
   (c) $\vec{u} = (-2; p + 3)$, $\vec{v} = (0; -1 + 2p)$, $p \in \mathbb{R}$ parameter.
2. Given line $p: \ 2x + 5y - 6 = 0$,
   (a) Find point $P \in p$; $P = [?, 2]$.
   (b) Write the parametric equation of the line.
   (c) Find line q; $q \perp p \quad \bigwedge \quad P \in q$.
3. Find the angle between given lines:
   (a) $p : x = 1 + 2t; y = 2 - 3t; \quad t \in \mathbb{R}$
   $q : x = 1 + k; y = 2 + 3k; \quad k \in \mathbb{R}$.
   (b) $p : x = 1 + 2t; y = 2 - 3t; \quad t \in \mathbb{R}$
   $q : x = -1 - 4k; y = 7 + 9k; \quad k \in \mathbb{R}$.
4. Find vector $\vec{x}$ which satisfies:
   $2(\vec{x} + \vec{u}) = 3\vec{v} + (0; 0; 2)$, $\vec{u} = (1; -3; 0)$ and $\vec{v} = (0; 2; 1)$

2 Linear Independence, basis, dimension.

Are the following vectors Linearly Independent? What is the vector space the vectors are generating (Write the basis and dimension)?

1. $\vec{u} = (2; 1)$, $\vec{v} = (-1; 3)$
2. $\vec{u} = (1; 4; 2)$, $\vec{v} = (3; 2; 2)$
3. $\vec{u} = (2; 0; 3)$, $\vec{v} = (1; 1; 0)$, $\vec{w} = (0; -2; 1)$
4. $\vec{u} = (2; 3; -2)$, $\vec{v} = (3; 0; 1)$, $\vec{w} = (0; 9; -8)$
5. $\vec{a} = (2; 4; 3; 0)$, $\vec{b} = (1; 1; 0; 0)$, $\vec{c} = (3; 5; 3; 0)$, $\vec{d} = (1; 0; 2; 0)$

Write the vectors $\vec{a}$ and $\vec{b}$ as a linear combination of vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$. Is the expression unique?

6. $\vec{u} = (1; 3; 2)$, $\vec{v} = (2; -1; 3)$, $\vec{w} = (5; 1; 8)$
   $\vec{a} = (3; 2; 5)$, $\vec{b} = (5; 6; 7)$
7. $\vec{u} = (3; 4; 5)$, $\vec{v} = (-6; 7; 0)$, $\vec{w} = (8; -9; 1)$
   $\vec{a} = (23; -19; 6)$, $\vec{b} = (-20; 23; -1)$
8. $\vec{u} = (4; 0; 7; 2)$, $\vec{v} = (3; 1; 0; 5)$, $\vec{w} = (5; 3; 1; 0)$
   $\vec{a} = (3; 1; 8; -8)$, $\vec{b} = (0; 0; 0; 1)$