(Triple integrals)

1. Given a prism M bounded by surfaces: $x = 1;\ y = 0;\ y = x;\ z = 0;\ z = \sqrt{2}$
   Sketch the projection to xy plane, sketch projection to xz plane and compute

$$\iiint\limits_M (x + y + z)\,\mathrm{d}x\mathrm{d}y\mathrm{d}z.$$

   Choose which approach is better (projection to xy plane or projection to xz plane).

Triple integrals: applications, cylindrical coords.

2. Given a body: $M = \{[x, y, z] \in \mathbb{R}^3 :\ 0 \le x \le 3\ \wedge\ 0 \le y \le 3\ \wedge\ 0 \le z \le xy\}$.
   Compute static moment about the xy-plane when $\rho(x, y, z) = x^2 + y^2$.

3. Given a homogeneous body (in 3D) bounded by surfaces: $z = \sqrt{3x^2 + 3y^2};\ z = 3$
   Sketch the projection to xy-plane and compute moment of inertia about the z-axes.
   $\rho(x, y, z) = \rho = const.$

4. Given a body: $M = \{[x, y, z] \in \mathbb{R}^3 :\ 0 \le z \le 4 - \sqrt{x^2 + y^2}\}$.

   (a) Transfer the following integral to cylindrical coordinates:

$$\iiint\limits_M \sqrt{x^2 + y^2}\,\mathrm{d}x\mathrm{d}y\mathrm{d}z.$$

   (b) Compute the integral.

   (c) Write one possible physical meaning of the integral, $\rho(x, y, z) =?$.

5. Given a body: $M = \{[x, y, z] \in \mathbb{R}^3 :\ 0 \le z \le 1\ \wedge\ 0 \le y \le x\ \wedge\ \frac{x^2}{3} + y^2 \le 1\}$.

   (a) Transfer the following integral to generalized cylindrical coordinates:

$$\iiint\limits_M 1\,\mathrm{d}x\mathrm{d}y\mathrm{d}z.$$

   (b) Compute the integral.

   (c) Write one possible physical meaning of the integral.

6. Sketch (in cuts) a body $M = \{[x, y, z] \in \mathbb{R}^3 :\ 0 \le z \le h - \sqrt{x^2 + y^2}\}$. $h = const.$
   Compute the center of mass z-coordinate for homogeneous body M ($\rho(x, y, z) = \rho = const.$).
   .
   $\left[C_z = \frac{m_{xy}}{m} = \frac{h}{4}\right]$