Chapter 1

Analytic geometry in E_2 - Results

1.1 the parametric equations of line p and q: $p : x(t) = 2 + t, y(t) = 5 + 3t; q : x(s) = 4 + 4s, y(s) = 7$
general equation of both lines: $p : 3x - y - 1 = 0 q : y = 7$
parametric equations of line segment AB: $x(w) = 2, y(w) = 5 - 4w, w \in \langle 0; 1\rangle$
parametric equations of ray AB with startpoint A: $x(q) = 2, y(q) = 5-4q, q \in \langle 0; +\infty)$

1.2 the parametric equation of line AB: $x(t) = -5 + 3t, y(t) = 18 + 4t$
general equation of line AB: $p : 4x - 3y + 74 = 0$
slope equation of line AB: $p : y = \frac{4}{3}x + \frac{74}{3}$
intercept equation of line AB: $-\frac{x}{\frac{74}{4}} + \frac{y}{\frac{74}{3}} = 1$

1.3 $p : x = -1$

1.4 $p : x(t) = 1 + t, y(t) = -1$

1.5 $p : y = -\frac{\sqrt{2}}{2}x - \sqrt{2}$

1.6 $d(A, p) = \frac{11\sqrt{5}}{5}$

1.7 lines p and q are intersecting lines, point of intersection is $R = [\frac{51}{5}, \frac{72}{5}]$, angle is $60°$
$r : x(w) = 18 - 7w, y(w) = 17 - 14w$

1.8 $C = [1, 2]$

1.9 $p : 3x - 7y + 15 = 0$ or $p' : 7x + 3y - 23 = 0$

1.10 ray q intersects line segment AB at point $R = [2, 4]$

1.11 $A = [1, -6], B = [-2, 3], C = [4, 5]$, $\alpha = 33°41'24''$, $\beta = 90°$, $\gamma = 56°18'36''$, altitude of $v_b = \frac{6\sqrt{130}}{13} = 5.26$, area of triangle ABC is $S = 30$

1.12 $B = [11, 1]$, $D = [3, 3]$

1.13 four results: $X_1 = [2, 0], Y_1 = [0, 1]$, $X_2 = [-2, 0], Y_2 = [0, -3]$, $X_3 = [-3 + \sqrt{13}, 0], Y_3 = [0, 4 - \sqrt{13}]$, $X_4 = [-3 - \sqrt{13}, 0], Y_4 = [0, 4 + \sqrt{13}]$

Conics:

1.14 it is a circle with centre $S = [3, -2]$ and radius $r = \sqrt{5}$
tangent lines are $t_1 : x + 2y - 4 = 0$, $t_2 : 2x - y - 3 = 0$

1.15 $c : (x - 9)^2 + (y - 7)^2 = 85$, centre $S = [9, 7]$ and radius $r = \sqrt{85}$

1.16 it is a parabola with vertex $V = [-\frac{23}{5}, \frac{3}{5}]$ and it intersects only the x-axis at point $[-\frac{89}{20}, 0]$

1.17 it is an ellipse with centre $S = [1, -2]$ and it intersects axes at points $X = [1, 0]$ and $Y = [0, -2]$

1.18 it is a hyperbola with centre $S = [2, -3]$ and it intersects axes at points $X = [\frac{4}{3}, 0]$ and $Y = [0, -2]$

Regions:

1.19 points of intersection: $[2, 2], [1, 1], [2, \frac{1}{2}]$
border curves are: two lines and hyperbola

1.20 points of intersection: $[8, -4], [2, 2], [18, -6], [8, 4]$
border curves are: two lines and parabola

1.21 points of intersection: $[6, 3], [-2, 3]$
border curves are: two rays and line

1.22 points of intersection: $[-2, -1], [2, -1], [-2, \sqrt{6}], [-2, -\sqrt{6}], [2, \sqrt{6}], [2, -\sqrt{6}]$
border curves are: line and hyperbola and strip

1.23 points of intersection: $[-2, 0], [2, 0], [-1, 0], [1, 0], [0, 2], [0, -2], [0, -1], [0, 1]$
border curves are: two circles and axes

1.24 points of intersection: $[0, 0], [1, 1], [4, 2]$
border curves are: two lines and parabola

Chapter 2

Analytic geometry in E_3

2.1 to prove it, use dot product of each two vectors AB, AC, BC, where it is zero there is right angle $\beta = 45°$.

2.2 $\mathbf{c} = (\frac{\sqrt{35}}{35}, -\frac{3\sqrt{35}}{35}, -\frac{\sqrt{35}}{7})$
$\mathbf{c}' = (\frac{-\sqrt{35}}{35}, \frac{3\sqrt{35}}{35}, \frac{\sqrt{35}}{7})$

2.3 area $S = \frac{13}{2}\sqrt{10}$

2.4 points A, B, C, D lie in same plane

2.5 angle between planes is $\alpha = 60°$

2.6 angle between planes ABC and ACF is $\rho = 54°44'8''$

2.7 $\rho : y - z - 1 = 0$

2.8 $\rho : y = -7$

2.9 $\rho : x + y + z - 2 = 0$

2.10 $d(\rho, \sigma) = 4$

2.11 $\rho : -5x + y + 3z + 5 = 0$

2.12 $t_c : x(s) = 4 + 5s, y(s) = -7 - 11s, z(s) = -2$

2.13 $\phi = 0°$

2.14 $d(A, p) = 7$

2.15 p and q are skew lines with shortest distance $d(p, q) = \sqrt{14}$

2.16 $\phi : 23x + 5y - 11z + 15 = 0$

2.17 $\alpha = 45°$

Quadrics

2.18 $\kappa : (x-3)^2 + y^2 + (z+2(^2 = 4$

2.19 $\omega : y^2 + z^2 = 9$

2.20 $\tau_1 : x - y + 2z - 3 = 0$, $\tau_2 : -x + y - 2z + 15 = 0$

2.21 with plane $y = 2$: curve is parabola $z = 4x^2 - 36$
with plane $z = 1$: curve is hyperbola $4x^2 - y^2 = 1$
with plane $z = 0$: curves are two intersecting lines $y = 2x$ and $y = -2x$

2.22 $R = [8, -6, 9]$ and $Q = [-2, 4, -1]$

2.23 line lies on the quadric

2.24 it is an oblate spheroid with centre $S = [-2, 1, -1]$ and axes $a = b = 2, c = 1$

2.25 it is a point $[1, 2, -1]$

2.26 it is a one-sheeted hyperboloid of revolution with centre $S = [2, -3, -1]$ and axes $a = c = 1, b = 2$

2.27 it is a two-sheeted hyperboloid of revolution with centre $S = [2, 3, 1]$ and axes $a = b = c = 1$

2.28 it is an elliptic cone with vertex $V = [0, 0, 1]$ and axis of revolution parallel to z-axis

2.29 it is a hyperbolic cylinder with centre $S = [2, -1, 0]$ and axes $a = b = \sqrt{3}$ and axis of revolution parallel to z-axis

2.30 it is a paraboloid of revolution with vertex $V = [3, -2, 2]$ and parameter $p = 1/2$ and axis of revolution parallel to z-axis

2.31 it is a hyperbolic paraboloid with vertex $V = [1, 0, 3]$ and axis parallel to z-axis

2.32 two planes $\rho : y = x$ and $\phi : y = -x - 1$

Regions in E_3:

2.33 border surfaces are: paraboloid of revolution and one half of cone of revolution
they intersect in circle $x^2 + y^2 = 1$ section with plane $z = 0$ are circle $x^2 + y^2 = 2$ and point $[0, 0, 0]$
section with plane $z = 2$ are point $[0, 0, 2]$ and circle $x^2 + y^2 = 4$

2.34 border surfaces are paraboloid of revolution (with vertex $V = [0, 0, 0)$ and z-axis as an axis of revolution) and cone of revolution (with vertex $V = [0, 0, 0]$ and z-axis as an axis of revolution)
they intersect in circle $x^2 + y^2 = 9$ and in origin
sections with plane (xy) and (yz) are same curves: two intersecting lines and parabola

2.35 border surfaces are planes
sections with planes (xy) and (yz) are lines

2.36 border surfaces are planes
sections with planes (xy) and (yz) are lines

2.37 border surfaces are one half of cone of revolution and sphere
curve of intersection is circle $x^2 + y^2 = \frac{1}{2}$ sections with planes (xy) and (yz) are