Repetition - dif. calculus

1. Given $f(x,y) = \sqrt{1+xy^2}$.

   (a) Where is the function differentiable?

   (b) $\vec{s} = \vec{AB}$ where A = [2;-2] and B = [5;2]. Find a directional derivative $\frac{\partial f}{\partial \vec{s}}(A)$.

   (c) Find a directional derivative of $f(x,y)$ in the point A in the direction of maximum grow.

   (d) Find an equation of a tangent plane to the graph of the function in the point A.

2. (a) Is $u(x,y) = e^{-x}\cos y - e^{-y}\cos x$ a solution of the Laplace equation

$$u_{xx} + u_{yy} = 0\ ?$$

   (b) Find local extrema of $f(x,y) = x^2 + xy - y^2 - 6\ln x$, determine its type and position.

3. (a) Find local extrema of $f(x,y) = 2y - y^2 - xe^x$.

   (b) Find absolute (global) extrema of $g(x,y) = xy - x^2 + 3y^2 - 4y$ on a set $M = \{x, y \in \mathbb{R}^2 : y = 1 - x \ \wedge\ -1 \leq x \leq 1\}$.

4. (a) Find partial derivatives of 1^{st} order: $g(x,y) = \left(\frac{x^3}{2} + \frac{2}{y^3}\right)e^{2x}$.

   (b) By the equation $F(x,y) = \frac{x^2}{2} + y^3 - xy - 1 = 0$ is around the point $A = [2;1]$ implicitly define function $y = f(x)$, verify.

   (c) Find the Taylor's polynomial of 2^{nd} order for the function $y = f(x)$ in the neighbourhood of $x_0 = 2$. Use the result to approximate $f(1.8)$.

5. By the equation $F(x,y) = x^2 - xy + 2y^2 + x - y - 1 = 0$ is around the point $A = [0;1]$ implicitly define function $y = f(x)$.

   (a) Verify that the $y = f(x)$ has continuous first and second derivative.

   (b) Find $y'(0)$ and $y''(0)$. Is the point $x_0 = 0$ a local extreme of $y = f(x)$?

   (c) Write a tangent line and a normal to the graph of $y = f(x)$ in the tangent point A.

6. By the equation $F(x,y,z) = x^3 + y^3 + z^3 + xyz - 6 = 0$ is around the point $T = [1;2;-1]$ implicitly define function $z = f(x,y)$.

   (a) Verify that $z = f(x,y)$ has continuous P.D. in $T_0 = [1;2]$ and compute them.

   (b) For the direction $\vec{u} = (-1;2)$ compute $\frac{\partial f}{\partial \vec{u}}(T_0)$.

   (c) Compute total differential of $z = f(x,y)$ in T_0.

Results

1. (a) diff. on $\Omega = \{x, y \in \mathbb{R}^2 : 1 + xy^2 > 0\}$ (b) -2/3 (c) $\frac{2\sqrt{5}}{3}$ (d) $z - 3 = \frac{2}{3}(x-2) - \frac{4}{3}(y+2)$

2. (a) yes (b) no extrema ($[2\sqrt{\frac{3}{5}}; \sqrt{\frac{3}{5}}]$ is a saddle p.)

3. (a) $1 + \frac{1}{e}$ is loc. max. in [-1;1] (b) $-\frac{5}{4}$ is min. in $\left[\frac{1}{2}; \frac{1}{2}\right]$ and 1 is max. in [-1;2]

4. (a) $g_x = 2e^{2x}(\frac{3x^2}{4} + \frac{x^3}{2} - \frac{2}{y^3})$, $g_y = -e^{2x}\frac{6}{y^4}$ (b) $T_2(x) = 1 - 1(x-2) - \frac{9}{2}(x-2)^2$, $f(1.8) \doteq 0.84$

5. (b) $y'(0) = 0$, $y''(0) = -\frac{2}{3}$; 1 is loc. max. in x_0 (c)$\tau:\ y = 1$, $\nu:\ x = 0$

6. (a) $z_x(T_0) = -\frac{1}{5}$, $z_y(T_0) = -\frac{11}{5}$ (b) $-\frac{21\sqrt{5}}{25}$ (c) $df = -\frac{1}{5}dx - \frac{11}{5}dy$