

## Line integral II

1. A curve is given as a segment of a function  $y = x^2$  from  $A = [0; 0]$  to  $B = [2; 4]$ .
  - (a) Suggest its parametrization (and determine bounds for the parameter).
  - (b) Compute line integral of a scalar function  $f(x, y) = x + 4xy$ .
  - (c) Compute line integral of a vector function  $\vec{f}(x, y) = (x; xy)$ .
2. A curve is given as a segment of a function  $x = 2y^2$  from  $A = [8; 2]$  to  $B = [2; 1]$ .
  - (a) Suggest its parametrization (and determine bounds for the parameter).
  - (b) Compute line integral of a vector function  $\vec{f}(x, y) = (\sqrt{x} + y; x + \sqrt{y})$ .
3. For a curve  $C = \{[x, y] \in \mathbb{R}^2 : y = e^x \wedge |x| \leq 1\}$  oriented with the starting point  $[1; e]$  compute integral:

$$\int_C x^3 dx + \frac{1}{y} \ln y dy.$$

4. For a curve  $C = \{[x, y] \in \mathbb{R}^2 : x^2 + y^2 = a^2 \wedge y \geq 0\}$  oriented clockwise ( $a \in \mathbb{R}^+$ ) compute:

$$\int_C \frac{1}{\sqrt{x^2 + y^2}} (y; -x) \cdot d\vec{s}.$$

5. For the line segment between points  $A = [7; -2; 0]$  a  $B = [3; 0; 1]$  compute integral:

$$\int_C (z^2 \vec{i} + z \vec{j} - y \vec{k}) \cdot d\vec{s}.$$

6. Compute line integral of a vector function  $\vec{f}(x, y, z) = (-yz; z\sqrt{9 - y^2}; xy)$  along one-quarter of a thread of a circular helix with radius 3 ( $z = \frac{3t}{2\pi}$ ). The helix starts in the lowest point  $z = 0$ .

7. Given force  $\vec{f}(x, y, z) = \frac{K}{\sqrt{(x^2 + y^2 + z^2)^3}} (x; y; z)$  where  $K > 0$  is a real constant.

Compute a work done by the force when it moves a mass point along a strait line  $AB$  where  $A = [2; 0; 0]$  and  $B = [2; 1; 5]$ .