Double integrals: applications, polar coordinates

1. Given 2D body: $D = \{[x,y] \in \mathbb{R}^2 :\ 0 \le x \le 1\ \wedge\ 0 \le y \le 2x+1\}$.
   Its (2D) density $\rho(x,y) = x$.
   (a) Compute its mass.
   (b) Compute the static moment according to y-axis ($m_y =?$).
   (c) Determine the x-coordinate of center of mass ($x_C =?$).
   (HW:) Determine the y-coordinate of center of mass ($y_C =?$). $[y_C = 17/14]$

2. Given 2D body bounded by curves: $y = \frac{2}{x} - 1;\quad y = x;\quad y = 0,$
   with (2D) density $\rho(x,y) = (y+1)^2$. Compute its moment of inertia relative to x-axis ($J_x =?$)

3. Given $D = \{[x,y] \in \mathbb{R}^2 :\ x^2 + y^2 \le 4\ \wedge\ y \ge 0\}$.
   (a) Transfer the following integral to polar coordinates:
   $$\iint\limits_D xy\,\mathrm{d}x\mathrm{d}y.$$
   (b) Compute the integral.
   (c) Write one possible physical meaning of the integral, $\rho(x,y) =?$.
   (HW:) Determine the center of mass ($C =?$) when $\rho(x,y) = y$. $[y_C = 3\pi/8]$.

4. Given $f(x,y) = \frac{1}{\sqrt{9-x^2-y^2}}$
   and $D = \{[x,y] \in \mathbb{R}^2;\ x \ge 0\ \wedge\ x^2 + y^2 \le 8\}$.
   $$\iint\limits_D f(x,y)\,\mathrm{d}x\mathrm{d}y = ?$$

5. Given $D = \{[x,y] \in \mathbb{R}^2 :\ \frac{x^2}{9} + \frac{y^2}{4} \le 1\ \wedge\ x \ge 0\ \wedge\ y \ge 0\}$.
   (a) Transfer the following integral to generalized polar coordinates:
   $$\iint\limits_D xy^2\,\mathrm{d}x\mathrm{d}y.$$
   (b) Compute the integral.
   (c) Write all possible physical meanings of the integral, $\rho(x,y) =?$

6. Given $D = \{[x,y] \in \mathbb{R}^2 :\ 1 \le y \le x^2\ \wedge\ (0) \le x \le 2\}$,
   compute volume of a body form above domain D under the graph of function $f(x,y) = 3 + \frac{x}{y^2}$.