Double integrals II

1. Given domain in $\mathbb{R}^2$ is bounded by curves: $y = x^3$; $\quad y = \sqrt{x}$.
   (a) Sketch the domain and express it as EDI relative to y-axis.
   (b) Express the domain as EDI relative to x-axis.
   (c) Compute area of the domain.
   (d) Compute $\iint_D (4xy - y^3)\,\mathrm{d}x\,\mathrm{d}y$.

2. Reverse the order of integration (a): $\int_0^{\pi} (\int_0^{y} x \sin y\,\mathrm{d}x)\,\mathrm{d}y$
   and compute the double integral (b).

3. Compute the double integral $\int_0^{2} (\int_{y/2}^{1} e^{x^2}\,\mathrm{d}x)\,\mathrm{d}y$.
   hint: Reversing the order of the integration could help you.

4. Given domain in $\mathbb{R}^2$ is bounded by curves: $y = 8 - x^2$; $\quad y = x^2$.
   (a) Sketch the domain and express it as EDI of your choice.
   (b) Compute area of the domain.
   (c) Compute $\iint_D xy\,\mathrm{d}x\,\mathrm{d}y$.

5. Given $D = \{[x, y] \in \mathbb{R}^2 :\ 0 \le x \le \sqrt{4 - y^2}\}$.
   (a) Sketch the domain and express it as EDI relative to y-axis.
   (b) Transfer the following integral to polar coordinates $\iint_D f(x, y)\,\mathrm{d}x\mathrm{d}y$.

6. Transfer the following integral to polar coordinates $\int_{-3}^{0} (\int_0^{\sqrt{9-x^2}} \sin(x^2 + y^2)\,\mathrm{d}y)\,\mathrm{d}x$ and compute it.