

Triple integrals

1. Given a body: $M = \{[x, y, z] \in \mathbb{R}^3 : 0 \leq x \wedge 0 \leq y \leq x \wedge 0 \leq z \leq 3 - x - y\}$.
Its density $\rho(x, y, z) = 3$.
 - (a) Determine the upper x -boundary and sketch the projection to xy plane.
 - (b) Compute its mass.
2. Given a body bounded by: planes $y = 2x$; $x = 0$; $y = 2$
and inequality $2 - y \leq z \leq 3$.
Sketch the projection to xy plane and compute its volume.
3. Given a body in the first octant, bounded by: $x + y = 1$; $z = xy$.
Sketch the projection to xy plane and compute its volume.
4. Sketch (in cuts) a cylindrical reservoir with radius $R = 2$, symmetrical about z -axes which is bounded by planes $z = 0$ and $z = -3 - x$. Compute its volume.
5. Sketch (in cuts) a body $M = \{[x, y, z] \in \mathbb{R}^3 : 0 \leq z \leq 4 - x^2 - y^2 \wedge y \geq 0\}$.
Compute its volume.
6. Given a body M bounded by surfaces: $y = \sqrt{x}$; $y = 0$; $z = 0$; $x + z = \frac{\pi}{2}$
Sketch the projection to xy plane and compute

$$\iiint_M y \cos(x + z) \, dx dy dz.$$

7. Given a prism M bounded by surfaces: $x = 1$; $y = 0$; $y = x$; $z = 0$; $z = \sqrt{2}$.
Sketch the projection to xy plane, sketch projection to xz plane and compute

$$\iiint_M (x + y + z) \, dx dy dz.$$

Choose which approach is better (projection to xy plane or projection to xz plane).