(tangent (hyper-)plane)

1. a) Find an equation of the plane tangent to the graph of $f(x, y) = \ln(x + y)$ at a point $[1; 0; ?]$.
   b) Use the result to approximate the functional value $f(A_1)$ in a point $A_1 = [1.1; 0.1]$.

2. Given $f(x, y) = 2x^2 - y^2$ and a plane $\sigma: \quad 8x - 6y - z + 12 = 0$.
   a) Find a plane (τ) tangent to the graph of f and parallel to the plane σ.
   b) Find a line (ν) normal to the graph of f and normal to the plane σ.

3. Find an equation of the hyper-plane (τ) tangent to the graph of $f(x, y, z) = \ln(x^2 - y + 3z)$ at a point $T = [2; 1; 1; ?]$.

4. Given $f(x, y, z) = \ln(z + \sqrt{9 - x^2 - y^2})$,
   a) Find Domain of definition of f and sketch it (at least in 2 cuts).
   b) Find an equation of the hyper-plane (τ) tangent to the graph of f at a point $T = [0; 0; 1; ?]$.

Gradient and directional derivative

5. Given $f(x, y) = \sqrt{1 - x^2} - \sqrt{1 - y^2}$,
   a) find Domain of definition of f and sketch it.
   b) Where is the function f differentiable? (Find the domain of differentiability.)
   c) Compute gradient of the function in a point $A = [1/2; 0]$

6. Given $f(x, y) = \frac{\sqrt{y - x^2}}{1 - x^2}$ and a point $A = [0; 1]$,
   a) find the Domain of definition of f and sketch it.
   b) Where is the function f differentiable? (Find the domain of differentiability.)
   c) Determine the direction in which the graph of the function is increasing the most at point A.

7. Given $f(x, y, z) = \sin xz + x + y - \frac{z}{y}$ and a point $A = [2; 1; 0]$,
   Determine the direction of maximal decrease of the function f at the point A.

8. Given $f(x, y) = x^2 + 2xy - 3y^2$ and a point $A = [1; 1]$,
   a) compute the (directional) derivative of f at point A in direction given by vector $\vec{s} = (3; 4)$.
   b) Describe the behavior of the function in this direction.
   c) Compute the derivative of f at point A in the direction given by the vector $\vec{t} = \frac{1}{\sqrt{2}}(1; 1)$. What can you say about the function in this direction at the point A?

9. Given $f(x, y) = \cos xy + e^{xy}$ and a point $A = [1; 0]$,
   a) determine the direction $\vec{s}$ of maximal increase of the function f at a point A.
   b) Compute the (directional) derivative of f at point A in the direction given by a vector $\vec{s}$.
   c) Compute the derivative of f at point A in the direction given by a vector $\vec{t} = (1; 2)$. What can you say about the function in this direction?

10. Given $f(x, y) = \sqrt{9 - x - y^2}$ and a point $A = [1; -2]$,
    a) compute gradient of the function at point A.
    b) Find the direction vector $\vec{u}$ in which the function doesn't change its value.

11. Given $f(x, y, z) = x^2 - 2y^2 - 3z^3 - 17$ and a point $A = [1; 1; 1]$,
    compute the directional derivative of f at point A in the direction given by a vector $\vec{s} = (1; 1; 1)$. What can you say about the function in this direction?