Partial derivatives

1. Find a domain of definition of following functions (and sketch it), compute all partial derivatives:
   (a) $f(x, y) = \ln(9 - x^2 - 9y^2)$
   (b) $f(x, y) = x^y$
   (c) $f(x, y, z) = \sqrt{x} + \sqrt{y} + \sqrt{z}$
   (d) $f(x, y, z) = xz - 5x^2y^3z^4$
2. To given function $f(x, y, z, t) = x^2 y \cos(\frac{z}{t})$ find the $\frac{\partial f}{\partial t}$.
3. Compute all partial derivatives of $f(x, y, z) = x \sin(y - z)$ in a point $A = [1; 0; 0]$. What does these values mean?
4. Compute all partial derivatives of $f(x, y, z) = ze^{xyz}$ in a point $A = [0; 2; -1]$. What does these values mean?
5. a) Compute all partial derivatives of $f(x, y) = \ln(2x - y) + 3x^3 - xy$ in a point $A = [1; 1]$.
   b) Write a tangent line of the function in a cut $x \equiv 1$ in tangent point A.
6.* Compute first and second order partial derivatives of following functions:
   (a) $f(x, y) = x^2 + 5xy + \sin(xy) + xe^{y^2/2}$
   (b) $f(x, y) = y + x^2y + 4y^3x - \ln(y^2 + x)$
7. a) Compute all partial derivatives of $f(x, y) = \ln(2x - y) + 3x^3 - xy$ in a point $A = [1; 1]$.
   b) Write a tangent line of the function in a cut $x \equiv 1$ in tangent point A.
8. a) Compute all partial derivatives of $f(x, y, z) = ze^{xyz}$ in a point $A = [0; 2; -1]$.
   b) Write a tangent line of the function in a cut $y \equiv 2 \wedge z \equiv -1$ in tangent point A.
9. Verify that a function $u(x, y) = e^y(y^2 - x^2)$ is a solution of an equation
$$y\frac{\partial u}{\partial x} + x\frac{\partial u}{\partial y} = xu.$$
10. Verify that a function $u(x, t) = \sin(x - ta)$ (with parameter $a \in \mathbb{R}$) is a solution of an equation
$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0.$$

Differential and tangent (hyper-)plane

11. a) Write (total) differential of a function $f(x, y) = \frac{y}{x}$ in a point $A_0 = [2; 1]$.
    b) Approximate the increment of the function between points A_0 and $A_1 = [2.1; 1.2]$
    (i.e. $\Delta f = f(A_1) - f(A_0) =?$)
12. By using the (total) differential, approximate the value of $f(0.97; 1.02; 0.99) = \frac{\sqrt[4]{0.97}}{1.02^3 \sqrt[3]{0.99}}$ (with 2 decimal places precision) hint: Use known value $f(1; 1; 1)$.