

(Gradient and directional derivative)

- Given $f(x, y) = x^2 + 2xy - 3y^2$ and a point $A = [1; 1]$,
 - compute the (directional) derivative of f at point A in direction given by vector $\vec{s} = (3; 4)$.
 - Describe the behavior of the function in this direction.
 - Compute the derivative of f at point A in the direction given by the vector $\vec{t} = \frac{1}{\sqrt{2}}(1; 1)$. What can you say about the function in this direction at the point A ?
- Given $f(x, y) = \cos xy + e^{xy}$ and a point $A = [1; 0]$,
 - determine the direction \vec{s} of maximal increase of the function f at a point A .
 - Compute the (directional) derivative of f at point A in the direction given by a vector \vec{s} .
 - Compute the derivative of f at point A in the direction given by a vector $\vec{t} = (1; 2)$. What can you say about the function in this direction?
- Given $f(x, y) = \sqrt{9 - x - y^2}$ and a point $A = [1; -2]$,
 - compute gradient of the function at point A .
 - Find the direction vector \vec{u} in which the function doesn't change its value.
- Given $f(x, y, z) = x^2 - 2y^2 - 3z^3 - 17$ and a point $A = [1; 1; 1]$, compute the directional derivative of f at point A in the direction given by a vector $\vec{s} = (1; 1; 1)$. What can you say about the function in this direction?

Local extrema

- Given $f(x, y) = x^2y + \cos y + y \sin x$,
Find all partial derivatives of first and second order. Decide if the origin ($O = [0; 0]$) is the critical point of the function f (verify). Find the Hesse matrix in this point.
- Given $f(x, y) = x^y$,
Find all partial derivatives of first and second order. Decide if $P = [1; 1]$ is the critical point of the function f (verify).
- Find the local extrema of the function $f(x, y) = \ln(1 - x^2 - y^2)$, i.e. find their position, type and value.
- Find the local extrema of the function $f(x, y) = 2xy - 5x^2 - 2y^2 + 4x + 4y$, i.e. find their position, type and value.
- Find the local extrema of the function $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$, i.e. find their position, type and value.
- Determine if the function $f(x, y) = 4xy - x^4 - y^4 - 11$ has local extremes at points $A_0 = [0; 0]$ or $A_1 = [1; 1]$. If the answer is YES, find its type and value.
- Has the function $f(x, y) = e^x \cos y$ local extrema?
- Find all critical points of the function $f(x, y) = y \cos x$.