

Functions and limits

1. For a function $f(x) = \frac{1}{x^2}$ find a limit $L = \lim_{x \rightarrow 0} f(x)$ and a number $\delta > 0$ such that $\forall x \in P_\delta(0)$ is $f(x) > 100$.
2. For a function $f(x) = 3x + 2$ find a limit $L = \lim_{x \rightarrow 1} f(x)$ and a number $\delta > 0$ such that $\forall x \in P_\delta(1)$ is $f(x) \in U_\epsilon(L)$. $\epsilon = 0.1$

Compute the following limits

3. $\lim_{x \rightarrow \infty} \sin \frac{1}{x}$
4. $\lim_{x \rightarrow \infty} \left(\frac{x^3}{x^2+1} - x \right)$
5. $\lim_{x \rightarrow \infty} (\sqrt{x^2 - 1} - x)$
6. $\lim_{x \rightarrow 2} \frac{x^2 - 4x + 1}{2x + 1}$
7. $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{2x^2 - x - 1}$
8. $\lim_{x \rightarrow 0} \frac{3x^2 + x^3}{x^2 + x^5}$
9. $\lim_{x \rightarrow 0} \arctan(x^{-2})$
10. $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{\sin^3 x}$

Limits of composed function

11. $\lim_{x \rightarrow \infty} \arcsin \frac{x}{x+1}$
12. $\lim_{x \rightarrow \infty} \arctan \frac{x}{x^3 + 3x^2}$
13. $\lim_{x \rightarrow 1} \cos^3 \left(\pi \frac{x^2 - 1}{x - 1} \right)$

One sided limits

Find both one sided limits and decide if the original limit exists:

14. $\lim_{x \rightarrow 2} \frac{x^2 + \sin(x^2)}{x - 2}$
i.e. find $\lim_{x \rightarrow 2^+} \frac{x^2 + \sin(x^2)}{x - 2}$ and $\lim_{x \rightarrow 2^-} \frac{x^2 + \sin(x^2)}{x - 2}$ and compare the results.
15. $\lim_{x \rightarrow 2} \frac{x^2 + \sin(x^2)}{(x - 2)^2}$
16. $\lim_{x \rightarrow \frac{\pi}{2}} \tan x$
17. $\lim_{x \rightarrow 0} \frac{\ln 3x}{3x}$

Find (a) Domain of definition ($\mathcal{D}(f)$) and Range of the given function, (b) compute limits in boundary points of $\mathcal{D}(f)$.

18. $f(x) = \ln(x + \sqrt{x + 1})$
19. $f(x) = \sqrt{x^2 - x - 20} - 1$
20. $f(x) = \ln(x + 3) + \sqrt{5 - 2x}$

Continuity

21. Where is the function $f(x) = \frac{1}{9-x^2}$ continuous?

22. Where is the function $f(x) = \frac{1}{\ln x}$ continuous?

23. Is the following function continuous in a point $x_0 = 0$?

$$f(x) = \begin{cases} \frac{(x+2)^2-4}{x} & \text{for } x \in \mathbb{R} - \{0\} \\ 4 & \text{for } x = 0 \end{cases}$$

24. Find parameter $a \in \mathbb{R}$ such that the following function is continuous:

$$f(x) = \begin{cases} ax & \text{for } x > 1 \\ 4 - ax & \text{for } x \leq 1 \end{cases}$$

25. Find parameter $a \in \mathbb{R}$ such that the following function is continuous:

$$f(x) = \begin{cases} e^{ax} & \text{for } x < 0 \\ a - x & \text{for } x \geq 0 \end{cases}$$