(Eigenvalues and eigenvectors)

(1. - 2.) Find all eigenvalues to the given matrix, choose one and find the corresponding eigenvector

1. $\begin{pmatrix} 4 & -5 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix}$

2. $\begin{pmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{pmatrix}$

(3.) You have a 3×3 matrix, which of following statements can be true:

(a) $\lambda_1 = 2,\ \lambda_2 = 3$

(b) $\lambda_1 = 3,\ \lambda_2 = 2 + i,\ \lambda_3 = -2 - i$

(c) $\lambda_1 = \lambda_2 = \lambda_3 = 1$

(d) $\lambda_1 = 0,\ \lambda_2 = i,\ \lambda_3 = -i$

(e) $\lambda_1 = 2,\ \lambda_2 = 1,\ \lambda_3 = 2 + i$

(f) given eigenvectors (on tutorial)

4. Find all eigenvalues of matrix A, choose two of them and find the corresponding eigenvectors.
   $A = \begin{pmatrix} -2 & -2 & -9 \\ -1 & 1 & -3 \\ 1 & 1 & 4 \end{pmatrix}$

5. (a) Check if $\lambda_1 = -1$ is an eigenvalue of the following matrix.
   $A = \begin{pmatrix} 2 & 1 & 2 \\ 5 & 2 & 6 \\ 1 & 0 & -1 \end{pmatrix}$
   (b) Choose one of the eigenvalues and find a corresponding eigenvector.
   (c) Find the eigenvalues to A^2, choose one of them and find the corresponding eigenvector.

Repetition

1. (a) Are vectors $(1;2)$, $(2;3)$ and $(0;4)$ linear independent, why?
   (b) Find the parameters $a \in \mathbb{R}$ for which the following vectors form a base:
   $\vec{u} = (a; 0; a+3)$, $\vec{v} = (1;0;0)$ and $\vec{w} = (0; a-1; a)$.
   (c) If possible, express vectors $\vec{b} = (1;0;-3)$ and $\vec{c} = (2;-5;3)$ as a linear combination of vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ for $a = 1$.

2. (a) Determine how many solutions will have the following system depending on parameter $a \in \mathbb{R}$:

$$\begin{aligned} -x - 2y + z &= 3 \\ az + x + 3y &= 2a \\ 5y + 3x &= a \end{aligned}$$

   (b) Find the solution for the parameter value $a = -4$.

3. Check if exists an inverse to A. If it exists, compute its determinant: $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 4 & 5 \end{pmatrix}$.

4. compute $A \cdot B$.
   $A = \begin{pmatrix} 2 & -1 & 3 \\ 0 & 5 & 2 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 1 \\ 0 & 3 \\ 5 & 0 \end{pmatrix}$.

5. $\begin{vmatrix} 1 & 0 & -1 & -1 \\ 0 & -1 & -1 & 1 \\ 0 & 2 & 0 & 0 \\ -1 & -1 & 1 & 0 \end{vmatrix} = ?$

Results: 1a) NOT LI,b) $a \in \mathrm{R} - \{1; -3\}$, c) E.g. $\vec{b} = \vec{u} - 7\vec{w}$; $\vec{c} =$ impossible; 2) ∞ sol. for $a = -4$, 1 sol. elsewhere,b) $\vec{x} = (7 - 5p; -5 + 3p; p)$, $\forall p \in \mathrm{R}$; 3) A^{-1} doesn't exist; 4) (21, -1 \\ 10, 15); 5) -2