(Conservative fields, potential)

1. Find a potential (and verify its existence) of a vector field $\vec{f}(x, y) = (x + y, x - y)$.
   Then compute a circulation of the vector field along a positively oriented circle $x^2 + y^2 = 4$.

2. Given a conservative vector field $\vec{f}(x, y) = (\sqrt{x} + y, \sqrt{y} + x)$.

   (a) Find the potential of the vector field (determine where it is possible).

   (b) Compute $\int_C \vec{f} \cdot \mathrm{d}\vec{s}$ where $C = \{[x, y] \in \mathrm{R}^2 : y = x^2 \ \wedge \ 1 \le x \le 2\}$.

3. Given an incomplete potential $\varphi(x, y) = 2xy^{3/2} + K(y)$, containing an unknown function $K(y)$ depending just on one variable (y). The corresponding conservative/potential vector field is $\vec{f}(x, y) = (U(x, y); V(x, y))$, where $V(x, y) = 3x\sqrt{y} + y$.

   (a) From the definition of potential determine the component U of the given vector field $\vec{f}$.

   (b) Finish the computation of potential $\varphi(x, y)$ by finding the unknown function $K(y)$.

   (c) Find the domain where the vector field $\vec{f}(x, y)$ is conservative.

   (d) Compute the work done by the force $\vec{f}$ acting along the oriented line segment C from the point $A = [1; 0]$ to the point $B = [3; 4]$.

4. $\vec{f}(x, y) = (\frac{x}{\sqrt{x^2+y^2}}, \frac{y}{\sqrt{x^2+y^2}})$.

   (a) Where is $\vec{f}$ conservative?

   (b) Is $\varphi(x, y) = \sqrt{x^2 + y^2}$ its potential?

   (c) Compute the work done along a curve $x^2 + y^2 = 4$ oriented counter-clockwise.

Line integral repetition

5. A curve is given as a line segment from $E = [1; 0; 2]$ to $F = [1; 2; 1]$.

   (a) Compute its mass when $\rho(x, y, z) = x^2 + y^2$.

   (b) For the given potential $\varphi(x, y, z) = x^2y + 2y^4z$ find the corresponding vector field and compute its work done along the curve.

6. Given curve is a boundary of domain $\{[x, y] \in \mathbb{R}^2 : \ y \ge x^2 \ \wedge \ x \ge 0 \ \wedge \ y \le 1\}$ oriented negatively. Compute the circulation of a vector field $\vec{f}(x, y) = (-y/2, x/2)$ over the curve.