(Systems of Linear Algebraic Equations (SLAE))

1. a) Determine the number of solution (of the following SLAE):

$$\begin{aligned} x_1 - 2x_2 + 3x_3 - 4x_4 &= 4 \\ x_2 - x_3 + x_4 &= -3 \\ x_1 + 3x_2 - 3x_4 &= 1 \\ -7x_2 + 3x_3 + x_4 &= -3 \end{aligned}$$

   b) Find all the possible solutions.

Determinants and applications

Compute determinants of the following matrices and decide if there are singular or not. Try to write down the rank of the matrices.

- $A_1 = \begin{pmatrix} -1 & -3 \\ -2 & 5 \end{pmatrix}$
- $A_2 = \begin{pmatrix} 2 & 5 & 0 \\ -1 & 7 & 1 \\ 4 & 1 & -4 \end{pmatrix}$
- $A_3 = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 4 & 5 \end{pmatrix}$
- $A_4 = \begin{pmatrix} 1 & 1 & -1 \\ 2 & 1 & 0 \\ 1 & -1 & 1 \end{pmatrix}$
- $A_5 = \begin{pmatrix} 1 & i & 1+i \\ -i & 1 & 0 \\ 1-i & 0 & 1 \end{pmatrix}$

Compute determinants of the following matrices and decide if there are singular or not:

- $A_6 = \begin{pmatrix} 0 & 5 & -2 & 3 \\ 1 & 2 & 0 & 0 \\ 5 & 2 & 3 & 2 \\ 2 & -1 & 2 & 3 \end{pmatrix}$
- $A_7 = \begin{pmatrix} 1 & 0 & -1 & -1 \\ 0 & -1 & -1 & 1 \\ a & b & 0 & 0 \\ -1 & -1 & 1 & 0 \end{pmatrix}$
- $A_8 = \begin{pmatrix} a & a & a \\ -a & a & x \\ -a & -a & x \end{pmatrix}$

Inverse matrix and its determinant

2. Find the inverse matrix (A^{-1}) and compute its determinant, $A = \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix}$

3. Compute the determinant of an inverse matrix A^{-1}:

   (a) $A = \begin{pmatrix} 3 & -5 & 0 \\ 0 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$ (b) $A = \begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

Linear Independence of vectors with parameters

4. Find the parameter $p \in \mathbb{R}$ for which the vectors are linear independent, $\vec{u} = (3+p; 7; 1)$, $\vec{v} = (-2; 2p; 4)$ and $\vec{w} = (1; 0; 1)$

5. Find the parameter $k \in \mathbb{R}$ for which the vectors are linear independent, $\vec{u} = (k; 1; 0)$, $\vec{v} = (0; k-1; 3)$ and $\vec{w} = (0; 2; k)$

Cramer's rule

6. Solve:

$$
\begin{aligned}
3x_1 + 2x_2 &= 0\\
4x_1 - 5x_2 &= 40
\end{aligned}
$$

7. Find the solution for z:

$$
\begin{aligned}
2x + 3y - 3z &= -1\\
4x - 4y - z &= 3\\
8x - 9z &= 0
\end{aligned}
$$

8. Find all solutions depending on parameter $m \in \mathbb{R}$

$$
\begin{aligned}
4x_1 + 2x_2 - 2x_3 &= 0\\
2x_1 + x_2 + 3x_3 &= 0\\
mx_1 + x_2 + mx_3 &= 0
\end{aligned}
$$

9. Find a solution for x_1 depending on parameter $m \in \mathbb{R}$

$$
\begin{aligned}
-7x_2 - 5x_3 &= -1\\
(2m-1)x_1 - x_2 &= 1\\
4mx_1 - 7x_2 - 5x_3 &= 0
\end{aligned}
$$

10. Find a parameter $p \in \mathbb{R}$ for which the system has non-trivial (not only zero) solution:

$$
\begin{aligned}
px + 4y + 7z &= 0\\
3x - 4y + 5z &= 0\\
x + py + 4z &= 0
\end{aligned}
$$