1)

$$\int \cos^2 2x\,dx = \frac{1}{2}\int (1+\cos 4x)\,dx = \frac{1}{2}x + \frac{1}{8}\sin 4x + C$$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 2\cos^2 x - 1$

$\boxed{\cos^2 x = \frac{1}{2}(1+\cos 2x)}$

2)

$$\int \sin^2 x \cos^2 x\,dx = \int \sin^2 x\,(1-\sin^2 x)\,dx = \int \sin^2 x\,dx - \int \sin^4 x\,dx =$$

$\boxed{\sin^2 x = \frac{1-\cos 2x}{2}}$

$$= \frac{1}{2}x - \frac{1}{4}\cancel{\sin 2x} - \int \left(\frac{1-\cos 2x}{2}\right)^2 dx =$$

$$= \text{—||—} - \frac{1}{4}\int (1 - 2\cos 2x + \cos^2 2x)\,dx =$$

$$= \text{—||—} - \frac{1}{4}x + \frac{1}{4}\cancel{\sin 2x} - \frac{1}{8}\int (1+\cos 4x)\,dx =$$

$$= \frac{1}{4}x - \frac{1}{8}x - \frac{1}{32}\sin 4x + C = \frac{1}{8}x - \frac{1}{32}\sin 4x + C$$

3)

$$\int \frac{\cos^3 x}{\sin^4 x}\,dx = \int \frac{1-\sin^2 x}{\sin^4 x}\cos x\,dx =$$

$$= \int \frac{1}{\sin^4 x}\cos x\,dx - \int \frac{1}{\sin^2 x}\cos x\,dx =$$

$\left| \begin{array}{l} t = \sin x \\ dt = \cos x\,dx \end{array} \right|$ $= -\frac{1}{3}\frac{1}{\sin^3 x} + \frac{1}{\sin x} + C$

$\boxed{x \neq k\pi}$

$$\int t^{-4}dt - \int t^{-2}dt = \frac{t^{-3}}{-3} - \frac{t^{-1}}{-1}$$

10)

$$\int \frac{\tan^3 x}{\cos^2 x}\,dx \left| \begin{array}{l} t = \tan x \\ dt = \frac{1}{\cos^2 x}dx \end{array} \right| = \int t^3\,dt = \frac{1}{4}\tan^4 x + C$$

5) $\int \cos^3 x \sin^3 x \, dx =$

$$= \int \cos^3 x \sin^2 x \sin x \, dx =$$

$$= \int \cos^3 x \left(1 - \cos^2 x\right) \sin x \, dx =$$

$$\left| \begin{array}{l} t = \cos x \\ dt = -\sin x \, dx \end{array} \right| = -\int t^3 \left(1 - t^2\right) dt =$$

$$= -\int t^3 \, dt + \int t^5 \, dt =$$

$$= -\frac{\cos^4 x}{4} + \frac{\cos^6 x}{6} + C$$

'221'

Ma 1 w 21 3.

$$\int \frac{9}{x^2-2x+5} = 9\int \frac{1}{(x-1)^2+4}\,dx =$$

$D = 4 - 20$

x^2-2x+1

$$= 9\int \frac{1}{4\left(1+\frac{(x-1)^2}{4}\right)}\,dx = \frac{9}{4}\int \frac{1}{1+\left(\frac{x-1}{2}\right)^2}\,dx =$$

$$\left| \begin{array}{l} \frac{x-1}{2} = t \\ \frac{dx}{2} = dt \end{array} \right| = \frac{9}{2}\arctan\left(\frac{x-1}{2}\right) + C$$

integrály sin, cos

$$\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx \quad \left| \begin{array}{l} t = 2x \\ dt = 2dx \end{array} \right|$$

$$= \frac{1}{2}x - \frac{1}{4}\sin 2x + C$$

$\cos 2x = \cos^2 x - \sin^2 x$

$\cos 2x = 1 - 2\sin^2 x$

$\Rightarrow \sin^2 x = \frac{1-\cos 2x}{2}$

$\cos 2x = 2\cos^2 x - 1$

$\cos^2 x = \frac{1+\cos 2x}{2}$

$$\int \cos^2 x\,dx = \int \frac{1+\cos 2x}{2}\,dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$$

$$\int \cos^3 x\,dx = \int \cos^2 x \cos x\,dx = \int (1-\sin^2 x)\cos x\,dx =$$

$$\left| \begin{array}{l} \sin x = t \\ \cos x\,dx = dt \end{array} \right| = \int 1 - t^2\,dt = t - \frac{t^3}{3} + C =$$

$$= \sin x - \frac{\sin^3 x}{3} + C$$

$$\int \sin^4 x\,dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 dx = \frac{1}{4}\int (1 - 2\cos 2x + \cos^2 2x)\,dx =$$

$$= \frac{1}{4}\int \left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right) dx = \frac{1}{4}\left(\frac{3}{2}x - \sin 2x + \frac{1}{8}\cos 4x\right) + C$$