

Surface integral I

- Given a surface $\sigma = \{[x, y, z] \in \mathbb{R}^3 : z = 4 - x^2 - y^2 \wedge z \geq 0\}$
 - Suggest its parametrization.
 - Verify that the suggested parametrization is correct.
 - Compute the length of the normal vector to the parametrization.
- Parametrize a parallelogram ABCD and verify that the suggested parametrization is correct.
 $A = [0;0;0]$ $B = [0;2;3]$ $C = [0;5;3]$ $D = [0;3;0]$
- For the surface $\sigma = \{[x, y, z] \in \mathbb{R}^3 : 2x + 2y + z = 4 \wedge x, y, z \geq 0\}$ compute $\iint_{\sigma} xy \, dp$.
- For the surface $\sigma = \{[x, y, z] \in \mathbb{R}^3 : x^2 + y^2 = 4 \wedge 0 \leq z \leq 1\}$ compute $\iint_{\sigma} xy \, dp$.
- Compute a mass of a metal sheet in a shape of $\sigma = \{[x, y, z] \in \mathbb{R}^3 : z = \sqrt{4 - x^2 - y^2}\}$ if $\rho(x, y, z) = z$.
- Compute an area of a surface $\sigma = \{[x, y, z] \in \mathbb{R}^3 : z = \sqrt{x^2 + y^2} \wedge x^2 + y^2 \leq 2x\}$.

Surface integral II

- Surface $\sigma = \{[x, y, z] \in \mathbb{R}^3 : x, y \in \langle 0; a \rangle \wedge z = a; a > 0\}$ is oriented by normal \vec{n}_{σ} that has positive third (i.e. z) component.
 - Find the parametrization and determine if it is oriented in agreement with the surface.
 - Compute the surface integral $\iint_{\sigma} (x, y, z) \cdot \vec{dp}$.
- Consider a surface $Q = \{[x, y, z] \in \mathbb{R}^3 : x + 2y + z = 6 \wedge x, y, z \geq 0\}$ oriented by normal which makes an acute angle with $\hat{k} = (0, 0, 1)$ and a vector function $\vec{f}(x, y, z) = (z, y, 2x)$.
 - Find the parametrization and determine if it is oriented in agreement with the surface.
 - Compute the flux of a vector field \vec{f} through the oriented surface Q .
- Consider a surface $\sigma = \{[x, y, z] \in \mathbb{R}^3 : x^2 + y^2 = 16 \wedge x \leq 0 \wedge y \geq 0 \wedge 0 \leq z \leq 1\}$ oriented by normal $\vec{n}_{\sigma} = \hat{i} = (1, 0, 0)$ in a point $[-4; 0; 0]$.
 - Find the parametrization and determine if it is oriented in agreement with the surface.
 - Compute the surface integral $\iint_{\sigma} (y, z, x^2) \cdot \vec{dp}$.
- For the surface $\sigma = \{[x, y, z] \in \mathbb{R}^3 : z = x^2 + y^2 \wedge z \leq 9\}$ oriented with outer normal compute

$$\iint_{\sigma} (x, 0, 2z) \cdot \vec{dp}$$
- Compute the flux of a vector field $\vec{f}(x, y, z) = (0, 0, z^2)$ through the surface $Q = \{[x, y, z] \in \mathbb{R}^3 : x^2 + y^2 + z^2 = 4 \wedge x \leq 0 \wedge y, z \geq 0\}$ oriented by normal which makes an acute angle with $\hat{k} = (0, 0, 1)$.