Triple integrals: spheres and spherical coords.

1. Given a body: $M = \{[x, y, z] \in \mathbb{R}^3 : \ 1 \leq z \leq \sqrt{9 - x^2 - y^2}\}$.
   Sketch (in cuts) the body and compute its volume.

2. Given a body: $M = \{[x, y, z] \in \mathbb{R}^3 : \ 1 \leq x^2 + y^2 + z^2 \leq 9 \ \wedge \ z \geq 0\}$.

   (a) Transfer the following integral to spherical coordinates:

   $$\iiint_M \sqrt{x^2 + y^2 + z^2}\, \mathrm{d}x\mathrm{d}y\mathrm{d}z.$$

   (b) Compute the integral.

3. Compute mass of a body $M = \{[x, y, z] \in \mathbb{R}^3 : \ x^2 + y^2 + z^2 \leq 4 \ \wedge \ x \geq 0\}$
   for $\rho(x, y, z) = x^2 + y^2$.

4. Compute volume of the body $M = \{[x, y, z] \in \mathbb{R}^3 : \ \sqrt{x^2 + y^2} \leq z \leq \sqrt{1 - x^2 - y^2}\}$

5. Sketch (in cuts) a body $M = \{[x, y, z] \in \mathbb{R}^3 : \ x^2 + y^2 + z^2 \leq 16 \ \wedge \ x^2 + y^2 \leq 9\}$.
   Compute its volume.

6. Compute the center of mass of a half-ball with radius R = 1 which is homogenenous ($\rho = \text{const.}$)
   $\left[z_C = \frac{3}{8}\right]$