

Double integrals II, polar coordinates

1. Given domain $D \subset \mathbb{R}^2$ lies in 1st quadrant and is bounded by curves: $y = x$ and $x = 3$.
 - (a) Sketch the domain and express it as Elementary Domain of Integration relative to y -axis.
 - (b) Express the domain as EDI relative to x -axis.
 - (c) Compute $\iint_D \left(\frac{x^2+y^2}{x}\right) dx dy$
 - (d) Begin a calculation (compute inner integral) of the previous integral with the other EDI .
2. Reverse the order of integration (a): $\int_0^\pi \left(\int_0^y x \sin y dx\right) dy$
and compute the double integral (b).
3. Compute the double integral $\int_0^2 \left(\int_{y/2}^1 e^{x^2} dx\right) dy$.
hint: Reversing the order of the integration could help you.
4. Given domain in \mathbb{R}^2 is bounded by curves: $y = 8 - x^2$; $y = x^2$.
 - (a) Sketch the domain and express it as EDI of your choice.
 - (b) Compute area of the domain.
 - (c) Compute $\iint_D xy dx dy$.
5. Given $D = \{[x, y] \in \mathbb{R}^2 : 0 \leq x \leq \sqrt{4 - y^2}\}$.
 - (a) Sketch the domain and express it as EDI relative to y -axis.
 - (b) Transfer the following integral to polar coordinates $\iint_D f(x, y) dx dy$.
6. Transfer the following integral to polar coordinates $\int_{-3}^0 \left(\int_0^{\sqrt{9-x^2}} \sin(x^2 + y^2) dy\right) dx$ and compute it.
7. Given $D = \{[x, y] \in \mathbb{R}^2 : x^2 + y^2 \leq 4 \wedge y \geq 0\}$.
 - (a) Transfer the following integral to polar coordinates: $\iint_D xy dx dy$.
 - (b) Compute the integral.
8. Given $f(x, y) = \frac{1}{\sqrt{9-x^2-y^2}}$
and $D = \{[x, y] \in \mathbb{R}^2; x \geq 0 \wedge x^2 + y^2 \leq 8\}$.

$$\iint_D f(x, y) dx dy = ?$$