(implicitly defined functions)

1. Prove that the equation $\ln(x+2y)+x-1=0$ imlicitly defines (in some neighborhood) a function $y=f(x)$ which satisfies $f(1)=0$. Aproximate the value $f(1.1)$ by the second order Taylor's polynomial.

2. Given $F(x,y)=x^3+y^3-2x^2-xy+1$,
   verify that by the equation $F(x,y)=0$ is implicitly defined function $y=f(x)$ near the point $A=[1;0]$.
   Compute the first and the second derivative of $y=f(x)$ at point $x_0=1$ and describe the behavior of $y=f(x)$ near point A (is it increasing or decreasing, convex or concave?).

3. a) Find equation of an iso-curve for $F(x,y)=xye^{x-y}$ at point $P=[1;2]$.
   b) Find a tangent line to this iso-curve at point P.

4. Given $F(x,y)=\ln(xy+4)-2\ln 2$ and a point $A=[0;2]$.
   Can the equation $F(x,y)=0$ defined correctly the implicitly defined function $y=f(x)$ near the point A?
   If not, suggest how to compute tangent to the iso-curve $F(x,y)=0$. (hint: switch the variables)

5. Given $F(x,y,z)=x^3+y^3+z^3+xyz-6$,
   a) verify that by the equation $F(x,y,z)=0$ is implicitly defined function $z=f(x,y)$ near the point $A=[1;2;-1]$.
   b) Compute all the partial derivatives of $z=f(x,y)$ at point $T=[1;2]$.
   c) Find an equation of the tangent plain which is tangent to the graph of $z=f(x,y)$ at tangent point A.

6. Verify that by the equation $xz^2-x^2y+y^2z+2x-y=0$ is implicitly defined function $z=f(x,y)$ near the point $A=[0;1;1]$.
   Find a direction in which is the function $z=f(x,y)$ increasing the most at point $[0;1]$.

7. Given $F(x,y,z)=z^3+3x^2z-2xy=0$
   (a) $\exists?$ $z=f(x,y)$ near the point $A=[-1;-2;1]$ defined implicitly?
   (b) Find its gradient ($\nabla f=?$) at point $A_0=[-1;-2]$.
   (c) Compute a tangent plane (τ) to $z=f(x,y)$ at point A.