Line integral - circulation

1. $\vec{f}(x, y) = (2x - y, x)$ and a curve $x^2 + y^2 = 4$ oriented counter-clockwise.
   (a) Compute circulation of $\vec{f}$ over the curve.
   (b) Transfer the integral to double integral using the Green's theorem (check the assumptions).
   (c) Compare the results.

2. Given curve is a boundary of domain $\{[x, y] \in \mathbb{R}^2 : \ x^2 + y^2 \leq 4 \ \wedge \ x \geq 0 \ \wedge \ y \geq 0\}$ oriented counter-clockwise.
   (a) Compute circulation of $\vec{f}(x, y) = (-xy, \ y^2 + 2y)$ over the curve.
   (b) Suggest another approach of the computation (just one equality).

3. For a vector function $\vec{f}(x, y) = (-\ln(x^2 + y^2), 1)$ compute the circulation over a curve:
   (a) $\{[x, y] \in \mathbb{R}^2 : \ x^2 + y^2 = 1\}$ oriented counter-clockwise.
   (b) $\{[x, y] \in \mathbb{R}^2 : \ (x - 1)^2 + y^2 = 1\}$ oriented counter-clockwise.
   (c) C_c is boundary of a square with a center [3,1] and side length $a = 2$ oriented positively.

4. Given a curve $C = C_1 \cup C_2$ where C_1 is a line between points $[1; \sqrt{2}]$ and $[1; -\sqrt{2}]$ and $C_2 = \{[x, y] \in \mathbb{R}^2 : \ (x - 1)^2 + y^2 = 2 \ \wedge \ x \leq 1\}$ oriented clockwise.
   Compute
   $$\oint_C (x, 1) \cdot \mathrm{d}\vec{s}.$$

5. Given curve is a boundary of a triangle $P = [1; 1]$, $Q = [2; 1]$, $R = [2; 3]$ oriented respectively.
   Compute
   $$\oint_C \left(\frac{1}{y}, -\frac{1}{x}\right) \cdot \mathrm{d}\vec{s}.$$

6. Given curve is a boundary of domain $\{[x, y] \in \mathbb{R}^2 : \ y \geq x^2 \ \wedge \ x \geq 0 \ \wedge \ y \leq 1\}$ oriented negatively. Compute the circulation of a vector field $\vec{f}(x, y) = (-y/2, x/2)$ over the curve.