Chain rule (derivatives of composite functions)

1. Given $f(u,v) = u^2 \ln v$, compute $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ when you know that $u(x,y) = \frac{x}{y}$ and $v(x,y) = 3x - 2y$.

2. Given unknown function $z(x,y) = f(u,v) = f(u(x,y), v(x,y))$ and functions $u(x,y) = x^2 - y^2$, $v(x,y) = e^{xy}$. Compute $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at point $A = [1;2]$ when you know (from physics) that $\frac{\partial f}{\partial u}(A) = 1$ and $\frac{\partial f}{\partial v}(A) = 0$.

Implicitly defined functions

3. Given $F(x,y) = x^3 + y^3 - 6xy + 4$,
   verify that by the equation $F(x,y) = 0$ is implicitly defined function $y = f(x)$ near the point $A = [1;1]$. Compute its derivative $\frac{\mathrm{d}f}{\mathrm{d}x}$ at point $x_0 = 1$ and find an equation of tangent to the graph of $f(x)$.

4. Verify that by equation $x^3y + y^3x + x^2y - 3 = 0$ is implicitly defined function $y = f(x)$ near the point $A = [1;1]$.
   Compute its derivative $\frac{\mathrm{d}f}{\mathrm{d}x}$ at point $x_0 = 1$ and find an equation of normal to the graph of $f(x)$.

5. Given $F(x,y) = \sin(x+y) - y^2 \cos x$,
   verify that by the equation $F(x,y) = 0$ is implicitly defined function $y = f(x)$ in the neighborhood of the point $A = [\pi;0]$.
   Compute its derivative $\frac{\mathrm{d}f}{\mathrm{d}x}$ at point $x_0 = \pi$ and describe the behavior of $f(x)$ near point A (is it increasing or decreasing, how fast?).

6. Given $F(x,y) = x^3 + 2x^2y + y^4$
   verify that by the equation $F(x,y) = 1$ is implicitly defined function $y = f(x)$ near the point $A = [2;-1]$.
   Compute the first and the second derivative of f at point $x_0 = 2$.