(Cramer's rule)

1. Find the solution for z:

$$\begin{aligned} 2x + 3y - 3z &= -1 \\ 4x - 4y - z &= 3 \\ 8x - 9z &= 0 \end{aligned}$$

2. Find all solutions depending on parameter $m \in \mathbb{R}$

$$\begin{aligned} 4x_1 + 2x_2 - 2x_3 &= 0 \\ 2x_1 + x_2 + 3x_3 &= 0 \\ mx_1 + x_2 + mx_3 &= 0 \end{aligned}$$

3. Find a solution for x_1 depending on parameter $m \in \mathbb{R}$

$$\begin{aligned} -7x_2 - 5x_3 &= -1 \\ (2m-1)x_1 - x_2 &= 1 \\ 4mx_1 - 7x_2 - 5x_3 &= 0 \end{aligned}$$

4. Find a parameter $p \in \mathbb{R}$ for which the system has non-trivial (not only zero) solution:

$$\begin{aligned} px + 4y + 7z &= 0 \\ 3x - 4y + 5z &= 0 \\ x + py + 4z &= 0 \end{aligned}$$

Eigenvalues and eigenvectors

(1. - 2.) Find the eigenvalues and eigenvectors to the given matrix

1. $\begin{pmatrix} 3 & 4 \\ 5 & 2 \end{pmatrix}$

2. $\begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix}$

(3. - 5.) Find all eigenvalues to the given matrix, choose one and find the corresponding eigenvector

3. $\begin{pmatrix} 3 & 1 & 0 \\ -13 & -1 & 0 \\ 4 & -8 & -2 \end{pmatrix}$

4. $\begin{pmatrix} 4 & -5 & 1 \\ 1 & 0 & -1 \\ 0 & 1 & -1 \end{pmatrix}$

5. $\begin{pmatrix} 2 & -3 & 1 \\ 1 & -2 & 1 \\ 1 & -3 & 2 \end{pmatrix}$

(6.) You have a 3×3 matrix, which of following statements can be true:

(a) $\lambda_1 = 2$, $\lambda_2 = 3$

(b) $\lambda_1 = 3$, $\lambda_2 = 2 + i$, $\lambda_2 = -2 - i$

(c) $\lambda_1 = \lambda_2 = \lambda_3 = 1$

(d) $\lambda_1 = 0$, $\lambda_2 = i$, $\lambda_2 = -i$

(e) $\lambda_1 = 2$, $\lambda_2 = 1$, $\lambda_2 = 2 + i$

(f) given eigenvectors (on tutorial)