potential + conservative $\vec{f}$

1. $\vec{f}$ conservative in Ω $\iff$ $\exists$ potential φ (scalar); $\boxed{\vec{f} = \operatorname{grad} \varphi}$ in Ω $\quad (\vec{f} = \nabla\varphi)$

2. — || — $\implies \displaystyle\int_A^B \vec{f} \cdot ds = \varphi(B) - \varphi(A)$

-> independence on path (curve)

i.e. $\boxed{\oint \vec{f} \cdot d\vec{s} = 0}$

3. is $\vec{f}$ conservative?

a) necessary cond. (usage: $\neq \implies \vec{f}$ isn't conservative)

$\boxed{2D}$ $\vec{f} = (f_1; f_2)$

$\vec{f}$ cons. $\implies$ $\boxed{\dfrac{\partial f_2}{\partial x} = \dfrac{\partial f_1}{\partial y}}$

$\boxed{3D}$ $\vec{f}$ cons. $\implies$ $\boxed{\operatorname{rot} \vec{f} = \vec{0}}$ $\quad (\nabla \times \vec{f} = \vec{0})$

b) sufficient cond. (usage: $\checkmark \implies \vec{f}$ is conservative.)

$\left.\begin{array}{l} \vec{f} \text{ has continuous P.D. in } \Omega \;\wedge\; (a) \\ \wedge\; \boxed{\Omega \text{ is simply connected domain (without holes)}} \end{array}\right\} \implies \vec{f}$ is conservative $\quad (\exists \varphi;\ \vec{f} = \nabla\varphi)$

$\left.\begin{array}{l} \dfrac{\partial f}{\partial x} \\ \dfrac{\partial f}{\partial y} \\ \left(\dfrac{\partial f}{\partial z}\right) \end{array}\right\}$ cont. in Ω