Double integrals, Elementary Domain of Integration (EDI)

1. Given $f(x,y) = \frac{1}{\sqrt{9-x^2-y^2}}$.
   $\exists? \iint\limits_D f(x,y)\,\mathrm{d}x\,\mathrm{d}y$

   (a) $D = \{x, y \in \mathbb{R}^2;\ x \geq 0 \ \wedge\ x^2 + y^2 < 9\}$

   (b) $D = \{x, y \in \mathbb{R}^2;\ x \geq 0 \ \wedge\ x^2 + y^2 \leq 8\}$

2. Given domain in $\mathbb{R}^2$, bounded by curves: $y = x - 1; \quad y = -1; \quad y = \ln(x)$.

   (a) Sketch the domain and express it as EDI relative to y-axis.

   (b) Express the domain as EDI relative to x-axis.

   (c) Compute area of the domain.

3. Given curves: $y^2 = x + 2; \quad y = x$.

   (a) Sketch a domain bounded between them and determine intersection points.

   (b) Express the domain as EDI relative to y-axis.

   (c) Compute area of the domain.

4. Given domain $D = \{x, y \in \mathbb{R}^2;\ x \geq 0 \ \wedge\ x + y \leq 2 \ \wedge\ y \geq \sqrt{x}\}$.
   To the given function $f(x,y) = xy$ compute $\iint\limits_D f(x,y)\,\mathrm{d}x\,\mathrm{d}y$.

5. Change the order of integration:

$$\int_1^e \Big(\int_0^{\ln(x)} f(x,y)\,\mathrm{d}y \Big)\,\mathrm{d}x$$

6. Reverse the order of integration (a):

$$\int_0^1 \Big(\int_1^{x+1} e^x\,\mathrm{d}y \Big)\,\mathrm{d}x$$

   and compute the double integral (b).

7. Given domain in $\mathbb{R}^2$ is bounded by curves: $y = x^3; \quad y = \sqrt{x}$.

   (a) Sketch the domain and express it as EDI relative to y-axis.

   (b) Express the domain as EDI relative to x-axis.

   (c) Compute area of the domain.

   (d) Compute $\iint\limits_D (4xy - y^3)\,\mathrm{d}x\,\mathrm{d}y$.