Repetition - $\int_C$

1. Compute the work done by a vector field $\vec{f}(x,y) = (xy^2, 2x^2y)$ along the curve which is positively oriented boundary of a triangle M = [0;0], N = [2;2], O = [2;4].
2. A curve is given as a segment of function $x = \sqrt{4-y^2}$ oriented clockwise.
   (a) Suggest its parametrization (and determine bounds for the parameter).
   (b) Compute the work done by a vector function $\vec{f}(x,y) = (x-2y, 2x)$ along the curve.
   (c) Compute the x-coordinate of the center of mass (of the curve) when $\rho(x,y) = 3$.
3. Given a vector field $\vec{f}(x,y,z) = (\frac{y^2}{z}, \frac{2xy}{z}, -\frac{xy^2}{z^2})$.
   (a) Compute the rotation of the vector field and decide where is the field conservative.
   (b) Check if a scalar function $\varphi(x,y,z) = \frac{xy^2}{z}$ is the corresponding potential.
   (c) Compute $\int_C \vec{f} \cdot \vec{ds}$ where C is a line segment from $E = [0;0;1]$ to $F = [1;2;1]$.
   (d) Compute the mass of the wire in the shape of the curve (c) when its linear density is $\rho(x,y,z) = y^2 + z^2$.
4. A curve is given as a segment of function $y = \tan x$ for $x \in \langle 0; \frac{\pi}{4} \rangle$.
   (a) Suggest its parametrization, compute the tangent vector and determine its length.
   (b) Compute line integral of a scalar function $f(x,y) = 4\cos^5 x \sin x$.
   (c) Compute line integral of a vector function $\vec{g}(x,y) = (x, \cos^3 x)$.
5. Given a conservative vector field $\vec{f}(x,y) = (2x^3y^2 + x, y^2 + yx^4)$.
   (a) Find the potential of the vector field (determine where it is possible).
   (b) Compute $\int_C \vec{f} \cdot \vec{ds}$ where $C = \{[x,y] \in \mathrm{R}^2 : y = (x+1)^2 - 2 \ \wedge \ 0 \le x \le 1\}$.
6. Compute the circulation of a vector field $\vec{f}(x,y) = (x+y, x-y)$ along a positively oriented circle $x^2 + y^2 = 4$.

Results

1. W = 12
2. (a) $P(t) = (2\cos(t); 2\sin(t))$, $t \in \langle -\frac{\pi}{2}; \frac{\pi}{2} \rangle$ (b) W $= -8\pi$ (c) $C = \left[\frac{4}{\pi}; 0\right]$
3. (a) $\nabla \times f = \vec{0}$, in domains $\{[x,y,z] \in \mathbb{R}^3; z > 0\}$ or $\{[x,y,z] \in \mathbb{R}^3; z < 0\}$ (b) yes (c) 4 (d) $\frac{7}{3}$
4. (a) $||\dot{P}(t)|| = \frac{\sqrt{\cos^4 t + 1}}{\cos^2 t}$ (b) $\frac{5\sqrt{5}+16\sqrt{2}}{12}$ (c) $\frac{\pi^2}{32} + \frac{\sqrt{2}}{2}$
5. (a) $\varphi(x,y) = \frac{1}{2}(x^4y^2 + x^2) + \frac{y^3}{3} + C$, (in R^2) (b) $\frac{11}{2}$
6. 0